# Lecture 14: Electrostatics

Poisson and Laplace Equations, Uniqueness Theorem Chapter 6, pages 215-239

# Poisson and Laplace Equations

starting with

$$\nabla \bullet \mathbf{D} = \rho_v$$

and using $\mathbf{E} = -\nabla V$ one gets,

$$\nabla \bullet (\varepsilon \nabla V) = -\rho_v$$

if $\varepsilon$ does not change within the considered region, we have

$$\nabla^2 V = -\rho_v / \varepsilon \quad \text{(Poisson Equation)}$$

$$\nabla^2 V = 0 \qquad \text{(Laplace equation, } \rho_v = 0\text{ )}$$

second order partial differential equation to be solved for $V$

# Uniqueness Theorem

if $V_1(\boldsymbol{r})$ is a solution for Laplace equation that satisfies the boundary conditions and if $V_2(\boldsymbol{r})$ is also a solution for Laplace equation that satisfies the boundary conditions, then we must have $V_1(\boldsymbol{r}) = V_2(\boldsymbol{r})$ (solution is unique)

# Laplacian Operator

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0 \quad \text{(Cartesian Coordinates)}$$

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right)+\frac{1}{\rho^2}\frac{\partial^2 V}{\partial \varphi^2}+\frac{\partial^2 V}{\partial z^2}=0 \qquad \text{(Cylindrical Coordinates)}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial V}{\partial \theta}\right)+\frac{1}{r^2\sin^2(\theta)}\frac{\partial^2 V}{\partial \varphi^2}=0$$

(Spherical coordinates)

# Procedure for Solving BVPs

use separation of variables, if needed, to solve for $V(\boldsymbol{r})$ in terms of unknown coefficients

apply boundary conditions to determine the unknown coefficients

obtain **E** and **D** as a by product using $V(\boldsymbol{r})$

we have now a third approach for solving for **E**!