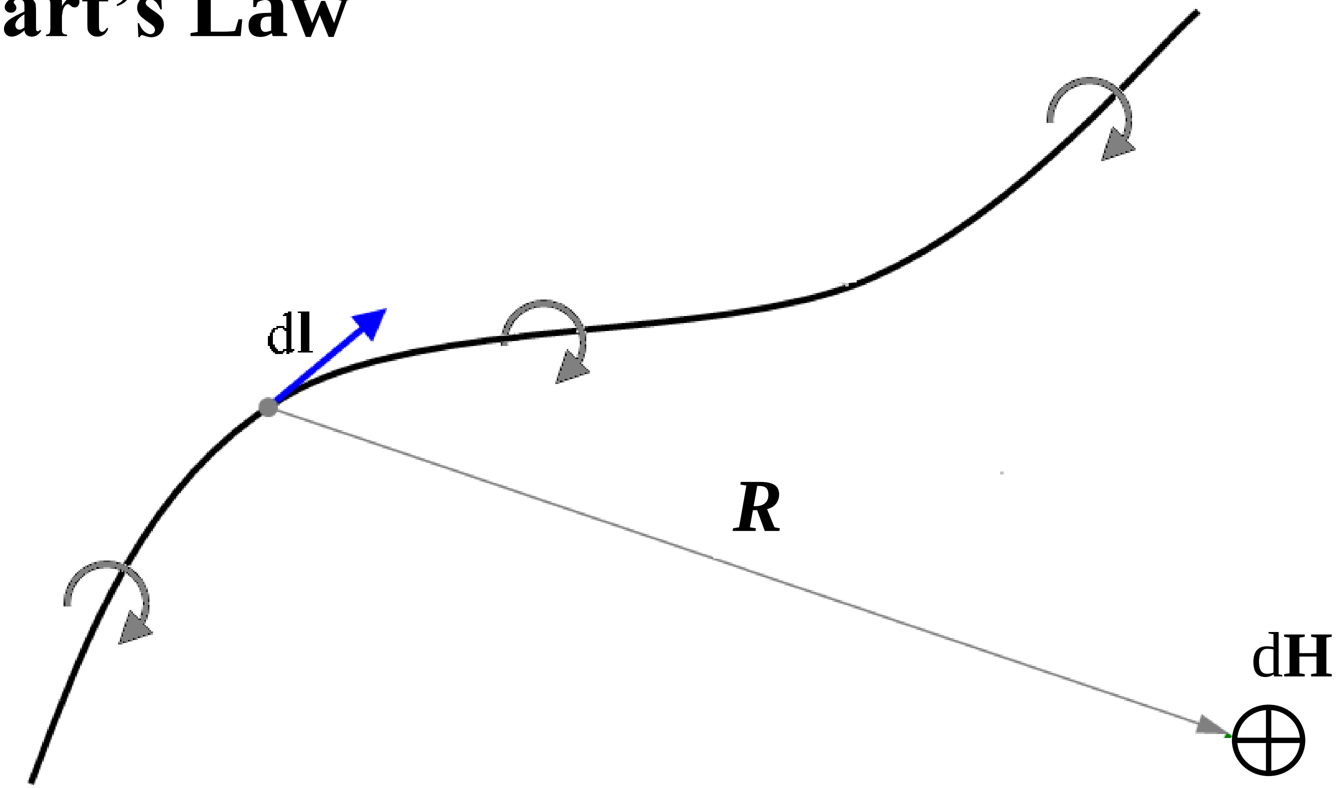
Lecture 18: Magnetostatics

Biot-Savart's law, applications, Chapter 7, pages 281-292

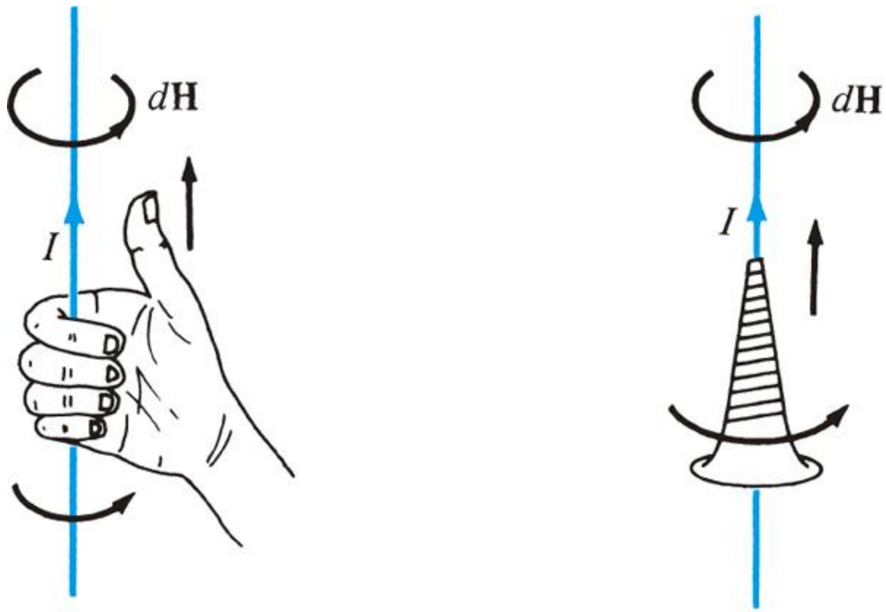
Biot-Savart's Law



$$d\mathbf{H} = \frac{I d\mathbf{l} \times \boldsymbol{a}_R}{4\pi R^2} = \frac{I d\mathbf{l} \times \mathbf{R}}{4\pi R^3} \; A/m$$

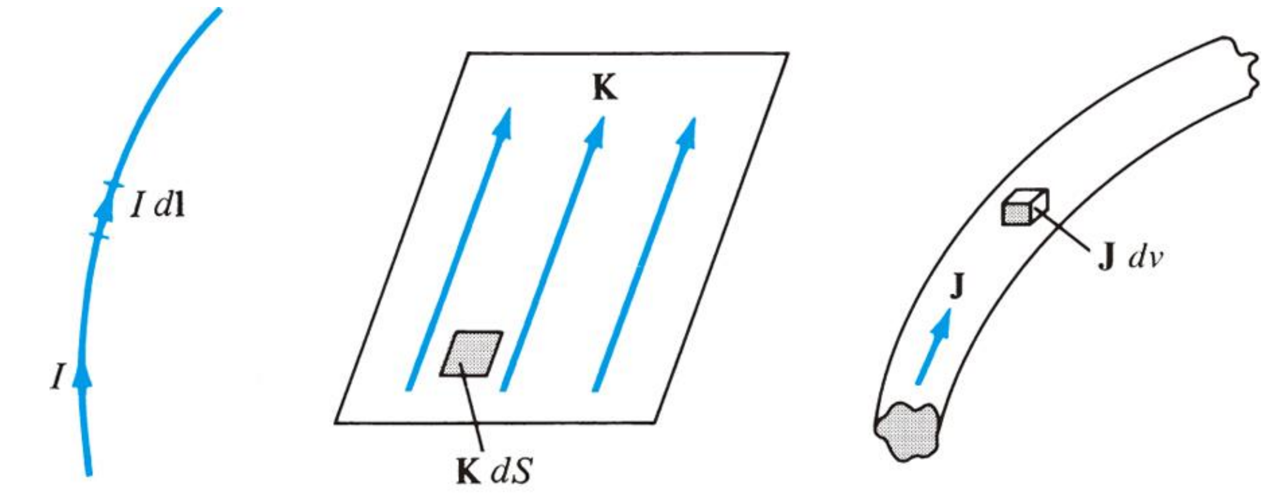
similar to Coulomb's law in electrostatics!

Right Hand Rule



magnetic field encircles the generating current according to the right hand screw rule

Types of Current Distributions



(a) (b) (c)

linear current element $I\,d\mathbf{l}$ (Ampere)

surface current element $\mathbf{K}\,dS$ (Ampere/meter)

volume current element $\mathbf{J}dv$ (Ampere/meter^2)

Forms of Biot-Savart's Law

$$\mathbf{H} = \int d\mathbf{H} = \int \frac{I d\mathbf{l} \times \boldsymbol{R}}{4\pi R^3}$$

line integral over all current-carrying differential linear elements

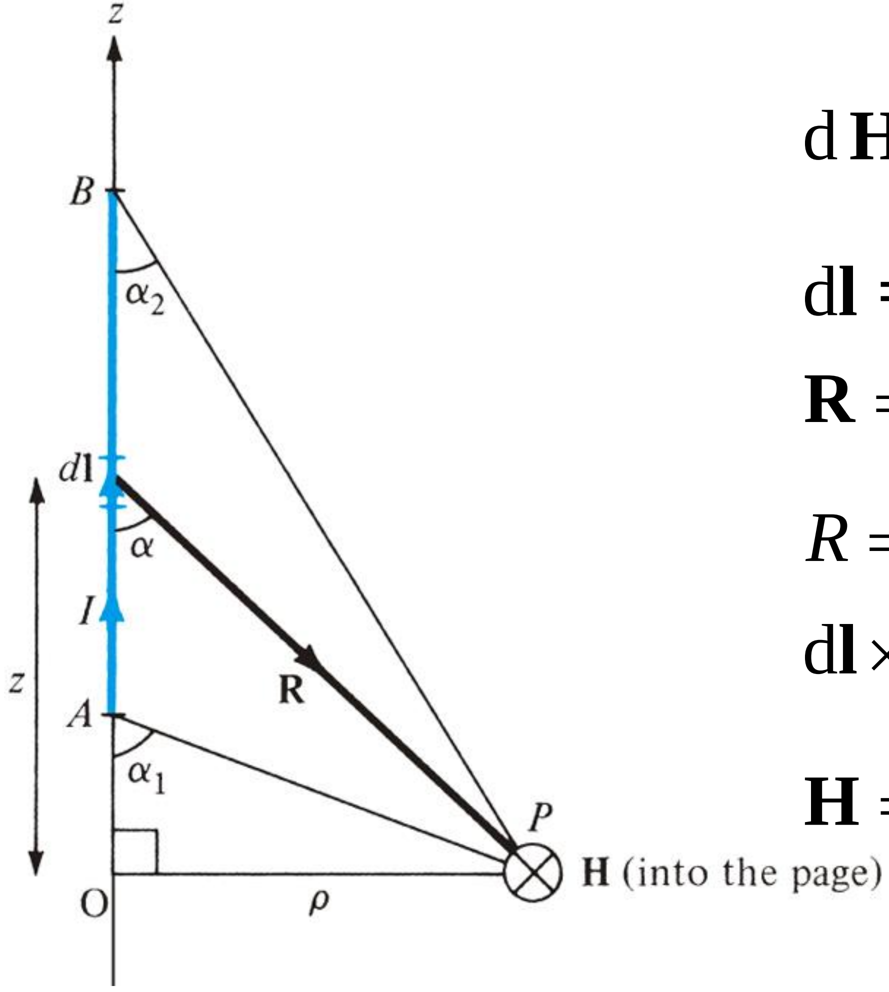
$$\mathbf{H} = \int d\mathbf{H} = \int \frac{\mathbf{K} \times \mathbf{R}}{4\pi R^3}\, ds$$

surface integral over all current-carrying differential surface elements

$$\mathbf{H} = \int d\mathbf{H} = \int \frac{\mathbf{J} \times \mathbf{R}}{4\pi R^3}\, dv$$

volume integral over all current-carrying differential volume elements

Example: Current-Carrying Straight Wire



$$d\mathbf{H} = \frac{I d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{I d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

$$d\mathbf{l} = dz' \, \mathbf{a}_z$$

$$\mathbf{R} = \rho \mathbf{a}_\rho - z' \mathbf{a}_z$$

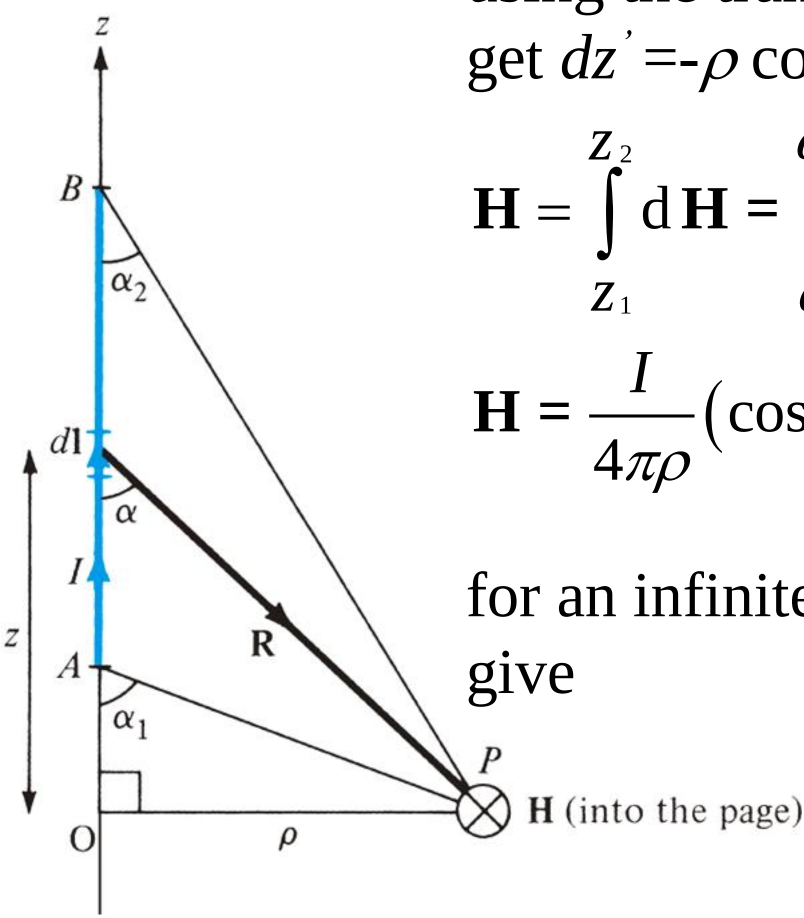
$$R = \sqrt{\rho^2 + z'^2}$$

$$d\mathbf{l} \times \mathbf{R} = \rho \, dz' \mathbf{a}_\varphi$$

$$\mathbf{H} = \int d\mathbf{H} = \int \frac{I \rho \, dz'}{4\pi \left(\rho^2 + z'^2\right)^{3/2}} \mathbf{a}_\varphi$$

notice that $\mathbf{a}_\rho$ and $\mathbf{a}_\varphi$ are constants for the considered observation point

Straight Wire (Cont'd)



using the transformation $z' = \rho \cot \alpha$, we get $dz' = -\rho \,\text{cosec}^2 \alpha \, d\alpha$

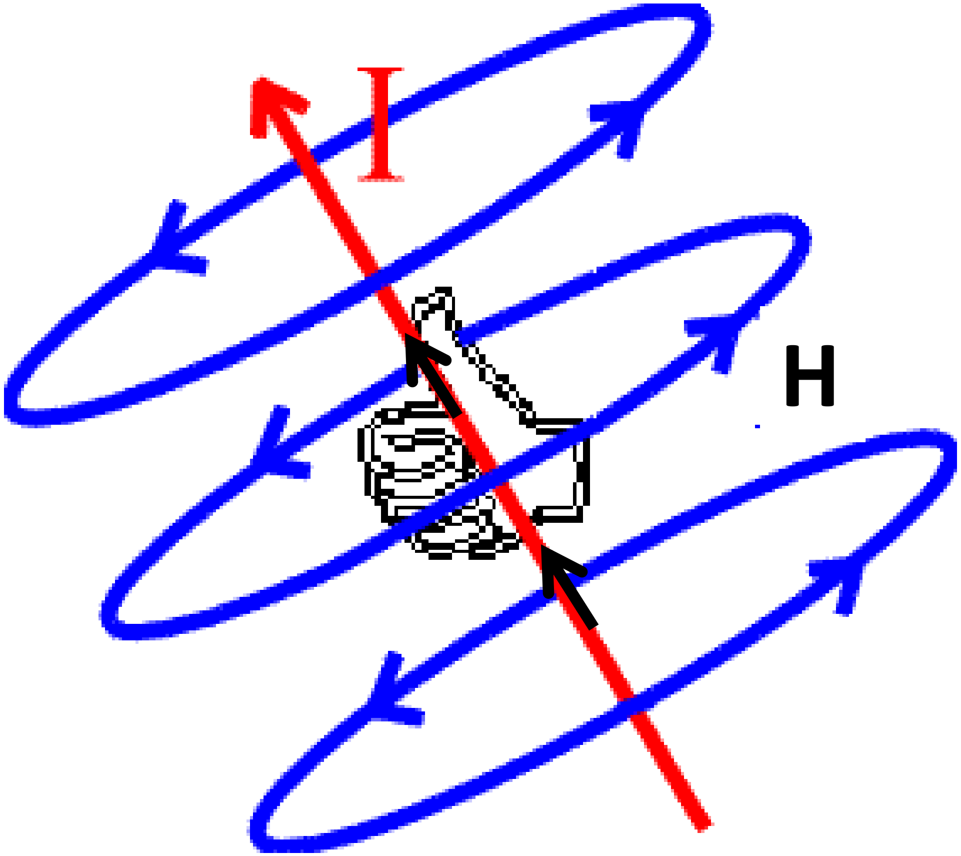
$$\mathbf{H} = \int_{z_1}^{z_2} d\mathbf{H} = \int_{\alpha_1}^{\alpha_2} \frac{I \rho (-\rho \;\text{cosec}^2 \alpha \; d\alpha)}{4\pi \rho^3 \text{cosec}^3 \alpha} \; \mathbf{a}_\varphi$$

$$\mathbf{H} = \frac{I}{4\pi\rho} \left(\cos \alpha_2 - \cos \alpha_1 \right) \mathbf{a}_\varphi$$

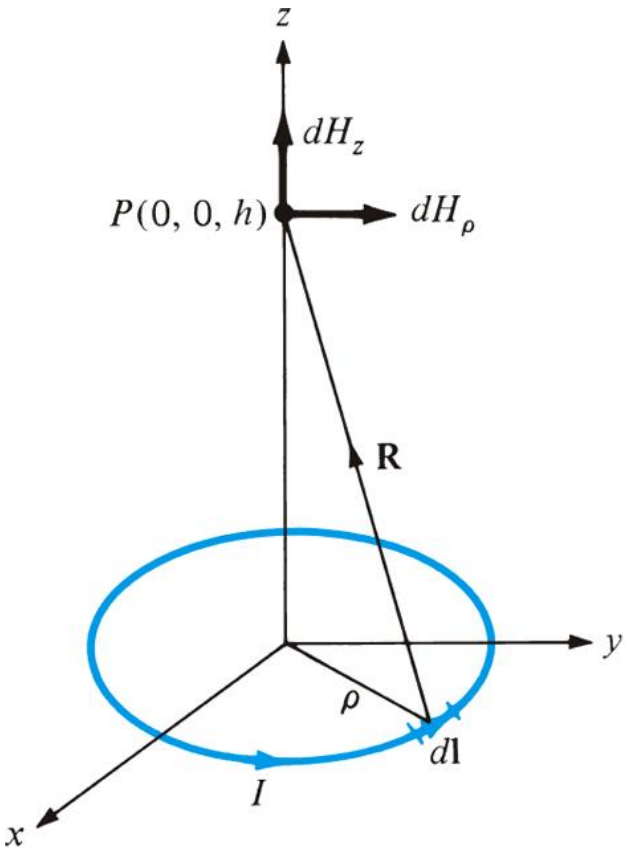
for an infinite wire, $\alpha_1 = \pi$ and $\alpha_2 = 0$ to give

$$\mathbf{H} = \frac{I}{2\pi\rho} \mathbf{a}_\varphi$$

Straight Wire (Cont'd)



Current Loop



$$d\mathbf{H} = \frac{I d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{I d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

$$d\mathbf{l} = \rho' d\varphi' \, \mathbf{a}_{\varphi'}$$

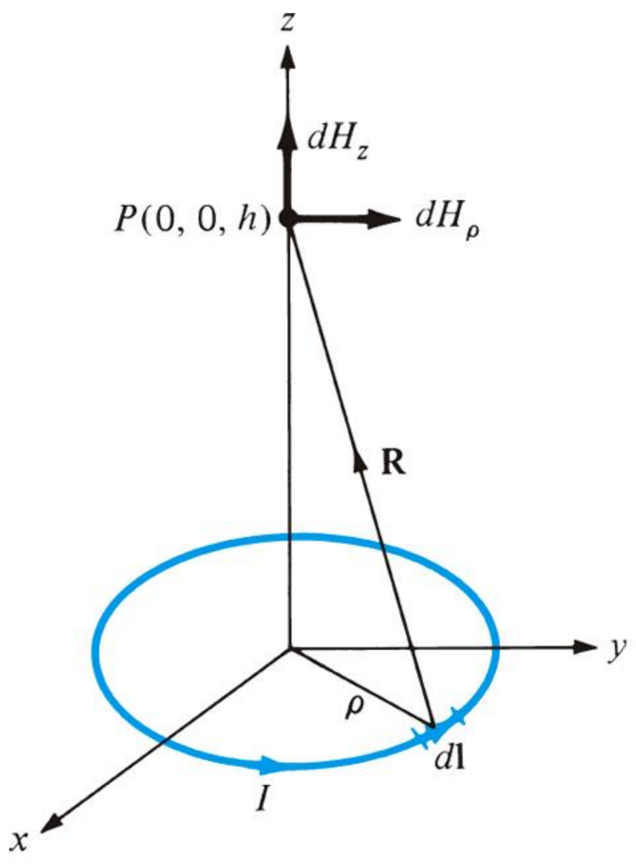
$$\mathbf{R} = h\mathbf{a}_z - \rho' \mathbf{a}_{\rho'}$$

$$R = \sqrt{\rho'^2 + h^2}$$

$$d\mathbf{l} \times \mathbf{R} = \rho' z d\varphi' \mathbf{a}_{\rho'} + \rho'^2 d\varphi' \mathbf{a}_z$$

all ρ-components cancel out leaving on the z-component of the magnetic field

Current Loop (Cont'd)



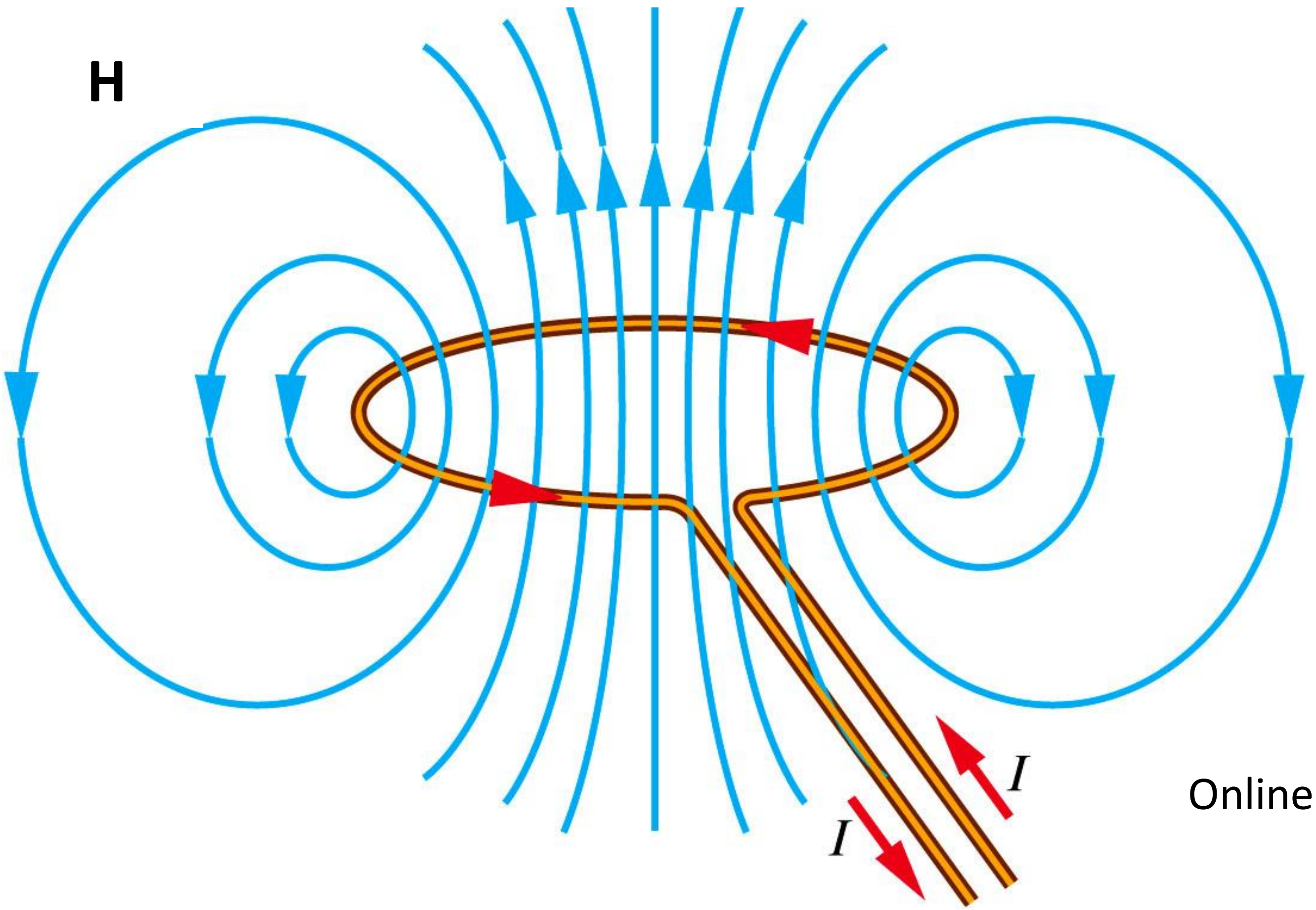
$$\mathbf{H} = \int_0^{2\pi} \frac{I\rho'^2 d\varphi'}{4\pi\left(\rho'^2 + h^2\right)^{3/2}} \mathbf{a}_z$$

$$\mathbf{H} = \frac{I\rho'^2}{2\left(\rho'^2 + h^2\right)^{3/2}} \mathbf{a}_z$$

$$\mathbf{H} = \frac{Ia^2}{2\left(a^2 + h^2\right)^{3/2}} \mathbf{a}_z$$

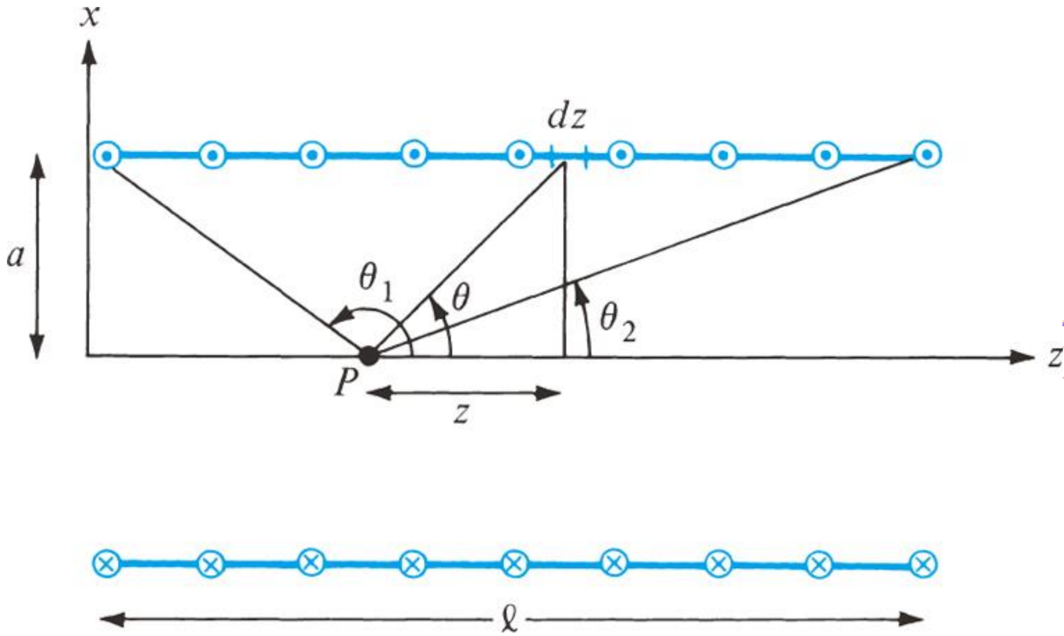
magnetic field increases with current value and decreases with *h*

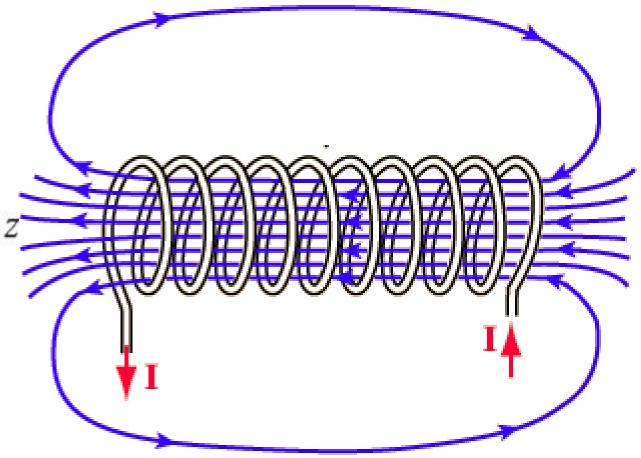
Current Loop (Cont'd)



Onlinephys.com

Magnetic Field of a Solenoid



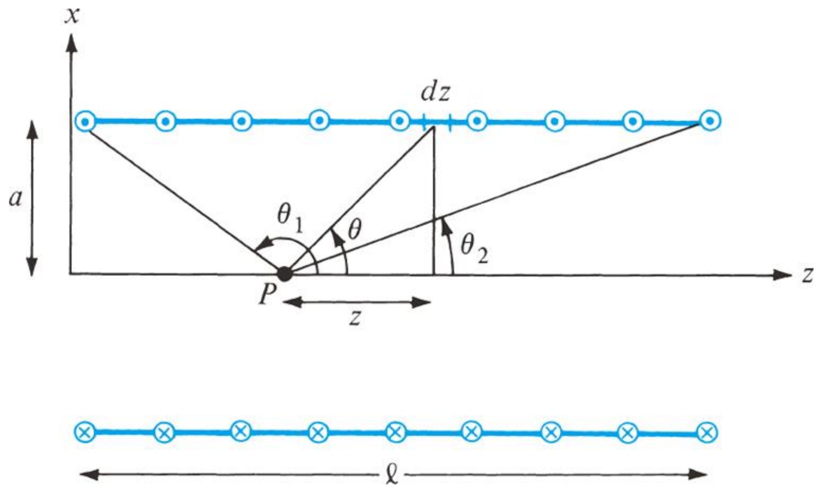


http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/solenoid.html

solenoid has length *l*, radius *a*, *N* number of turns, and carrying a current *I*

the solenoid is analyzed as a number of loops and superposition is applied

Magnetic Field of a Solenoid (Cont'd)



in a differential element of length *dz*, the current flowing is given by $I(N/l)\,dz$

$$d\mathbf{H} = \frac{INa^2 dz}{2l\left(a^2+z^2\right)^{3/2}}\mathbf{a}_z$$

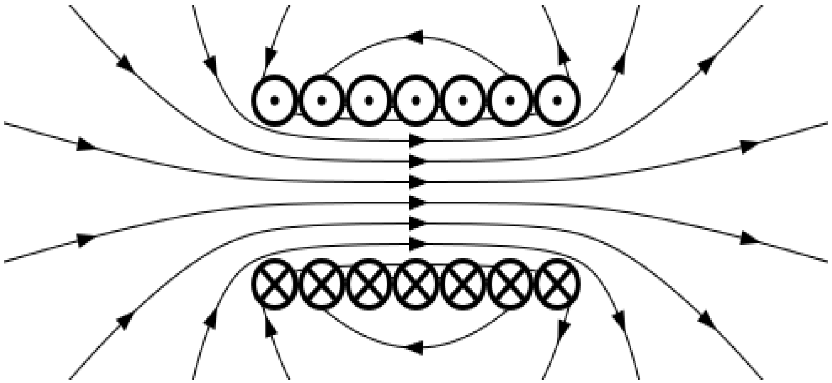
$$\mathbf{H} = \int_{-z_1}^{z_2} \frac{INa^2 dz}{2l\left(a^2+z^2\right)^{3/2}}\mathbf{a}_z$$

utilizing the transformation $z=a\cot(\theta)$, we have

$$dz=-a\,\text{cosec}^2(\theta)\,d\theta = -\frac{\left(z^2+a^2\right)^{3/2}}{a^2}\sin(\theta)d\theta$$

Solenoid (Cont'd)

$$\mathbf{H} = -\frac{NI}{2l}\int_{\theta_1}^{\theta_2} \sin(\theta)\, d\theta\ \mathbf{a}_z$$



wikipedia.org

$$\mathbf{H} = \frac{NI}{2l}(\cos(\theta_2) - \cos(\theta_1))\ \mathbf{a}_z$$

for a very long solenoid, $l >> a$

$$\mathbf{H} = \frac{NI}{l}\mathbf{a}_z$$