Lecture 19: Magnetostatics

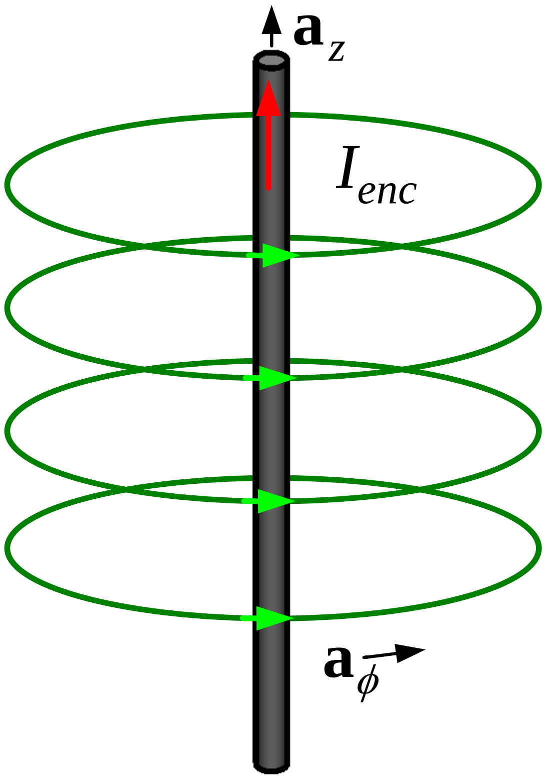
Ampere's law, applications, Chapter 7, pages 293-300

@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

Ampere's Circuital Law

Ampere's law in magnetostatics plays the same role Gauss law plays in electrostatics

it states that the line integral of the magnetic field strength **H** around a closed path is equal to the current enclosed I_{enc} by that path



$$\oint \mathbf{H}.\mathbf{dl} = I_{enc}$$ integral form

@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

Differential Form

using Stoke's Theorem, we can write

$$\oint \mathbf{H.dl} = \iint_S \left(\nabla \times \mathbf{H}\right).\mathrm{d}\mathbf{s} = I_{enc} = \iint_S \mathbf{J}.\mathrm{d}\mathbf{s}$$

$$\Rightarrow \nabla \times \mathbf{H} = \mathbf{J}$$

magnetostatic field is, in general, not conservative!

@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

Ampere's Law for Field Solution

similar to Gauss' law, Ampere's law is used in solving problems with high degree of symmetry

The integral $\oint \mathbf{H.dl}$ is carried over an Amperian path over which **H** is either normal or tangential to the integration contour

when **H** is tangential to the contour, it must have a constant value so that it can be taken outside the integral

$$\oint \mathbf{H.dl} = \oint H\mathrm{dl} = H\oint \mathrm{dl} = Hl = I_{enc}$$

$$\Rightarrow H = I_{enc} / l$$

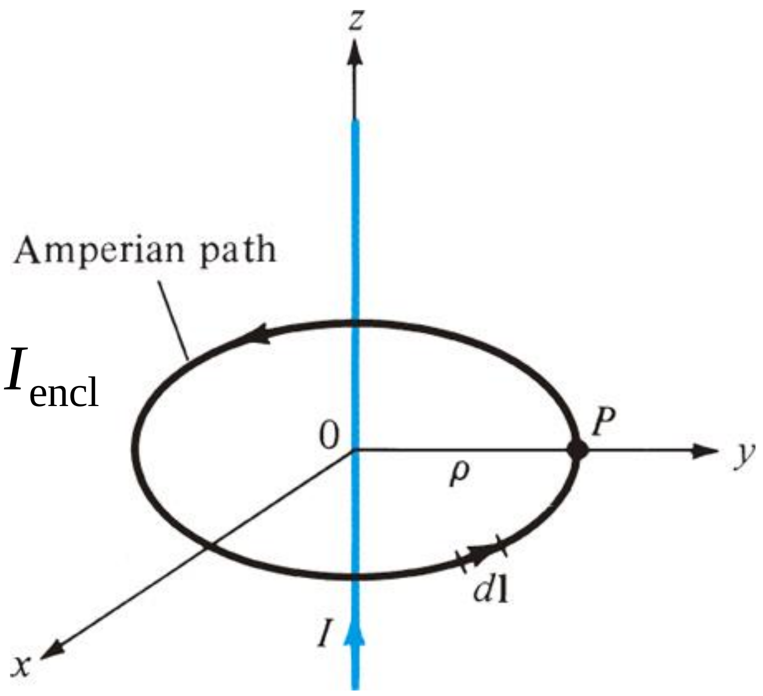
@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

Infinite Line Current

from symmetry, **H** is in the φ direction and depends only on ρ

$$\oint_C \mathbf{H}\cdot d\mathbf{L} = I_{\text{encl}} \;\Rightarrow\; \int_0^{2\pi} H_\phi \mathbf{a}_\phi \cdot \underbrace{\mathbf{a}_\phi(\rho d\phi)}_{d\mathbf{L}} = I_{\text{encl}}$$

$$\Rightarrow H_\phi 2\pi\rho = I \Rightarrow H_\phi = \frac{I}{2\pi\rho},\ \text{A/m}$$

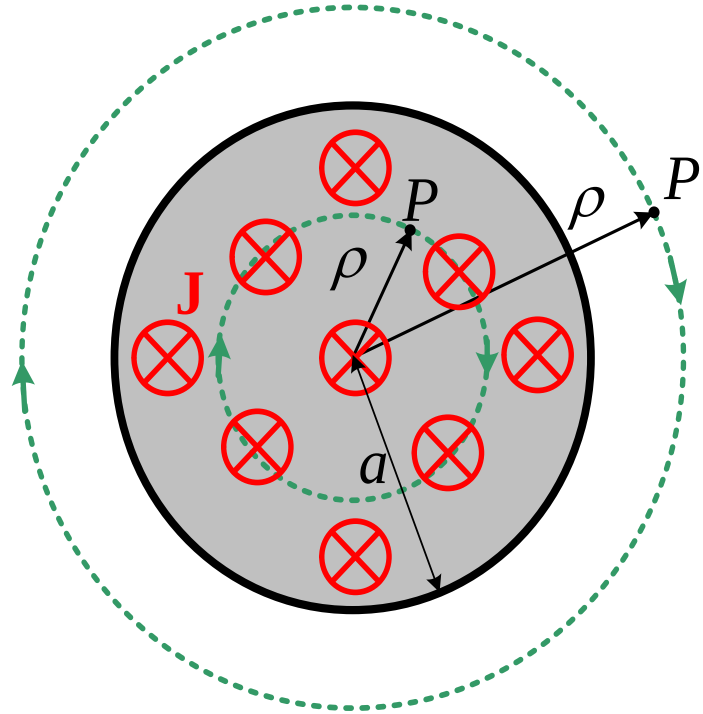


same result obtained earlier using Biot-Savart's law

@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

A Thick Conductor

a current I is uniformly flowing into the page. Find magnetic field inside and outside the conductor.



from symmetry, **H** is in the φ direction and depends only on ρ

inside conductor $\rho \leq a$

$$\mathbf{J} = \frac{I}{\pi a^2}\mathbf{a}_z, \quad \text{A/m}^2$$

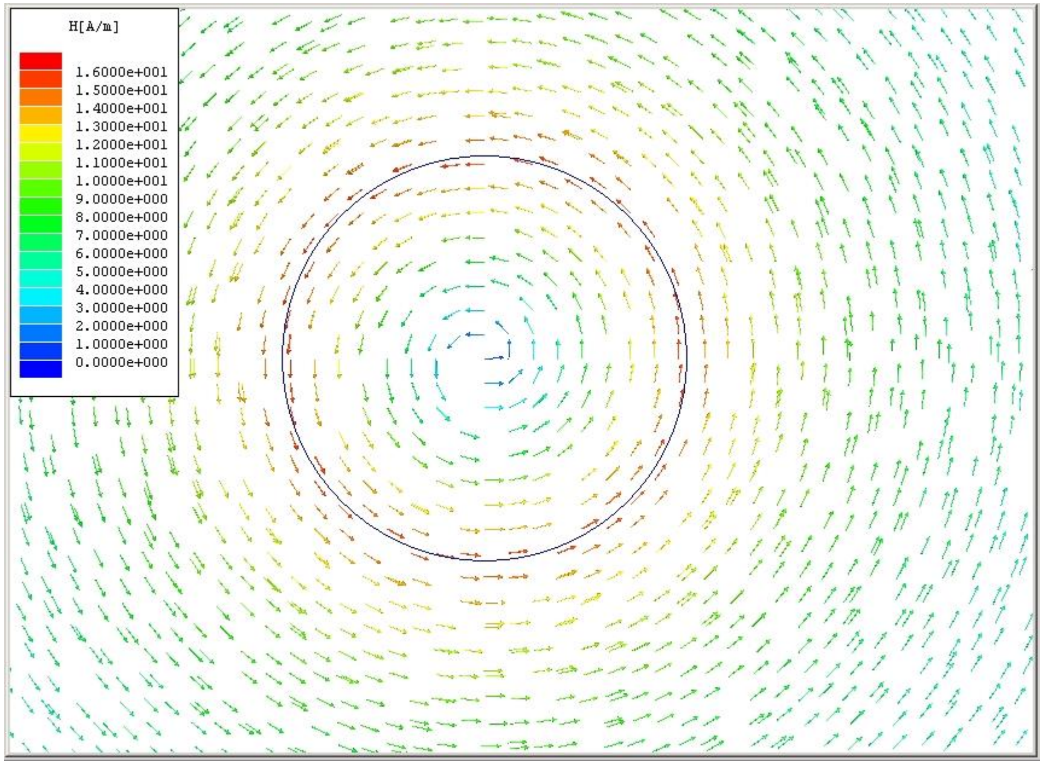
$$H_\phi \cdot 2\pi\rho = J \cdot \pi\rho^2 = I \cdot \frac{\rho^2}{a^2} \Rightarrow H_\phi(\rho) = \frac{I\rho}{2\pi a^2}, \text{ A/m}$$

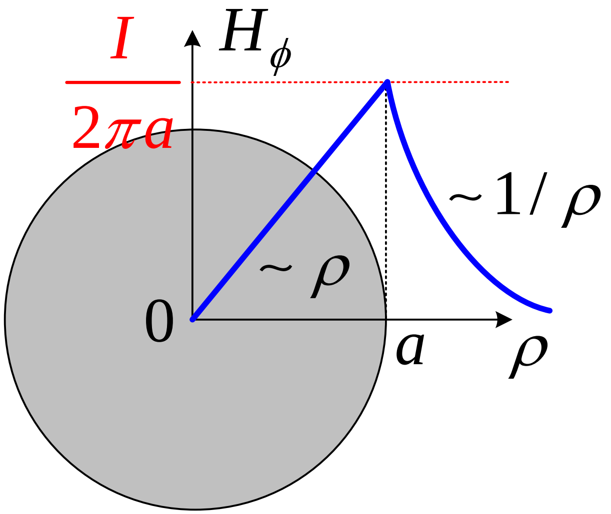
@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

A Thick Conductor (Cont'd)

outside the conductor

$$H_\phi \cdot 2\pi\rho = I \Rightarrow H_\phi(\rho) = \frac{I}{2\pi\rho}, \text{ A/m}$$

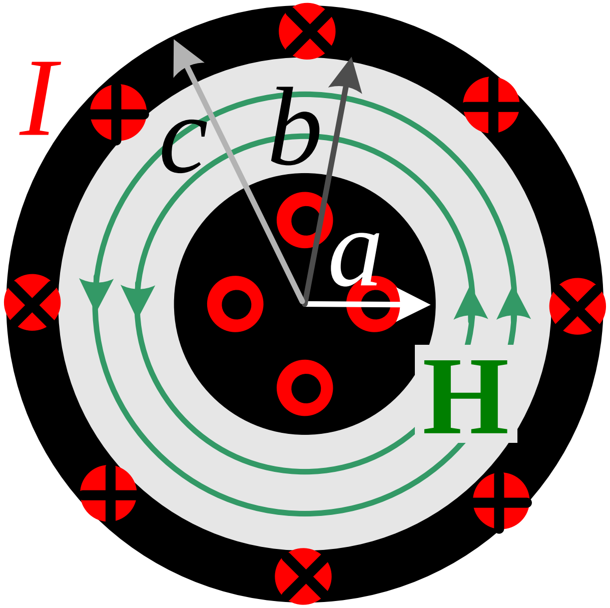




@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

A Coaxial Cable

current I is flowing out of the page in the inner conductor and into the page in the external conductor



from symmetry, **H** is in the φ direction and depends only on ρ

inside inner conductor, $\rho \leq a$,
similar to the thick conductor case, we have

$$H_\phi = \frac{I\rho}{2\pi a^2}, \ \text{A/m}$$

field strengthens with the radius of the Amperian path!

@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

Coaxial Cable (Cont'd)

between wire and shield, $a \le \rho \le b$, enclosed current is I

$$\Rightarrow H_\phi = \frac{I}{2\pi\rho}, \text{ A/m}$$

inside shield, $b \le \rho \le c$

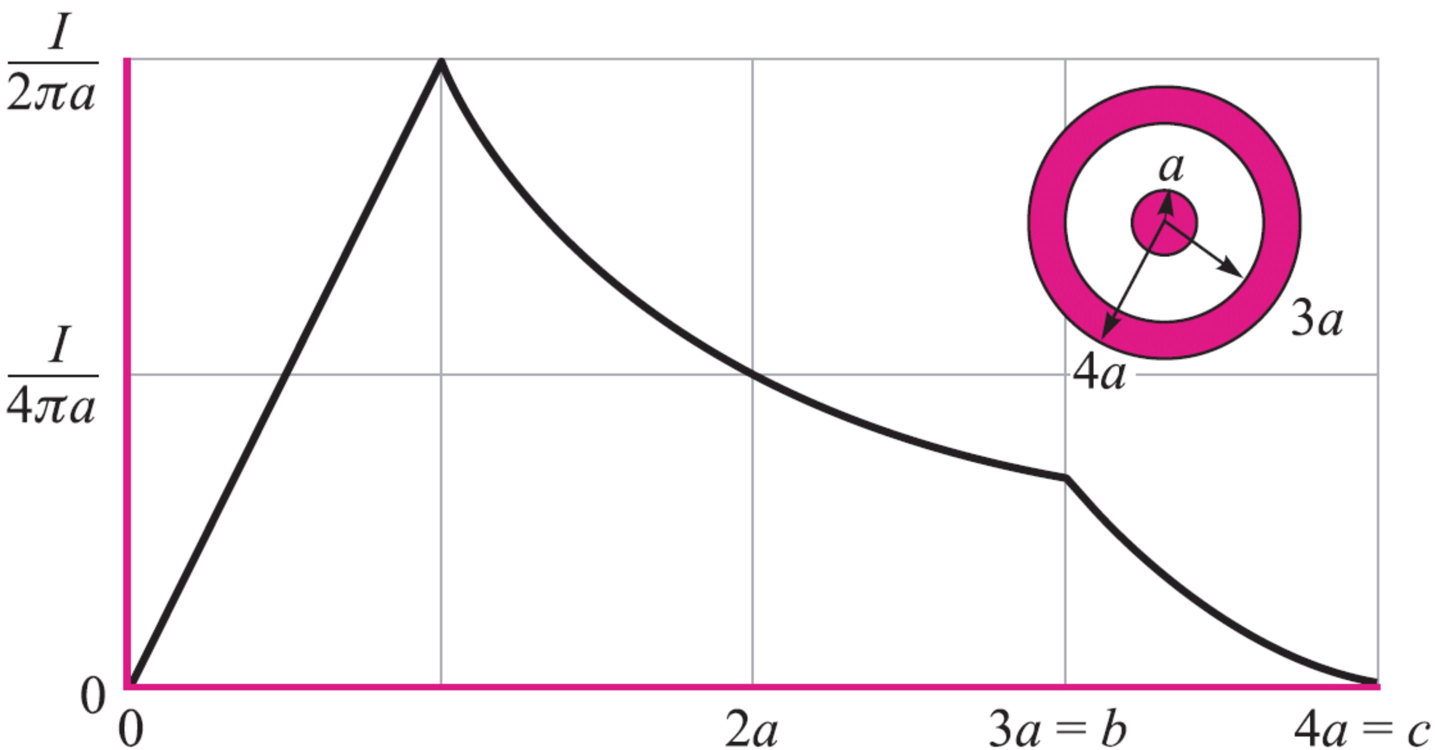
$$2\pi\rho\, H_\phi = I - J \cdot \pi(\rho^2 - b^2) = I - \frac{I}{\pi(c^2 - b^2)} \cdot \pi(\rho^2 - b^2)$$

$$\Rightarrow H_\phi = \frac{I}{2\pi\rho} \cdot \frac{(c^2 - \rho^2)}{(c^2 - b^2)}$$

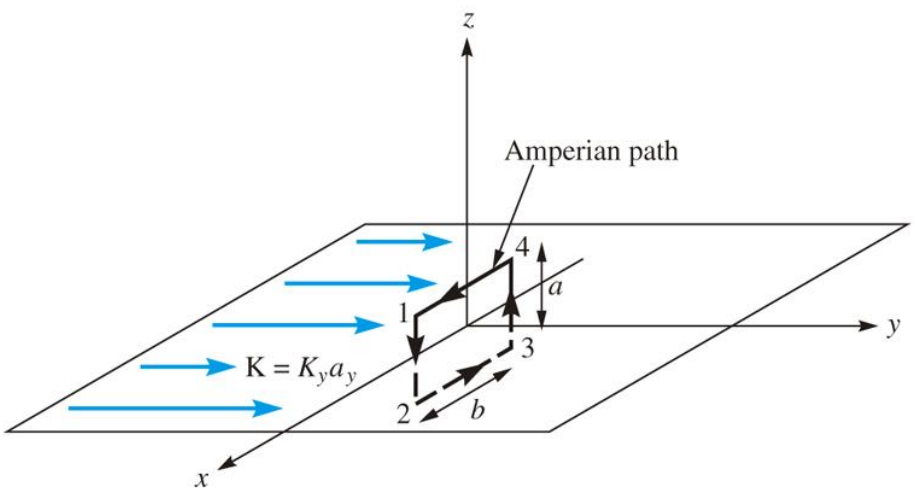
outside coaxial, $c \le \rho$, $\mathbf{H} = \mathbf{0}$

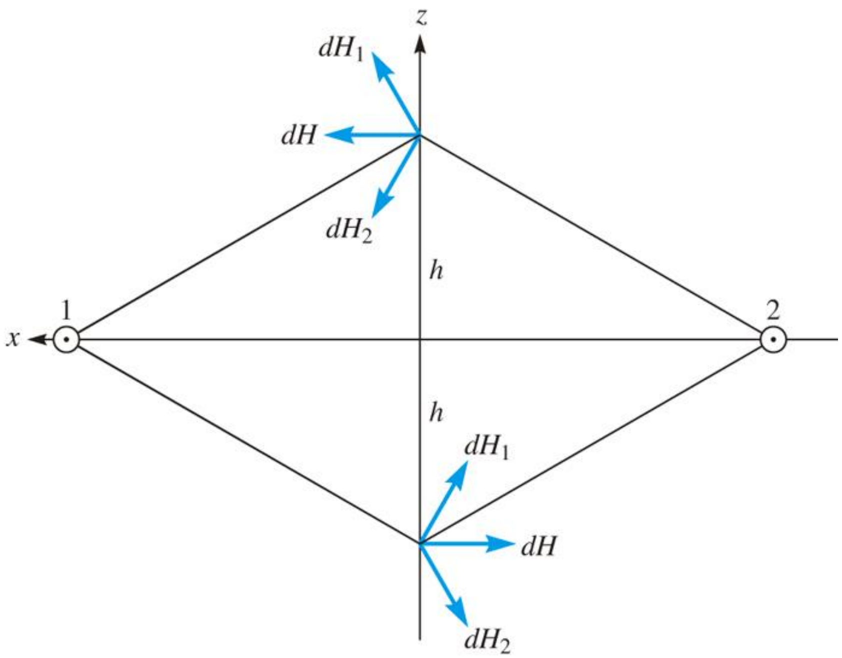
@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

Coaxial Cable (Cont'd)



Infinite Sheet of Current





because of symmetry, magnetic field will be in the *x* direction and should depend only on *z*!

the Amperian path is selected as shown

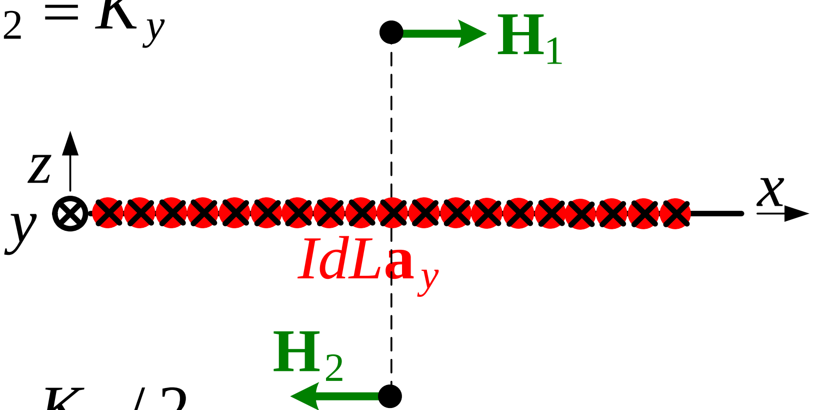
@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

Infinite Sheet (Cont'd)

$$\oint_C \mathbf{H}\cdot d\mathbf{L} = I_{\text{encl}}$$

$$\Rightarrow \int_1^{1'} H_{x1}dx + \underbrace{\int_{1'}^{2'}(-H_z)dz}_{0} + g\int_{2'}^{2}(-H_{x2})dx + \underbrace{\int_2^{1} H_z dz}_{0} = K_y \cdot L$$

$$\Rightarrow H_{x1}L - H_{x2}L = K_y L \Rightarrow H_{x1} - H_{x2} = K_y$$



from Biot-Savart's law, field is anti-symmetrical *wrt* the current sheet

$$H_{x2} = -H_{x1} \Rightarrow H_{x1} = K_y/2,\; H_{x2} = -K_y/2$$

magnetic field does not depend on the distance from the plane!

Infinite Sheet (Cont'd)

for an infinite sheet with a y-directed current $K_y\mathbf{a}_y$, we have

$$\mathbf{H} = \begin{cases} 0.5K_y\mathbf{a}_x, & z>0 \\ -0.5K_y\mathbf{a}_x, & z<0 \end{cases}$$

in general,

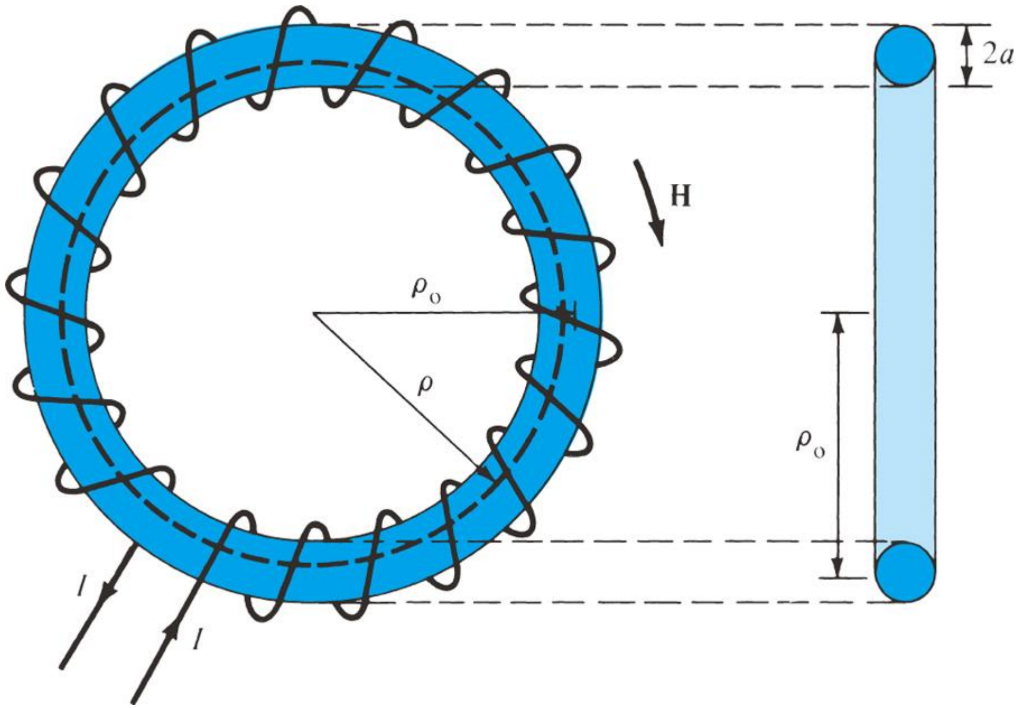
$$\mathbf{H} = 0.5(\mathbf{K} \times \mathbf{a}_n)$$

where $\mathbf{a}_n$ is the unit normal to the sheet directed from the sheet to the observation point

@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014

A Toroid

the toroid has a radius ρ_o, core radius a, number of turns N, and carries a current I



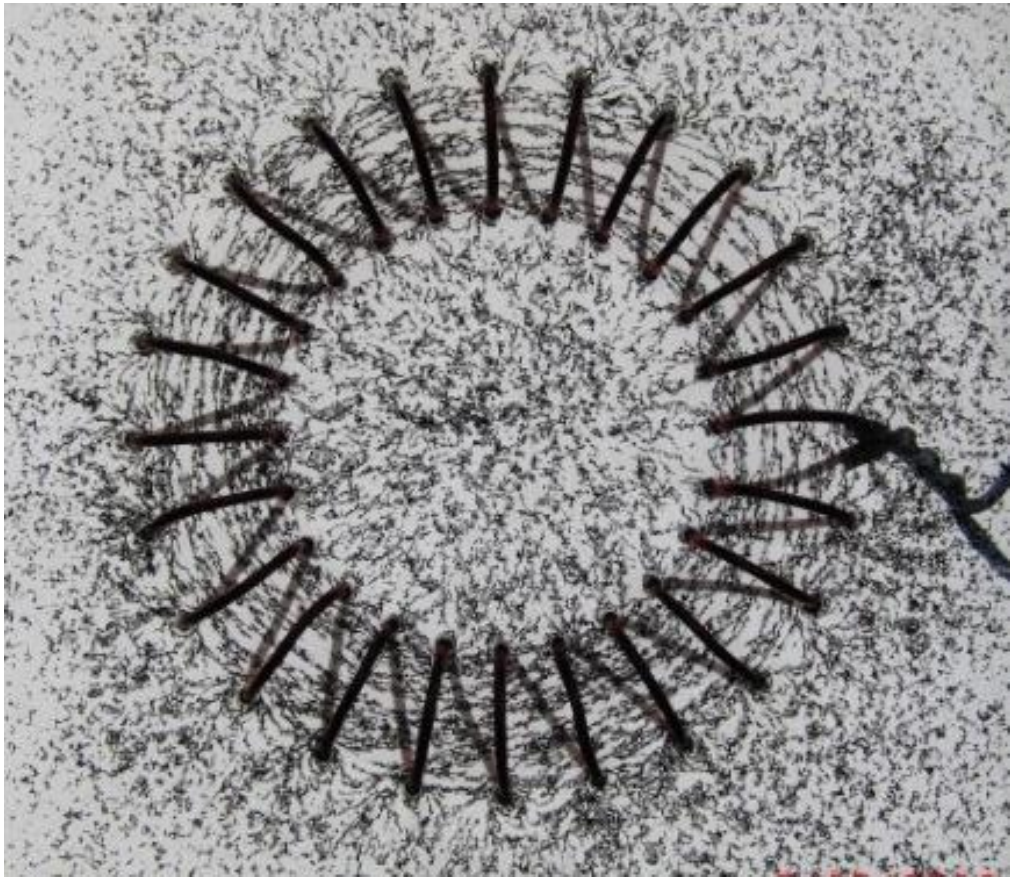
because of symmetry, **H** is in the φ direction and is a function of ρ

$$\oint_C \mathbf{H}\cdot d\mathbf{L} = I_{\text{encl}}$$

$$\Rightarrow H_\phi 2\pi\rho = N\cdot I \qquad \Rightarrow H_\phi = \frac{N\cdot I}{2\pi\rho},\ \rho_1 \le \rho \le \rho_2$$

what is the field outside the toroid?

Toroid (Cont'd)



sharepoint.umich.edu

@Copyright Dr. Mohamed Bakr, EE 2FH3, 2014