Lecture 21: Magnetostatics

Forces due to magnetic Fields, forces due to two current elements, Chapter 8, pages 331-343

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Forces on an Electric Charge

Lorentz' force law describes the force affecting a **moving** electric charge in a magnetic field

$$\mathbf{F}_m = Q\boldsymbol{u} \times \mathbf{B} \quad \text{N}$$

this magnetic force does not exert any work (why!)

remember that the force affecting a charge (moving or not moving) in an electric field is given

$$\mathbf{F}_e = Q\mathbf{E} \quad \text{N}$$

the total force affecting a charge is thus given by

$$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = Q(\mathbf{E} + \boldsymbol{u} \times \mathbf{B}) \quad \text{N}$$

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Ampère's Force Law (motor equation)

$$\mathbf{F}_m = Q\boldsymbol{u}\times\mathbf{B} \qquad \text{N}$$

but as $dQ=\rho_v dv$, we have

$$\Rightarrow d\mathbf{F}_m = \underbrace{(\rho_v dv)}_{dQ}(\boldsymbol{u}\times\mathbf{B}), \quad \text{N}$$

$$\rho_v \boldsymbol{u} dv = \mathbf{J}dv = \mathbf{J}\underbrace{\ dsdL\ }_{dsdL=dv} = \underbrace{\mathbf{J}ds}_{I\mathbf{a}_L}\, dL = Id\mathbf{L}, \quad \text{A}\times\text{m}$$

$$\Rightarrow d\mathbf{F}_m = I(d\mathbf{L}\times\mathbf{B}), \quad \text{N}$$

a current element $d\mathbf{L}$ carrying a current I in a magnetic field **B** experiences a force normal to both the current and the magnetic field

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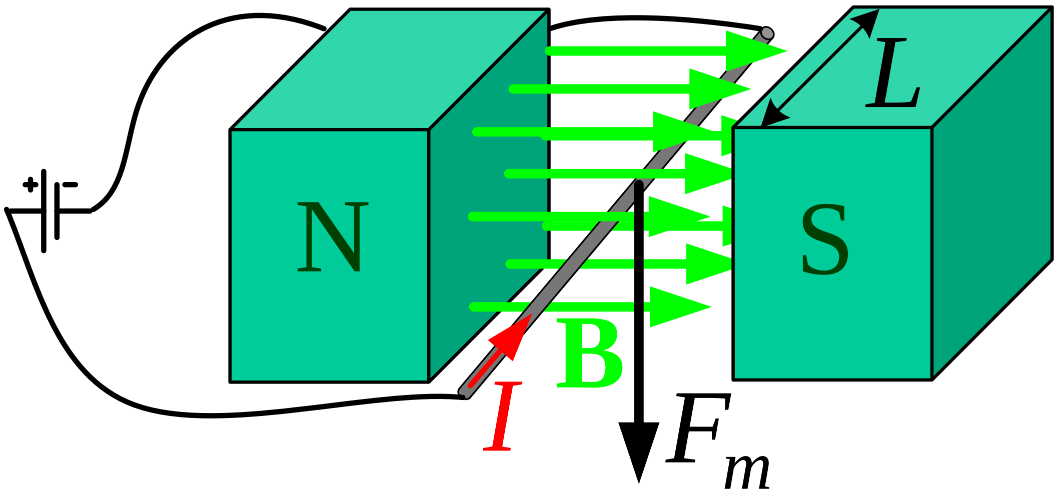
Motor Equation (Cont'd)



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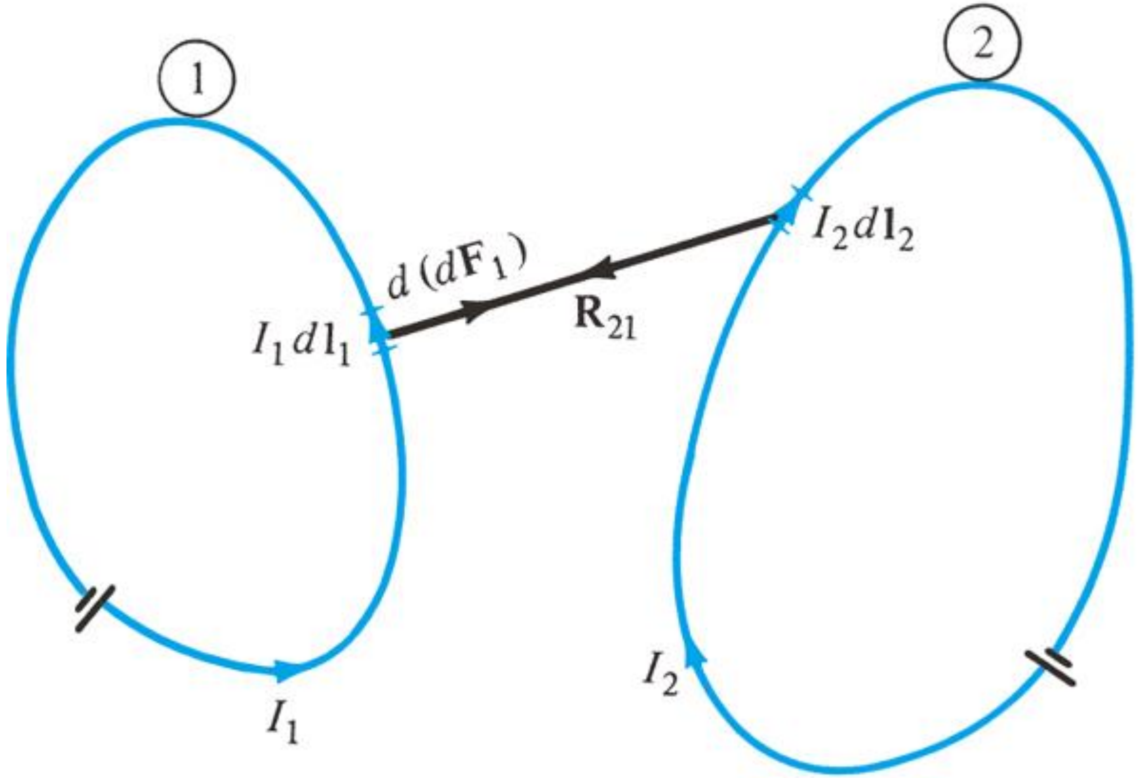
Force Between Two Current Loops



every element of one of the two loops creates a magnetic field that affects every current element in the other loop

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Two Current Loops (Cont'd)

the magnetic field resulting from a small current element of the second loop at an element of the first loop is given by

$$d\mathbf{B}_2 = \frac{\mu_o I_2 \, d\mathbf{L}_2 \times \boldsymbol{a}_{21}}{4\pi R_{21}^2}$$

the force affecting a small element of the first wire due to the field of a small current element in the second wire is thus given by

$$d(d\mathbf{F}_{21}) = I_1 d\mathbf{L}_1 \times \left(\frac{\mu_o I_2 \, d\mathbf{L}_2 \times \boldsymbol{a}_{21}}{4\pi R_{21}^2} \right)$$

total force affecting a differential element of the first wire is thus

$$d\mathbf{F}_{21} = \oint_{C2} I_1 d\mathbf{L}_1 \times \left(\frac{\mu_o I_2 \, d\mathbf{L}_2 \times \boldsymbol{a}_{21}}{4\pi R_{21}^2} \right) = \frac{\mu_o I_1 I_2}{4\pi} \oint_{C2} d\mathbf{L}_1 \times \left(\frac{d\mathbf{L}_2 \times \boldsymbol{a}_{21}}{R_{21}^2} \right)$$

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Two Current Loops (Cont'd)

total force affecting the first wire is

$$\mathbf{F}_1 = \frac{\mu_o I_1 I_2}{4\pi} \oint_{C1} \oint_{C2} d\mathbf{L}_1 \times \left(\frac{d\mathbf{L}_2 \times \boldsymbol{a}_{21}}{R_{21}^2} \right)$$

similarly, the total force affecting the second loop due to the magnetic field of the first loop is given by

$$\mathbf{F}_2 = \frac{\mu_o I_2 I_1}{4\pi} \oint_{C2} \oint_{C1} d\mathbf{L}_2 \times \left(\frac{d\mathbf{L}_1 \times \boldsymbol{a}_{12}}{R_{12}^2} \right)$$

it can be shown that $\mathbf{F}_2 = -\mathbf{F}_1$

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