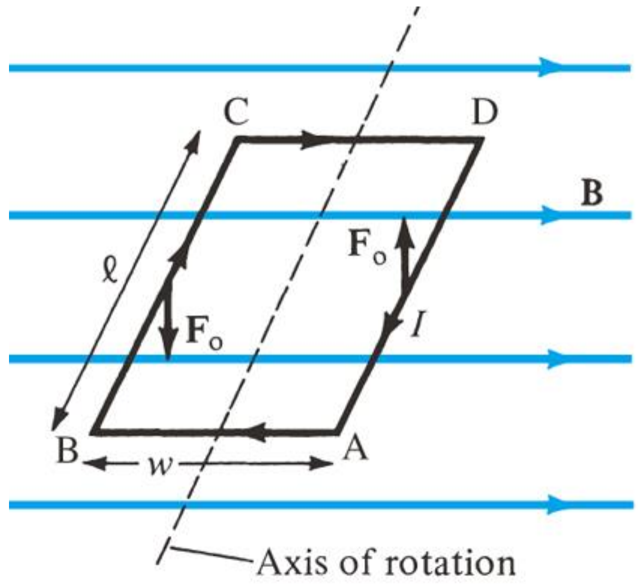
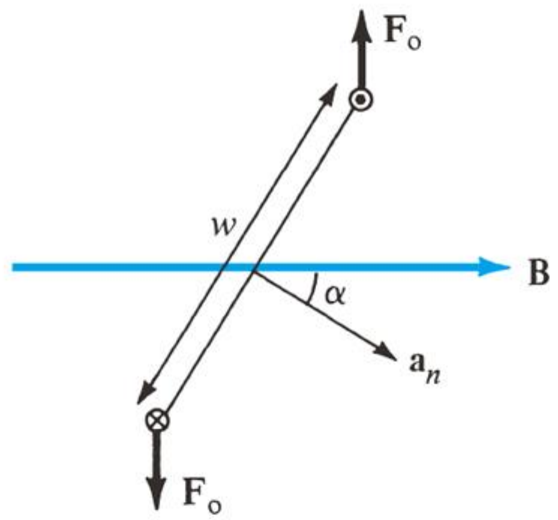
Lecture 22: Magnetostatics

Magnetic torque, magnetic dipole, applications,
Chapter 8, pages 343-350

Torque Due to Magnetic Field





$$|\mathbf{T}| = |\mathbf{F}_o| w \sin\alpha$$

$$T = BIlw\sin\alpha \quad \Rightarrow T = BIS\sin\alpha$$

$$\mathbf{T} = \mathbf{m}\times\mathbf{B}, \quad \mathbf{m} = IS\boldsymbol{a}_n$$

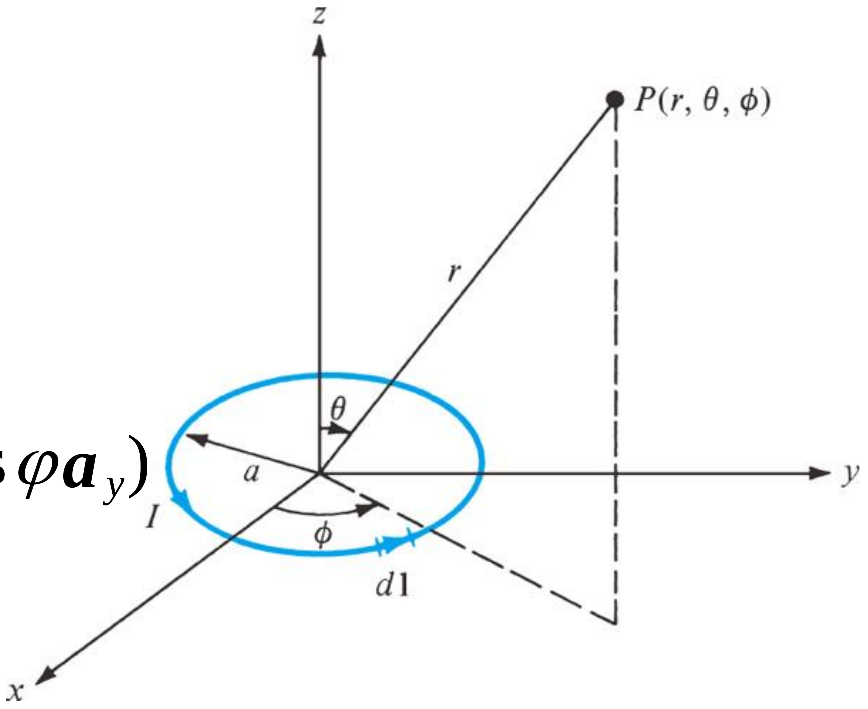
this definition of magnetic moment is general for any planar current loop

Magnetic Dipole

the magnetic vector potential of the current loop is obtained using

$$\mathbf{A} = \frac{\mu_o I}{4\pi} \oint \frac{d\mathbf{l}}{R}$$

$$d\mathbf{l} = a d\varphi \boldsymbol{a}_\varphi = a d\varphi(-\sin\varphi \boldsymbol{a}_x + \cos\varphi \boldsymbol{a}_y)$$



select coordinate system such that *P* has zero *y*-coordinate

$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = (x\mathbf{a}_x + z\mathbf{a}_z) - (a\cos\varphi \mathbf{a}_x + a\sin\varphi \mathbf{a}_y)$$

$$\mathbf{R} = (x - a\cos\varphi)\mathbf{a}_x + (-a\sin\varphi)\mathbf{a}_y + (z)\mathbf{a}_z$$

Magnetic Dipole (Cont'd)

$$R^2 = x^2 + z^2 + a^2 - 2ax\cos\varphi$$

$$R^2 \approx (x^2 + z^2)\left(1 - \frac{2ax\cos\varphi}{(x^2+z^2)}\right)$$

$$R^{-1} = (x^2 + z^2)^{-1/2}\left(1 + \frac{ax\cos\varphi}{(x^2+z^2)}\right)$$

as the *x*-component of the vector potential vanishes (why?), we have

$$\mathrm{A}_y = \frac{\mu_o I a}{2\pi(x^2+z^2)^{1/2}}\int_0^{\pi}(\cos\varphi + \frac{ax\cos^2\varphi}{x^2+z^2})d\varphi = \frac{\mu_o I a^2 x}{4(x^2+z^2)^{3/2}}$$

using $x=r\sin\theta$, $z=r\cos\theta$, $\mathbf{a}_y=\mathbf{a}_\varphi$, we have

$$\mathrm{A}_\varphi = \frac{\mu_o I\pi a^2\sin\theta}{4\pi r^2}$$

Magnetic Dipole (Cont'd)

magnetic vector potential can be written as

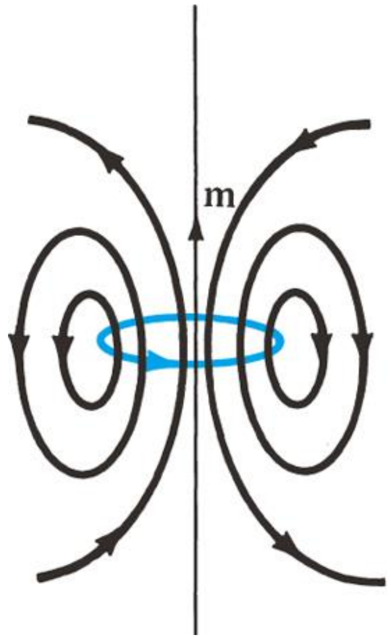
$$\mathbf{A} = \frac{\mu_o \mathbf{m} \times \mathbf{a}_r}{4\pi r^2}, \ \mathbf{m} = I\pi a^2 \mathbf{a}_z$$

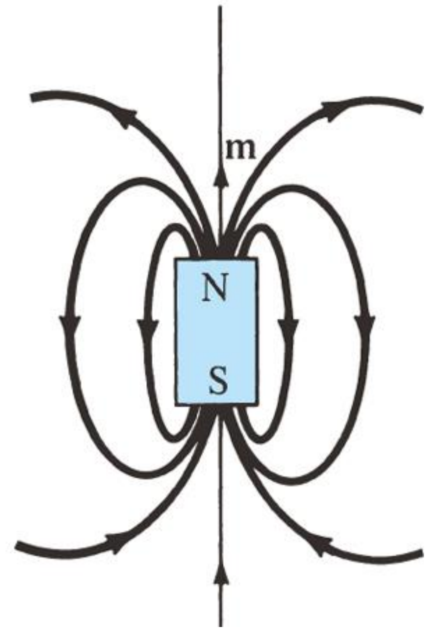
the corresponding magnetic flux density vector is given by

$$\boldsymbol{B} = \frac{\mu_o m}{4\pi r^3}(2\cos\theta \boldsymbol{a}_r + \sin\theta \boldsymbol{a}_\theta)$$

these two expressions are very similar to the ones for the electric dipole!

Small Bar Magnet

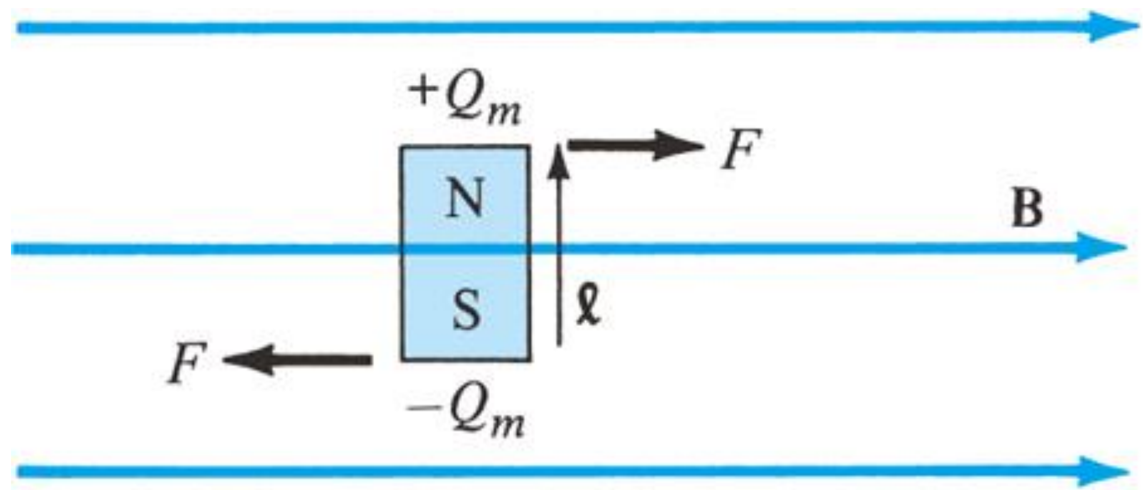




the magnetic field resulting from a magnetic loop and a small bar magnet are similar

the bar magnet can be used to define an equivalent magnetic dipole

Small Bar Magnet (Cont'd)



$\mathbf{T}=\mathbf{m}\times\mathbf{B}=Q_m\mathbf{l}\times\mathbf{B}$

torque tends to align the bar in the direction of the external magnetic field!

same effect can be observed for small current loops