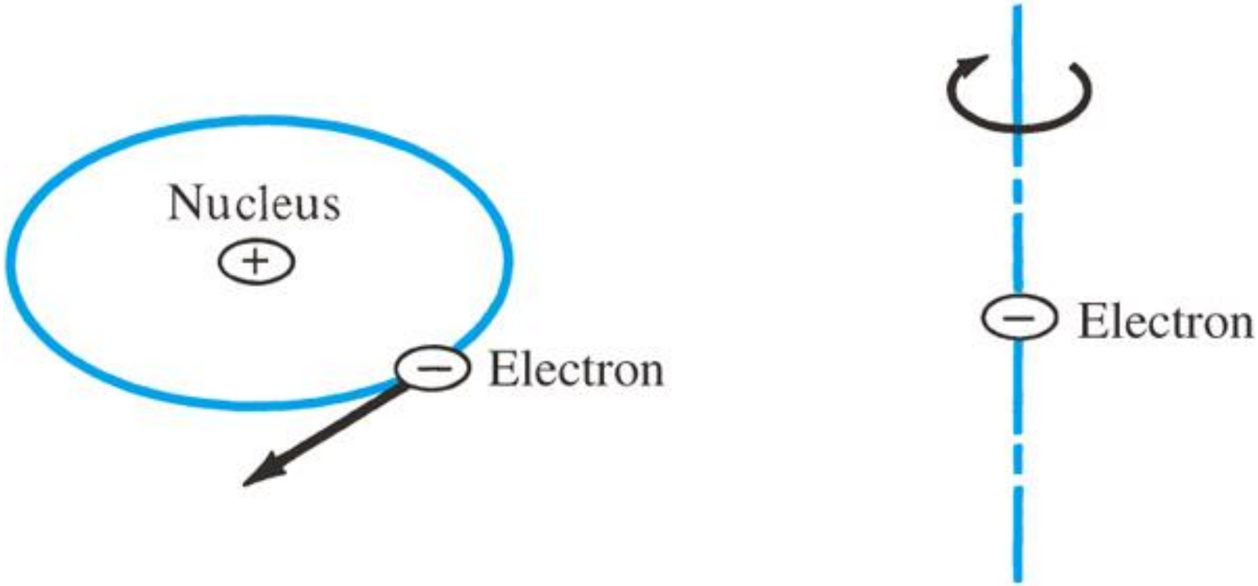
Lecture 23: Magnetostatics

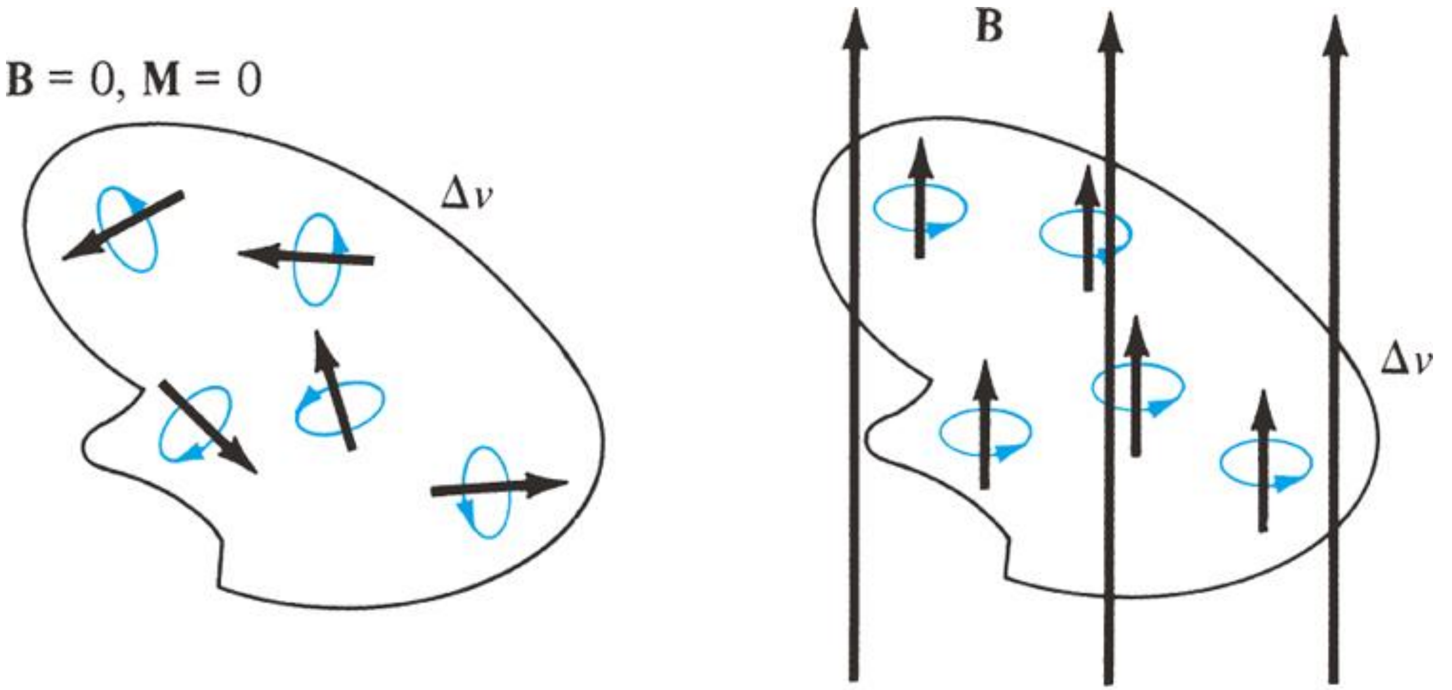
Magnetization in materials, classification of matter, magnetic boundary conditions, Chapter 8, pages 350-362

Magnetization



electron spinning around the nucleus is equivalent to a small current loop with a magnetic moment

Magnetization (Cont'd)



external magnetic field creates torque on all atomic magnetic dipoles and align them along its direction

Magnetization (Cont'd)

define the magnetization **M** as the magnetic dipole moment per unit volume

$$\mathbf{M} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} \mathbf{m}_k}{\Delta v}$$

M has units of Amperes/meter (similar to **H**)

the net magnetic field inside the matter is **H**+**M**

for a linear medium we have $\mathbf{M} = \chi_m \mathbf{H}$

Relative Permeability

for a non magnetic material, we have

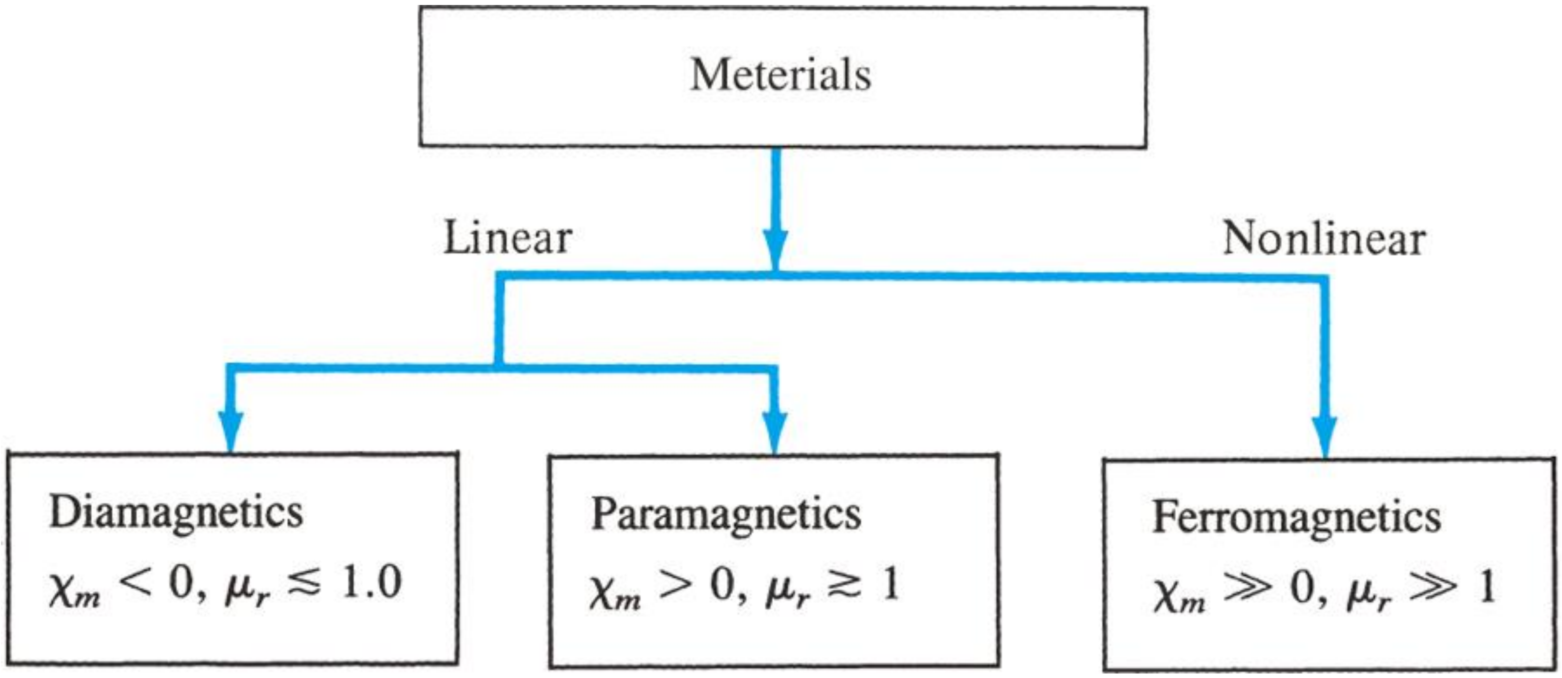
$$\frac{\mathbf{B}}{\mu_o} = \mathbf{H}$$

for a magnetic material, we have

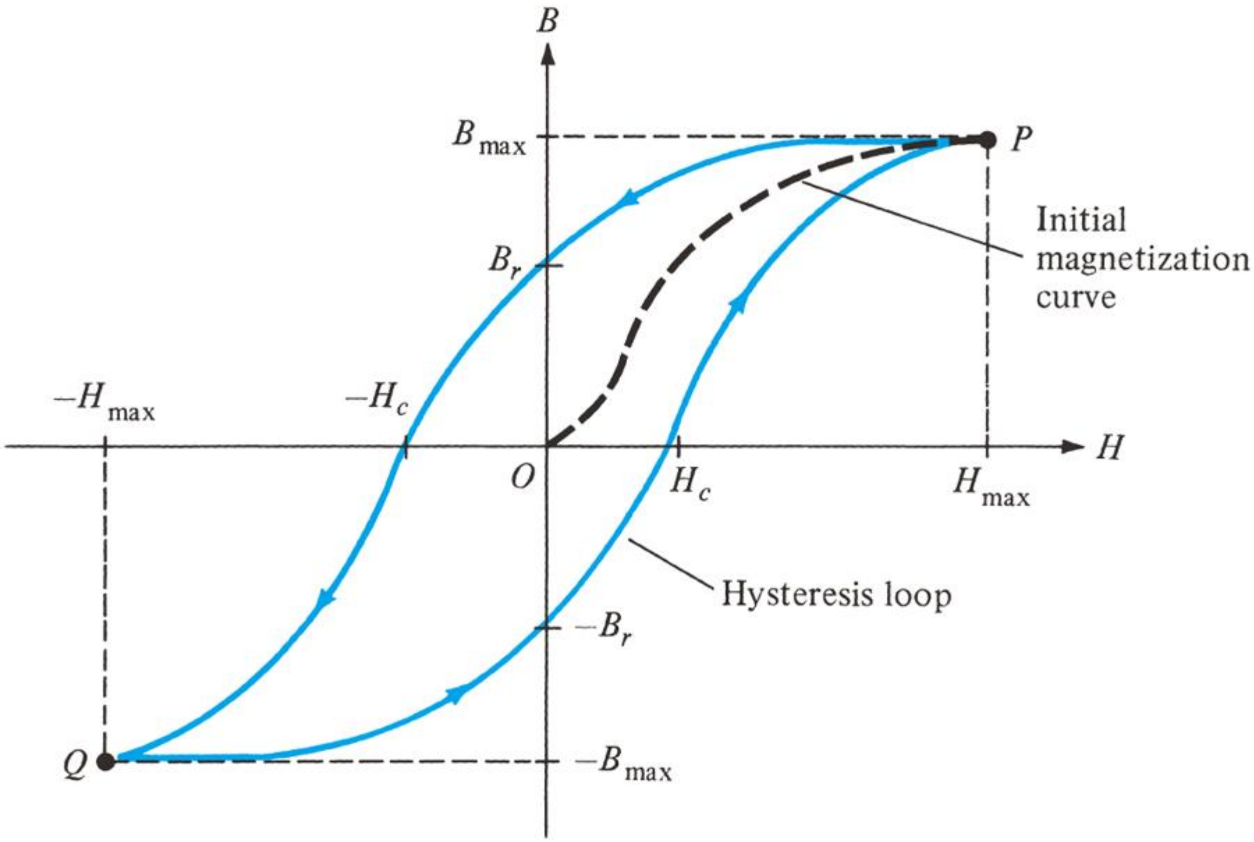
$$\frac{\mathbf{B}}{\mu_o} = \mathbf{H} + \mathbf{M} \;\Rightarrow\; \frac{\mathbf{B}}{\mu_o} = \mathbf{H} + \chi_m \mathbf{H} \Rightarrow\; \mathbf{B} = \mu_o (1 + \chi_m)\mathbf{H}$$

$$\mathbf{B} = \mu_o \mu_r \mathbf{H} \Rightarrow \mathbf{B} = \mu\, \mathbf{H}$$

Classification of Magnetic Materials

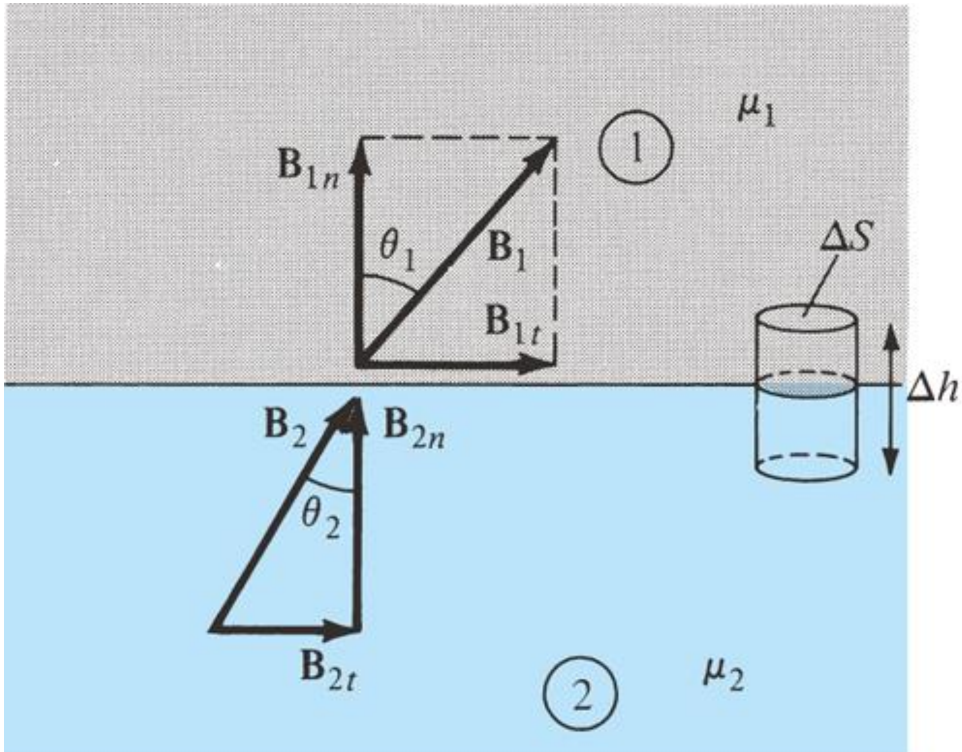


Hysteresis Curves of Ferromagnetic Materials



$\mathbf{B} = \mu_o(\mathbf{H} + \mathbf{M})$ with **M** depending on "magnetic history"

Magnetic Boundary Conditions



$$\oint \mathbf{B}.d\mathbf{s} = 0$$

$$B_{1n}\Delta s = B_{2n}\Delta s$$

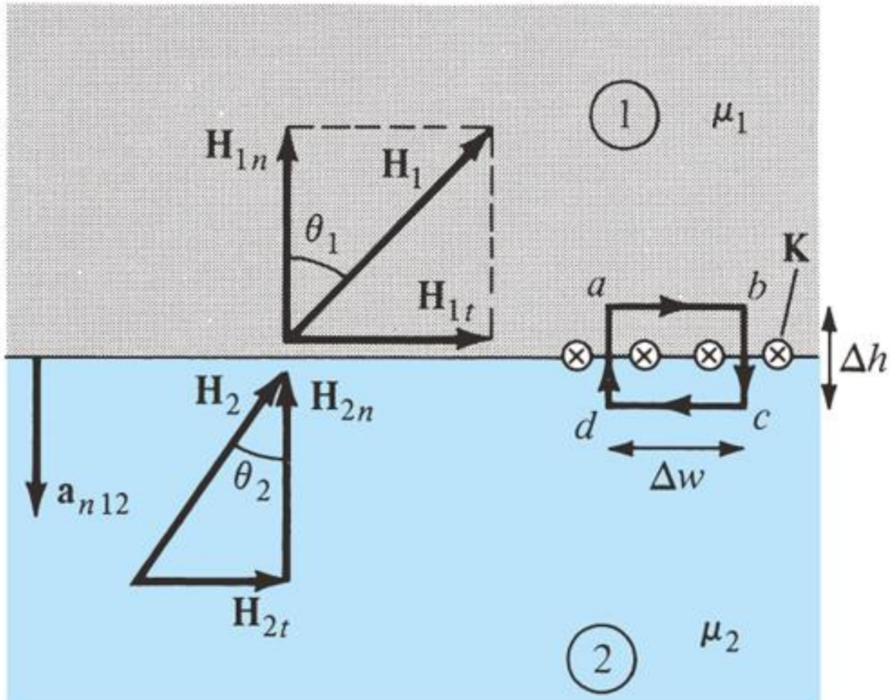
$$\Downarrow$$

$$B_{1n} = B_{2n}$$

$$\mathbf{B}_{1n} = \mathbf{B}_{2n}$$

normal component of the magnetic flux density vector is continuous across any interface

Boundary Conditions (Cont'd)



$$\oint \mathbf{H}.d\mathbf{l} = I$$

as $\Delta h \to 0$, we have

$$K\Delta w = H_{1t}\Delta w - H_{2t}\Delta w$$

$$\Downarrow$$

$$H_{1t} - H_{2t} = K$$

in general

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}$$

if there is a surface current, the tangential component of the magnetic field strength vector is non continuous across the interface