Lecture 24: Magnetostatics

Inductors and inductances, Chapter 8, 362-366

Flux Linkage

Flux linkage is the sum of all fluxes piercing the surfaces bounded by all turns (the total flux "linking" the turns)

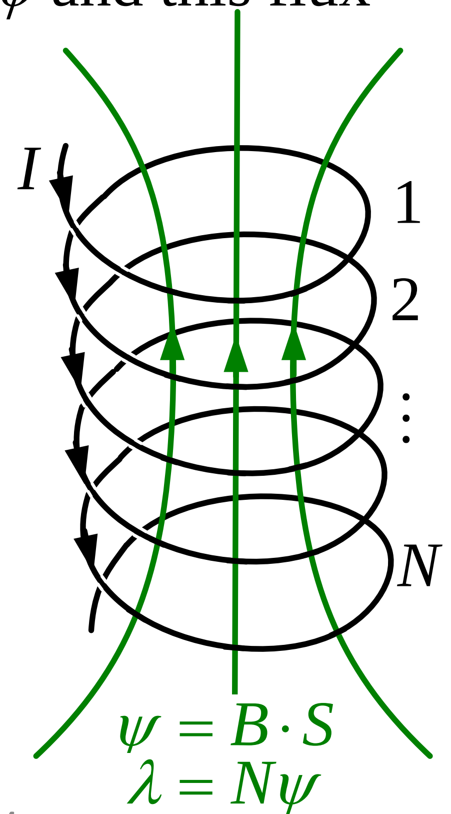
ideal scenario: each turn creates the same flux ψ and this flux "links" all turns (no leakage)

$$\lambda = N\psi, \quad \text{Wb}$$

if flux density **B** is uniform inside the coil, the flux linkage is

$$\lambda = NBS, \quad \text{Wb}$$

$$B \sim NI \Rightarrow \lambda \sim N^2 I$$



Self Inductance

for a linear medium, flux linkage is linearly proportional to the current $\lambda = LI$, where L is the self inductances

$$L = \frac{\lambda_{\text{self}}}{I}$$

self inductance (or simply inductance) is defined for a single inductor

the inductance is related to the stored energy through the expression

$$W_m = \frac{1}{2} L I^2 \Rightarrow L = \frac{2W_m}{I^2}$$

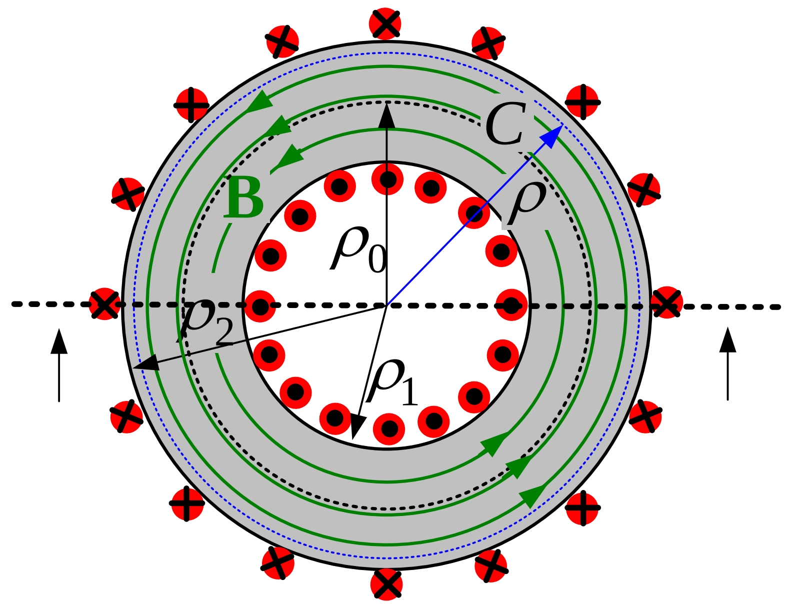
self inductance may be defined in terms of energy!

Example: A toroid

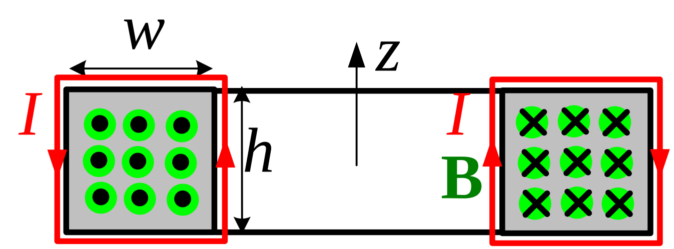
$$H_\phi = \frac{N \cdot I}{2\pi\rho}, \text{ H/m}, \rho_1 \le \rho \le \rho_2$$

$$\psi = \int_0^h \int_{\rho_1}^{\rho_2} \mu H_\phi d\rho dz = \mu \frac{NIh}{2\pi} \ln\left(\frac{\rho_2}{\rho_1}\right)$$

$$\lambda = N\psi = \mu \frac{N^2 Ih}{2\pi} \ln\left(\frac{\rho_2}{\rho_1}\right), \text{ Wb}$$



$$L = \mu \frac{N^2 h}{2\pi} \ln\left(\frac{\rho_2}{\rho_1}\right), \text{ H}$$



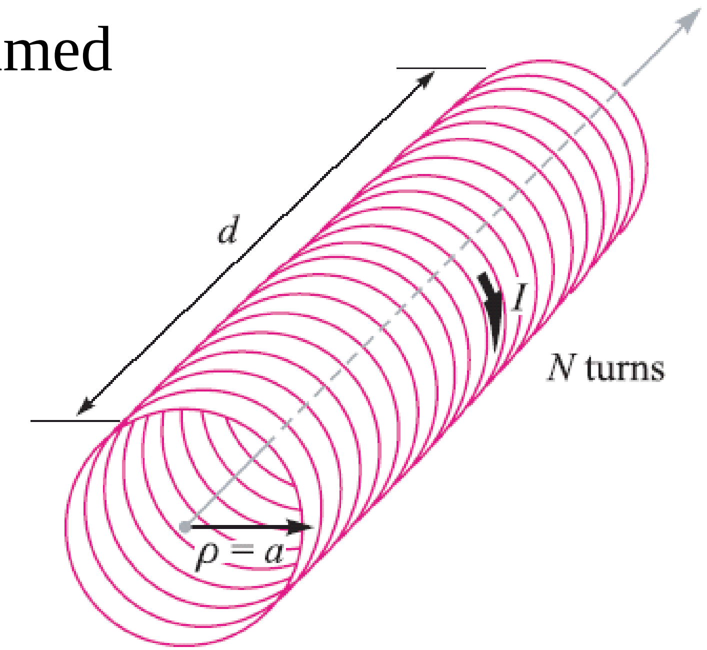
if toroid is thin, field is mostly uniform

$$H_\phi = \frac{N \cdot I}{2\pi\rho_0} \Rightarrow \psi = \mu \frac{NIA}{2\pi\rho_0} \Rightarrow L = \mu \frac{N^2 A}{2\pi\rho_0}$$

$$\rho_0 = 0.5(\rho_1 + \rho_2)$$
$$A = h\underbrace{(\rho_2 - \rho_1)}_{w}$$

Inductance of a Solenoid

the field of a long thin solenoid is assumed uniform inside the solenoid



$$H_z = \frac{NI}{d},\ \text{A/m}$$

$$\psi = \mu \cdot \frac{NI}{d} \cdot A$$

$$\Rightarrow \lambda = N\psi = \mu \frac{N^2 IA}{d},\ \text{Wb}$$

$$\Rightarrow L = \mu \frac{N^2 A}{d},\ \text{H}$$

Mutual Flux Linkage

mutual-flux linkage is due to currents in other inductors

mutual-flux linkage implies two coupled coils

$$\psi_{12} = \int_{S1} \mathbf{B}_2 . d\mathbf{S}$$

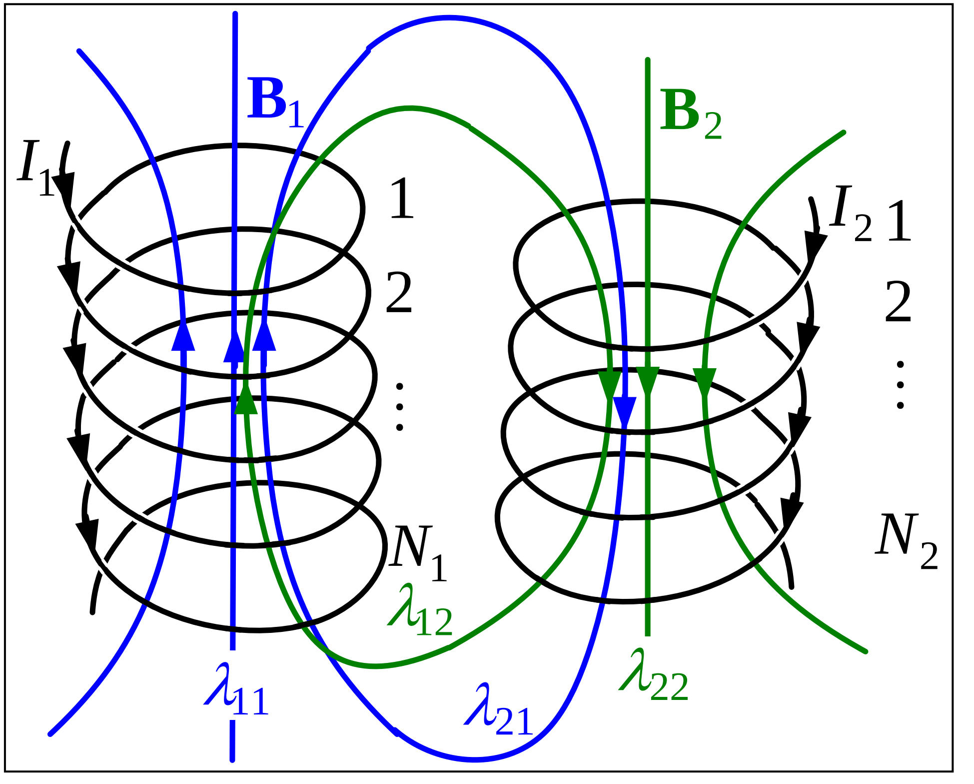
$$\lambda_1 = \lambda_{11} + \lambda_{12},\ \lambda_2 = \lambda_{22} + \lambda_{21}$$

$$\lambda_{11} = N_1 \psi_{11} \sim N_1^2 I_1$$

$$\lambda_{12} = N_1 \psi_{12} \sim N_1 N_2 I_2$$

$$\lambda_{22} = N_2 \psi_{22} \sim N_2^2 I_2$$

$$\lambda_{21} = N_2 \psi_{21} \sim N_2 N_1 I_1$$



Mutual Inductance

mutual inductance M_{12} is the flux linking the first conductor due to the current in the second conductor

$$M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \psi_{12}}{I_2}, \text{ H}$$

similarly, the mutual inductance M_{21} is the flux linking the second conductor due to the current in the first conductor

$$M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \psi_{21}}{I_1}, \text{ H}$$

it can be shown that for a linear surrounding medium $M_{12}=M_{21}$

the magnetic energy of the system is

$$W_m = W_1 + W_2 + W_{12} = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M_{12} I_1 I_2$$

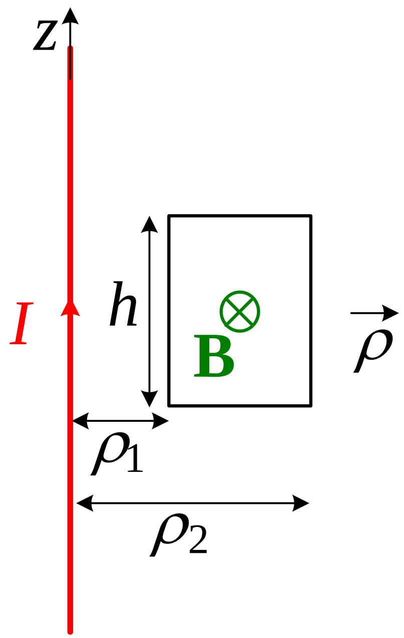
Mutual Inductance Between a Wire and a Loop

straight wire with current and rectangular loop

$$B_\phi = \frac{\mu}{2\pi} \cdot \frac{I}{\rho}, \text{ T}$$

$$\psi_{21} = \int_0^h \int_{\rho_1}^{\rho_2} B_\phi d\rho dz = \frac{\mu I}{2\pi} \cdot h \cdot \ln\left(\frac{\rho_2}{\rho_1}\right), \text{ Wb}$$

$$M_{21} = \frac{\psi_{21}}{I} = \frac{\mu h}{2\pi} \ln\left(\frac{\rho_2}{\rho_1}\right), \text{ H}$$



if rectangular loop consists of N tightly wound turns

$$M_{21} = \frac{N\psi_{21}}{I} = \frac{\mu h N}{2\pi} \ln\left(\frac{\rho_2}{\rho_1}\right)$$