Lecture 25: Magnetostatics

Magnetic Energy, Magnetic Circuits, Chapter 8, 365-375

Magnetic Energy

we would like to derive equations governing EM energy and power

starting with Maxwell's equations

$$(\nabla \times \mathbf{E}) = -\frac{\partial \mathbf{B}}{\partial t} \quad = -\boldsymbol{\mu}_d \qquad (.\boldsymbol{H})$$

$$(\nabla \times \mathbf{H}) = \mathbf{J}_i + \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_i + \mathbf{J}_c + \mathbf{J}_d \qquad (.\boldsymbol{E})$$

subtracting we get

$$\mathbf{H} \bullet (\nabla \times \mathbf{E}) - \mathbf{E} \bullet (\nabla \times \mathbf{H}) = -\mathbf{H} \bullet \boldsymbol{\mu}_d - \mathbf{E} \bullet (\mathbf{J}_i + \mathbf{J}_c + \mathbf{J}_d)$$

or alternatively,

$$\nabla \bullet (\mathbf{E} \times \mathbf{H}) = -\mathbf{H} \bullet \boldsymbol{\mu}_d - \mathbf{E} \bullet (\mathbf{J}_i + \mathbf{J}_c + \mathbf{J}_d)$$

Magnetic Energy

integrating over the volume of interest

$$\iiint_V \nabla \bullet (\mathbf{E}\times\mathbf{H})dV = -\iiint_V \mathbf{H}\bullet\boldsymbol{\mu}_d \, dV - \iiint_V \mathbf{E}\bullet(\mathbf{J}_i + \mathbf{J}_c + \mathbf{J}_d)dV$$

utilizing the divergence theorem, we get

$$\oiint_S (\mathbf{E}\times\mathbf{H})\bullet d\mathbf{S} + \iiint_V \mathbf{H}\bullet\boldsymbol{\mu}_d \, dV + \iiint_V \mathbf{E}\bullet(\mathbf{J}_i + \mathbf{J}_c + \mathbf{J}_d)dV = 0$$

explanation of different terms

$\mathbf{P} = \mathbf{E}\times\mathbf{H}$ is the Poynting vector (W/m^2)

$\mathbf{P}_o = \oiint_S \mathbf{P}.d\mathbf{S}$ is the power flowing out of the surface S

$\mathbf{P}_S = -\iiint_V \mathbf{E}\bullet\mathbf{J}_i \, dV$ is the supplied power (W)

Magnetic Energy (Cont'd)

$$P_d = \iiint_V \mathbf{E} \bullet \mathbf{J}_c \, dV = \iiint_V \sigma \mathbf{E} \bullet \mathbf{E} \, dV = \iiint_V \sigma |\mathbf{E}|^2 dV$$

=dissipated power (W)

$$P_m = \iiint_V \mathbf{H} \bullet \boldsymbol{\mu}_d \, dV = \iiint_V \mathbf{H} \bullet \frac{\partial \mathbf{B}}{\partial t} dV = \iiint_V \mu \mathbf{H} \bullet \frac{\partial \mathbf{H}}{\partial t} dV$$

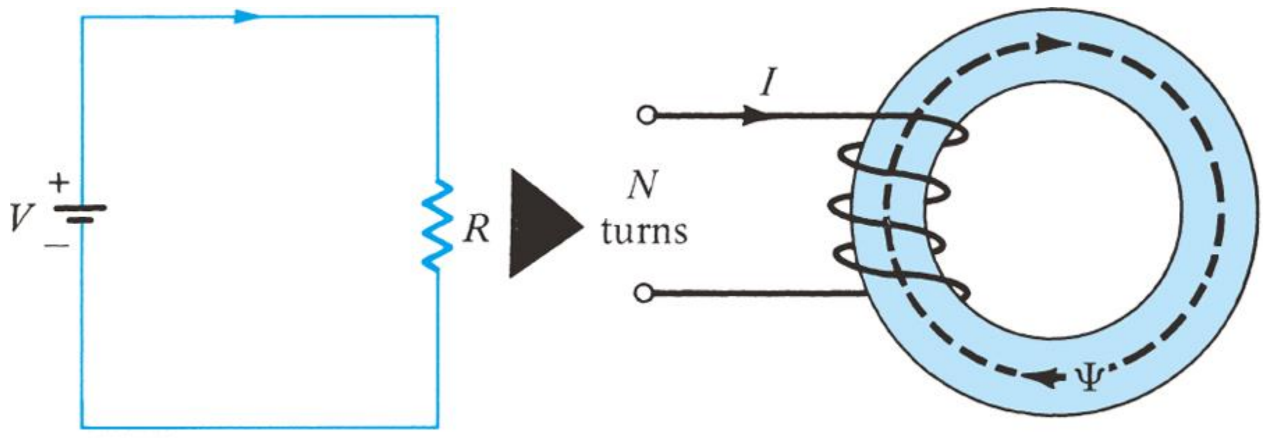
$$P_m = \frac{\partial}{\partial t} \iiint_V \frac{1}{2} \mu |\mathbf{H}|^2 dV = \frac{\partial}{\partial t} W_m$$ = magnetic power

$$P_e = \iiint_V \mathbf{E} \bullet \mathbf{J}_d \, dV = \iiint_V \mathbf{E} \bullet \frac{\partial \mathbf{D}}{\partial t} dV = \iiint_V \varepsilon \mathbf{E} \bullet \frac{\partial \mathbf{E}}{\partial t} dV$$

$$P_e = \frac{\partial}{\partial t} \iiint_V \frac{1}{2} \varepsilon |\mathbf{E}|^2 dV = \frac{\partial}{\partial t} W_e$$ =electric power

$$P_s = P_o + P_d + \partial (W_e + W_m) / \partial t$$ conservation of EM energy

Magnetic Circuits



voltage V in electric circuit corresponds to Magneto Motive Force (MMF) $\mathfrak{J}$ in a magnetic circuit; $\mathfrak{J}=NI$

current in an electric circuit I corresponds to magnetic flux ψ in a magnetic circuit

resistance R corresponds to reluctance $\mathfrak{R}$

Magnetic Circuit (Cont'd)

Conductivity σ

Field intensity E

Current $I = \iint \mathbf{J}.d\mathbf{S}$

Current density $J=I/S=\sigma E$

Electromotive force (emf) V

Resistance R

Permeability μ

Field intensity H

Flux $\psi = \iint \mathbf{B}.d\mathbf{S}$

Flux density $B=\psi/S=\mu H$

Magnetomotive force (mmf) $\mathfrak{I}$

Reluctance $\mathfrak{R}$

Magnetic Circuits (Cont'd)

Ohm's law $R=V/I=l/(\sigma S)$

$\Re = \Im / \psi = l/(\mu S)$

$V=El=IR$

$\Im = Hl = \psi \Re = NI$

Kirchhoff's laws

$$\sum I = 0$$

$$\sum \psi = 0$$

$$\sum V - \sum IR = 0$$

$$\sum \Im - \sum \psi \Re = 0$$