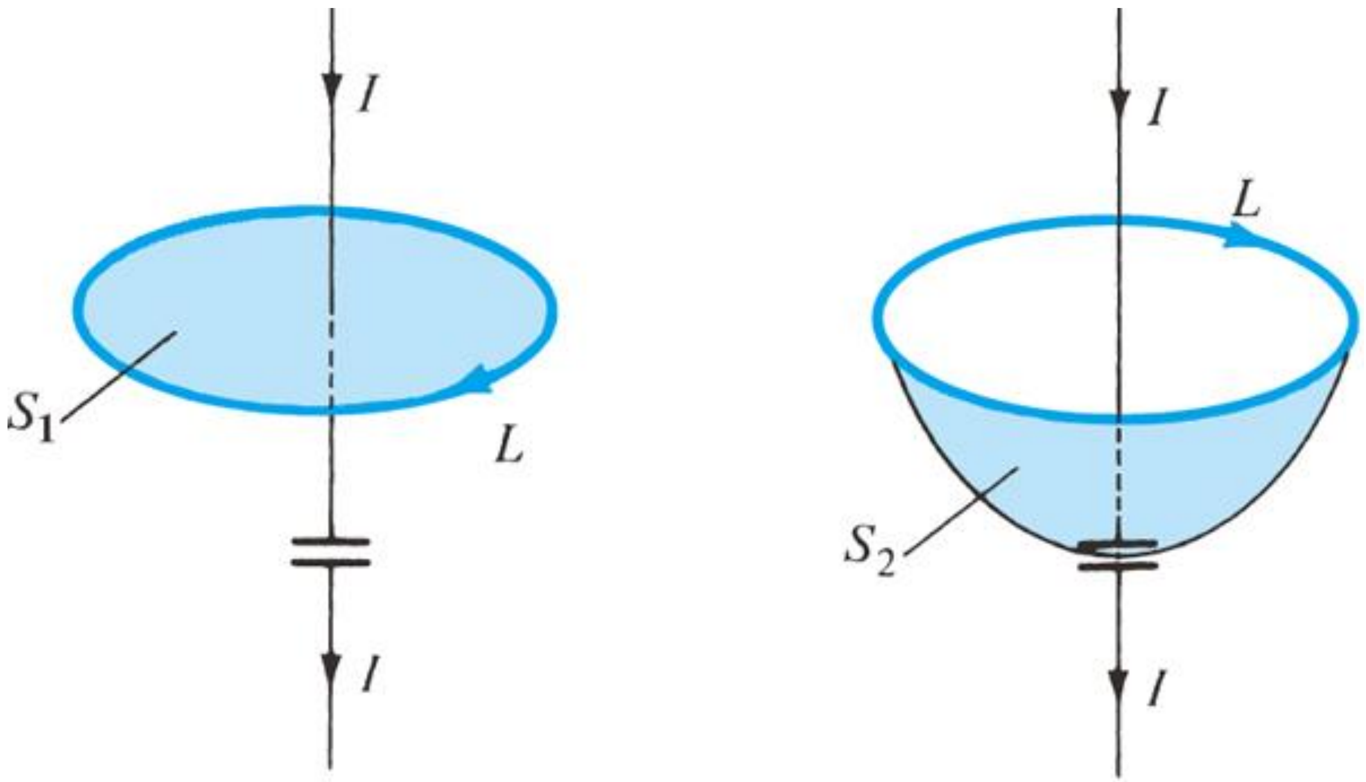
Lecture 27:Time-Varying Fields

Displacement current, Maxwell's equations,
Chapter 9, 411-416

Ampere's Law



Ampere's original law $\oint \mathbf{H}.d\mathbf{l} = I$ does not represent the general case when field are time-varying!

Ampere's Law (Cont'd)

as $\nabla\times\mathbf{H}=\mathbf{J}\ \Rightarrow \nabla.(\nabla\times\mathbf{H})=\nabla.\mathbf{J}=0$

but from the continuity equation

$$\nabla.\mathbf{J}=-\frac{\partial\rho_v}{\partial t}=-\frac{\partial(\nabla.\mathbf{D})}{\partial t}=-\nabla.\frac{\partial\mathbf{D}}{\partial t}\neq 0 \text{ for the time varying case!}$$

Maxwell corrected Ampere's law by adding the displacement current

$$\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\ \Rightarrow\ \oint\mathbf{H}.\mathbf{dl}=\iint_S\left(\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\right).d\mathbf{s}$$

The Complete Maxwell's Equations

Integral form	Differential form
$\oiint_S \mathbf{D}.d\mathbf{S} = \iiint_V q_v \, dV = Q$	$\nabla.\mathbf{D} = q_v$
$\oiint_S \mathbf{B}.d\mathbf{S} = 0$	$\nabla.\mathbf{B} = 0$
$\oint_C \mathbf{E}.\mathbf{dl} = -\frac{\partial}{\partial t}\iint_S \mathbf{B}.d\mathbf{S}$	$(\nabla \times \mathbf{E}) = -\frac{\partial \mathbf{B}}{\partial t}$
$\oint_C \mathbf{H}.\mathbf{dl} = \iint_S \mathbf{J}.d\mathbf{S} + \frac{\partial}{\partial t}\iint_S \mathbf{D}.d\mathbf{S}$	$(\nabla \times \mathbf{H}) = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$

Boundary Conditions

$$(\mathbf{E}_1 - \mathbf{E}_2) \times \mathbf{a}_n = \mathbf{0}$$

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_n = \mathbf{K}$$

$$\mathbf{a}_n \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$$

$$\mathbf{a}_n \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$$

Constitutive relations

$$\mathbf{B} = \mu \mathbf{H}, \quad \mathbf{D} = \varepsilon \mathbf{E}, \quad \mathbf{J}_c = \sigma \mathbf{E}$$