Lecture 7: Electrostatics

Coulomb's law, definition of electric field, superposition of discrete charges, Chapter 4 pages 107-115

Electric Charge

fundamental property of matter

measured in *coulombs* (C)

ampere (A) is a basic unit in SI and *coulomb* is a secondary unit derived from it

$$i = -\frac{dQ}{dt} \Rightarrow \text{C=A}\times\text{s}$$

i is the electric current in *amperes* (A)

Q is the electric charge in *coulombs* (C)

t is time in *seconds* (s)

Electric Charge (Cont'd)

charged particles in atoms: electrons and protons

these particles react in opposite way to the influence of external electric fields – they have opposite charges

protons have *positive* charge, and electrons have *negative* charge

electron charge is equal in magnitude to the charge of a proton

electron charge is the smallest <u>indivisible</u> amount of charge

$$e \approx -1.602 \times 10^{-19}, \ \ \mathrm{C}$$

we are concerned with charge distributions much larger than the dimensions of the largest atomic nucleus ($\propto 10^{-15}$ m)

Point Charge

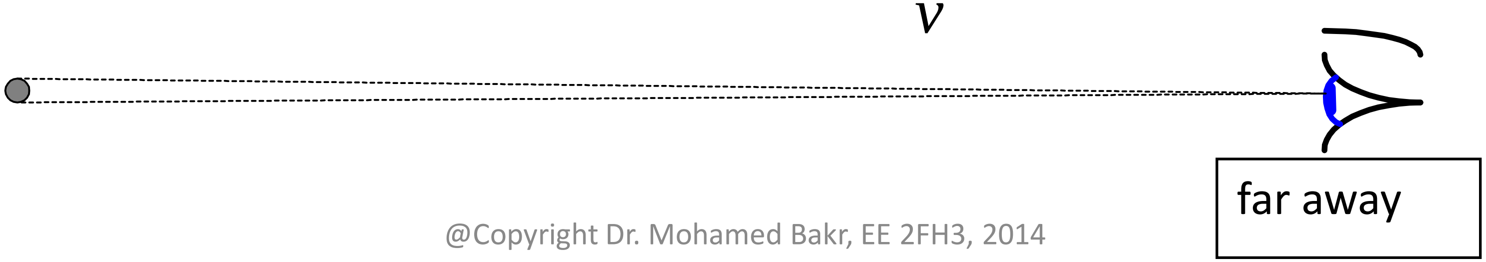
charge occupies a finite volume and may have varying density

a volumetric charge can be always considered made of smaller charges, so small that they tend to a point

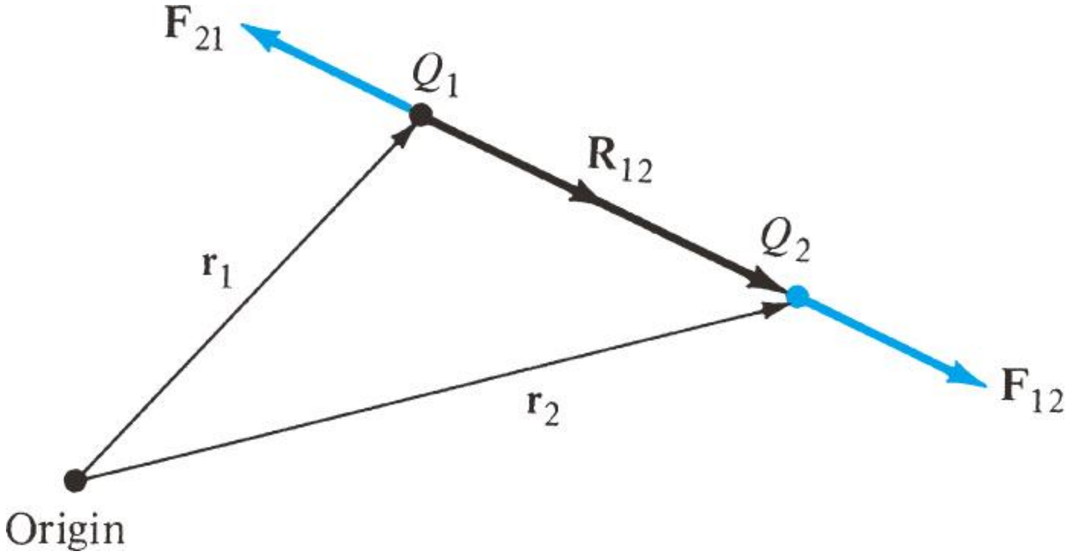
<u>point charge</u> features a volume which can be considered infinitesimal (a point) relative to the distance from its center to the observation point

a point charge is then an infinitesimal sphere of *homogeneous charge distribution* (charge density ρ_v is constant)

$$Q = \rho_v \cdot v \ [\text{C}] \Leftrightarrow \rho_v = \frac{Q}{v} \ [\text{C/m}^3]$$



Coulomb's Law



Force affecting Q_2 due to charge Q_1

$$\mathbf{F}_{12} = \underbrace{\frac{1}{4\pi\varepsilon}}_{k} \frac{Q_1 Q_2}{R_{12}^2} \mathbf{a}_{12} = -\mathbf{F}_{21}, \quad \text{N}$$

$$\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1,$$
$$R_{12} = |\mathbf{R}_{12}|,$$
$$\mathbf{a}_{12} = \mathbf{R}_{12} / R_{12}$$

Coulomb's Law

Similar charges rebel and different charges attract

$$\mathbf{F}_{12} = \underbrace{\frac{1}{4\pi\varepsilon}}_{k} \frac{Q_1 Q_2}{R_{12}^3} \mathbf{R}_{12} = \underbrace{\frac{1}{4\pi\varepsilon}}_{k} \frac{Q_1 Q_2}{\left|\mathbf{r}_2 - \mathbf{r}_1\right|^3} \left(\mathbf{r}_2 - \mathbf{r}_1\right)$$

in vacuum, if the force is measured in *newtons*, the distance in *meters*, and the charge in *ampere-seconds* (coulombs):

$$k \approx 9.0 \times 10^9$$

Coulomb's Law (Cont'd)

the constant

$$\varepsilon = \frac{1}{4\pi k}$$

is called *dielectric permittivity*. In *vacuum*, it is equal to

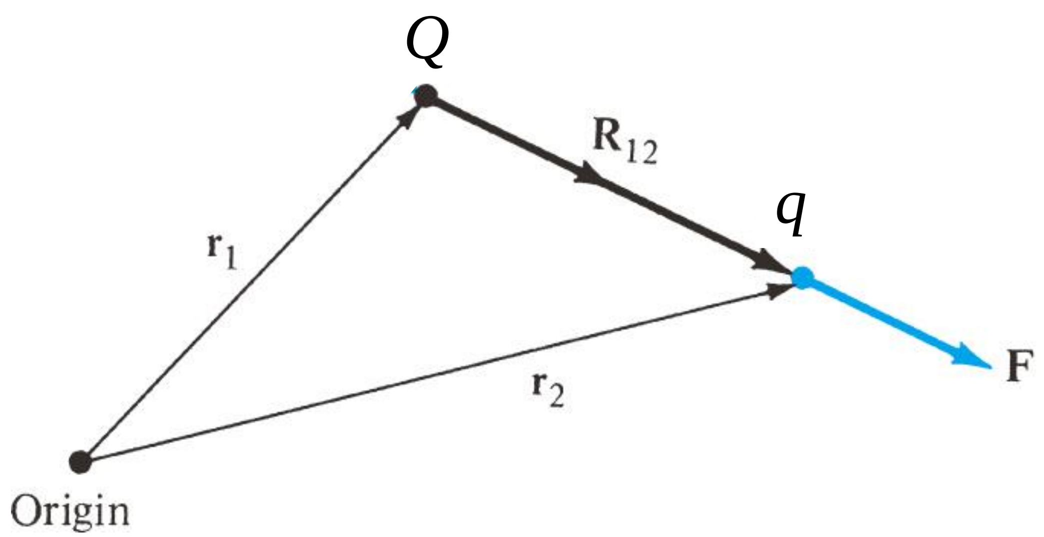
$$\varepsilon_0 \approx \frac{10^{-9}}{36\pi} = 8.856\times10^{-12}\ \text{F/m} = \text{C/(V}\times\text{m)}$$

For a general medium, we have $\varepsilon = \varepsilon_0 \varepsilon_r$, relative dielectric constant >1

Air: $\varepsilon_r \approx 1.0006$ Water: $\varepsilon_r \approx 80$

Urban (dry) ground: $\varepsilon_r \approx 3$

Electric Field



electric field vector – the force exerted on a unit charge

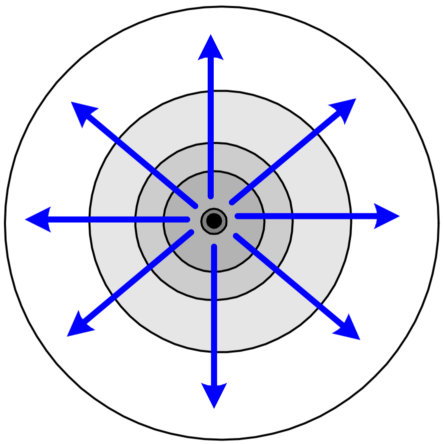
$$\mathbf{E} = \lim_{q \to 0} \frac{\mathbf{F}}{q}, \quad \text{N/C=V/m} \quad \Leftrightarrow \mathbf{F} = q\mathbf{E}, \quad \text{N}$$

q is a *test (probe) charge*, the value and size of which are small enough not to disturb the measured original field of the source.

Electric Field (Cont'd)

electric field of a positive point charge located at the origin of a spherical coordinate system

$$\mathbf{E} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\mathbf{a}_r, \quad \text{V/m}$$



notice that in this expression the charge is at the origin!

Superposition

superposition means adding the individual contributions of sources

superposition holds in a linear medium

<u>vector</u> superposition implies adding/subtracting <u>vectors</u>

the ESF of multiple charges at any point is a *vectorial sum* of the fields created by each individual charge.

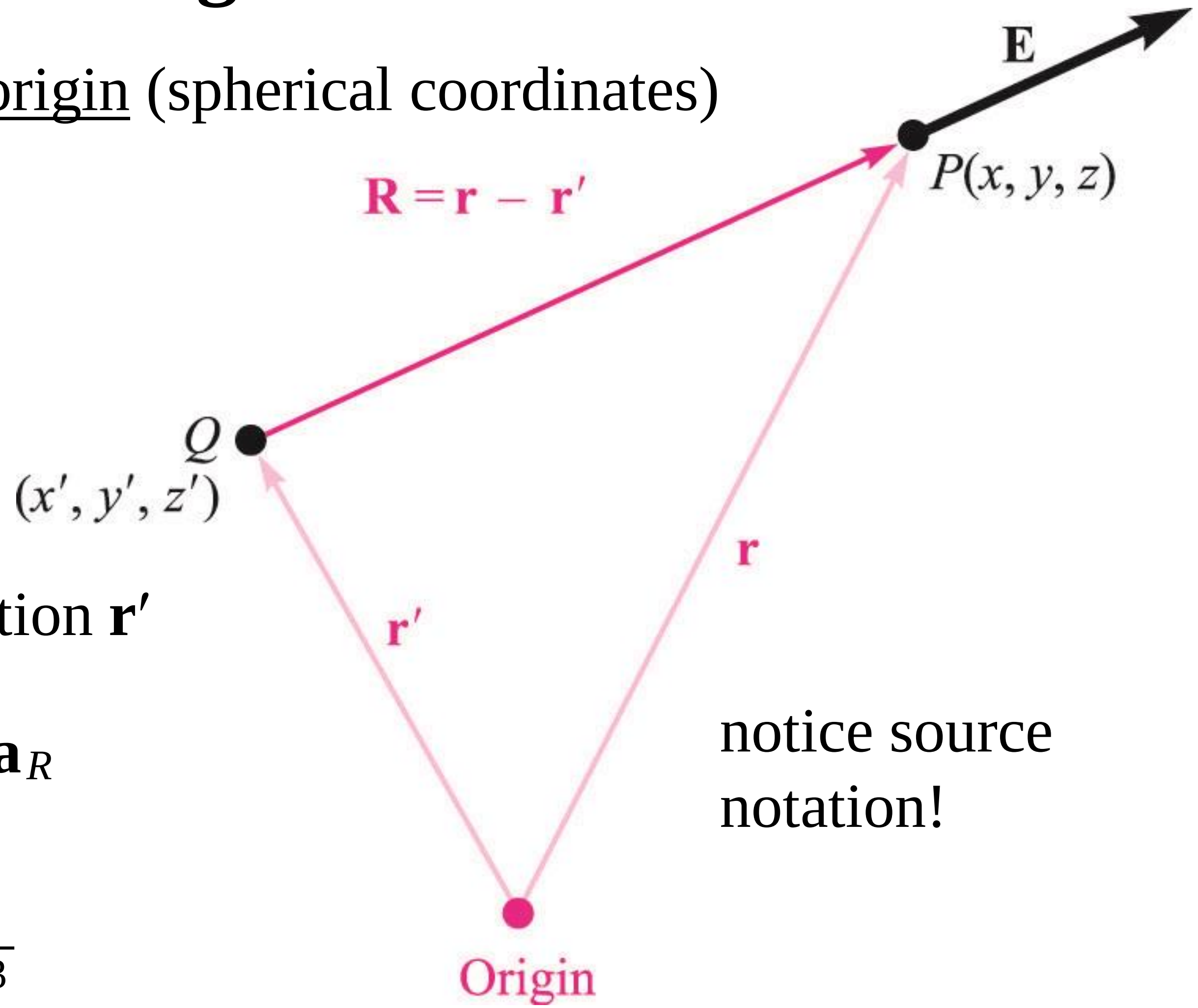
$$\mathbf{E} = \sum_i \mathbf{E}_i$$

this implies that the force affecting a test charge at a point is the vector sum of forces caused by all charges

Field of a Point Charge

a point charge at the origin (spherical coordinates)

$$\mathbf{E} = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\mathbf{a}_r$$



a point charge at position $\mathbf{r}'$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon}\cdot\frac{Q(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\mathbf{a}_R$$

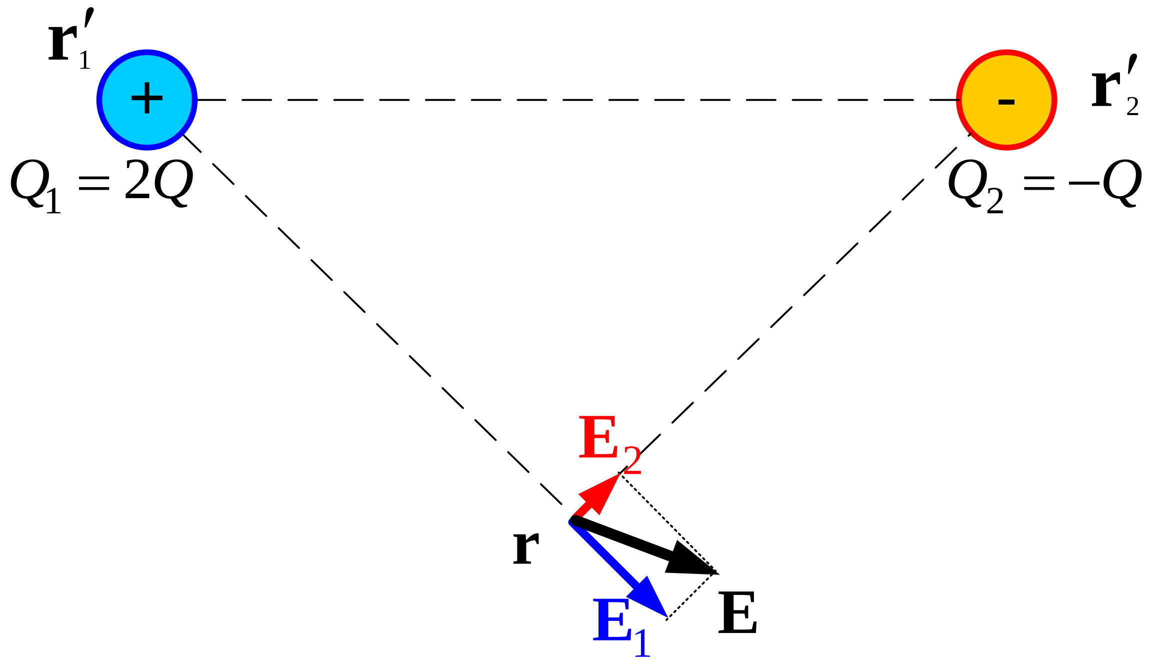
$$\mathbf{E}(\mathbf{r}) = \frac{Q(\mathbf{r}')}{4\pi\varepsilon}\cdot\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

notice source notation!

Field Resulting from Discrete Charges

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon} \cdot \sum_{n=1}^{N} \frac{Q_n(\mathbf{r}_n')}{|\mathbf{r} - \mathbf{r}_n'|^2} \mathbf{a}_n$$

for 2 charges:



Discrete Charges (Cont'd)

$$\mathbf{E}_1(x,y,z)=\frac{Q_1}{4\pi\varepsilon}\cdot\frac{(x-x_1)\mathbf{a}_x+(y-y_1)\mathbf{a}_y+(z-z_1)\mathbf{a}_z}{|(x-x_1)^2+(y-y_1)^2+(z-z_1)^2|^{3/2}}$$

$$\mathbf{E}_2(x,y,z)=\frac{Q_2}{4\pi\varepsilon}\cdot\frac{(x-x_2)\mathbf{a}_x+(y-y_2)\mathbf{a}_y+(z-z_2)\mathbf{a}_z}{|(x-x_2)^2+(y-y_2)^2+(z-z_2)^2|^{3/2}}$$

$$\mathbf{E}(x,y,z)=\mathbf{E}_1(x,y,z)+\mathbf{E}_2(x,y,z)$$