Lecture 9: Electrostatics

Electric Flux Density, Gauss Law, Applications of Gauss Law, **Chapter 4:** pages 126-137

Electric Flux Density D (C/m^2)

the electric field is material-dependent and this makes it not suitable for calculating flux which should be material-independent

$$\mathbf{E} = \frac{1}{4\pi\varepsilon} \cdot \frac{Q}{r^2} \mathbf{a}_r$$

define $\mathbf{D} = \varepsilon\mathbf{E}$ ⇨ $\mathbf{D} = \frac{1}{4\pi} \cdot \frac{Q}{r^2} \mathbf{a}_r$

in vacuum $\mathbf{D} = \varepsilon_0 \mathbf{E}$

the principle of superposition applies to **D** as well

$$\mathbf{D} = \frac{1}{4\pi} \iiint_v \frac{\rho_v \mathbf{a}_R}{R^2} dv$$

Electric Flux Density (Cont'd)

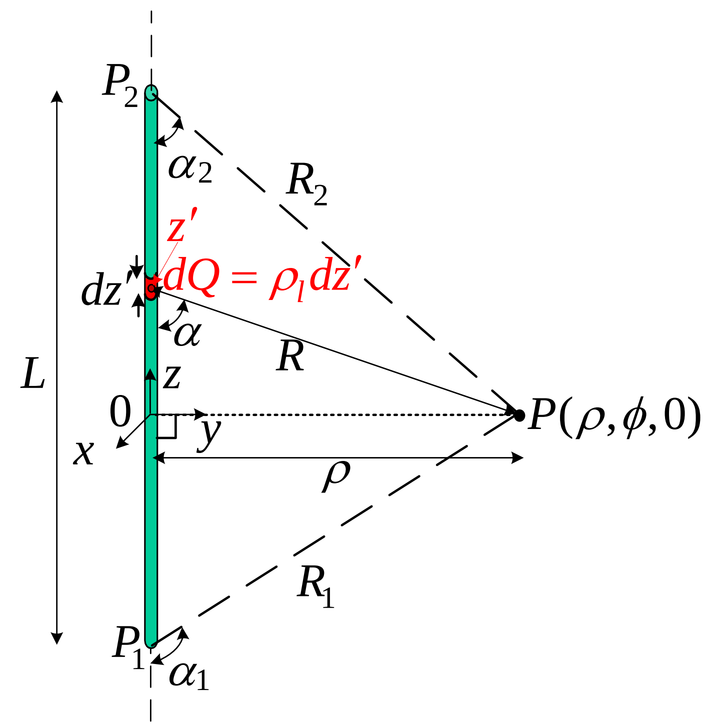
multiple point charges

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon} \cdot \sum_{n=1}^{N} \frac{Q_n(\mathbf{r}_n')}{|\mathbf{r}-\mathbf{r}_n'|^2} \mathbf{a}_n \Rightarrow \mathbf{D}(\mathbf{r}) = \frac{1}{4\pi} \cdot \sum_{n=1}^{N} \frac{Q_n(\mathbf{r}_n')}{|\mathbf{r}-\mathbf{r}_n'|^2} \mathbf{a}_n$$

line charge

$$D_\rho = \frac{\rho_l}{4\pi\rho} \cdot (\cos\alpha_2 - \cos\alpha_1)$$

$$D_z = \frac{\rho_l}{4\pi\rho} \cdot (\sin\alpha_2 - \sin\alpha_1)$$



infinite sheet of uniform charge:

$$\mathbf{D} = \frac{\rho_s}{2} \mathbf{a}_n$$

The Electric Flux (C)

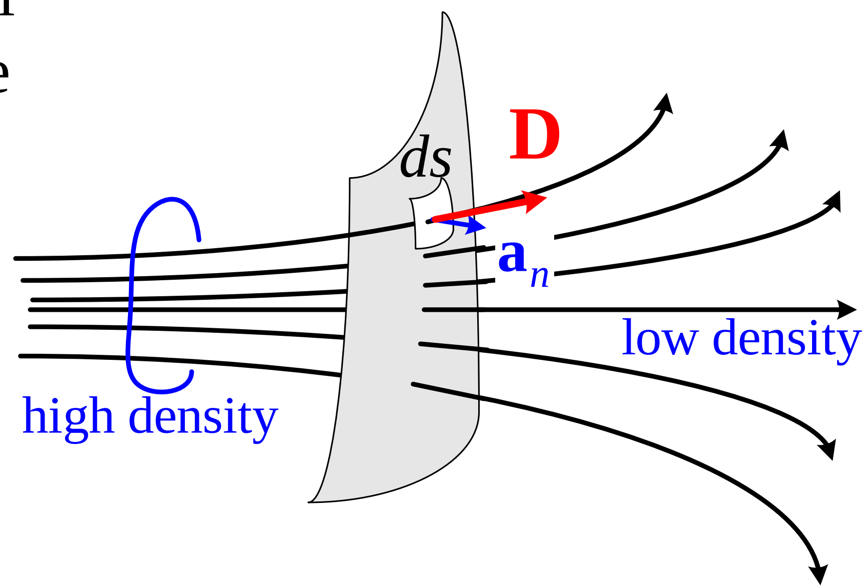
general expression for differential flux through a differential surface

$$d\Psi = \mathbf{D}\cdot d\mathbf{s}$$

the total flux through a surface is given by $\Psi = \iint \mathbf{D}\cdot d\mathbf{s}$

flux through a closed surface

$$\Psi = \oiint_S \mathbf{D}\cdot d\mathbf{s}$$

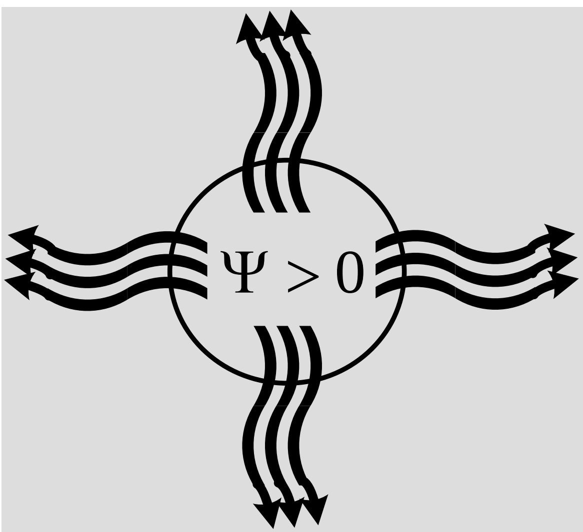


flux lines show the direction and density of the flux

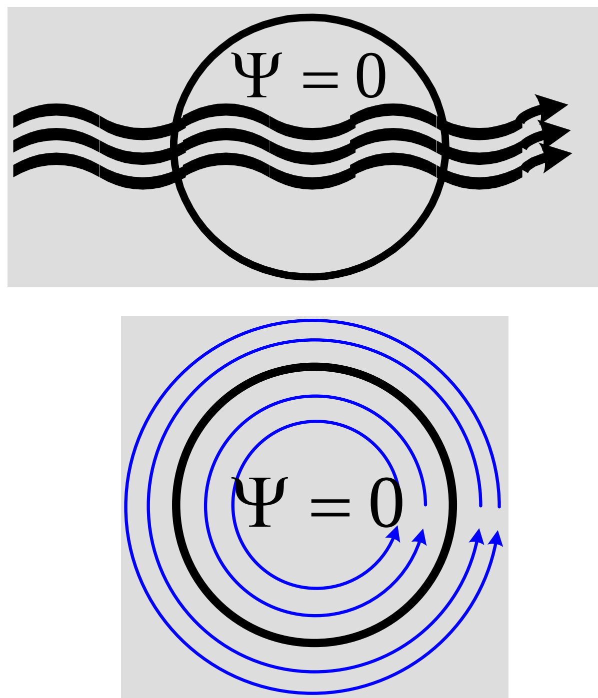
Electric Flux (Cont'd)

electric flux through a closed surface is a measure of the electric sources in the enclosed volume.

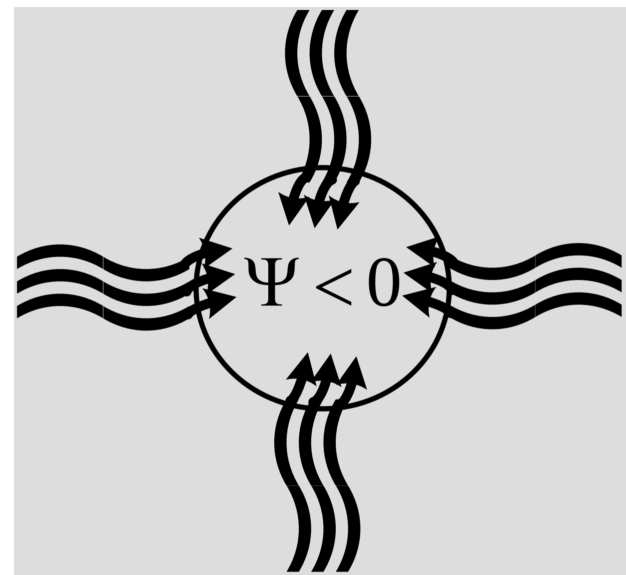
$$\Psi = \oiint_S \mathbf{D} \cdot d\mathbf{s}$$



source (positive flux)



no source (zero net flux)



sink (negative flux)

Gauss' Law

the electric flux through a closed surface is equal to the electric charge enclosed by that surface

$$\Psi = \oiint_S \mathbf{D} \cdot d\mathbf{s} = Q = \iiint_v \rho_v dv \qquad \text{integral form}$$

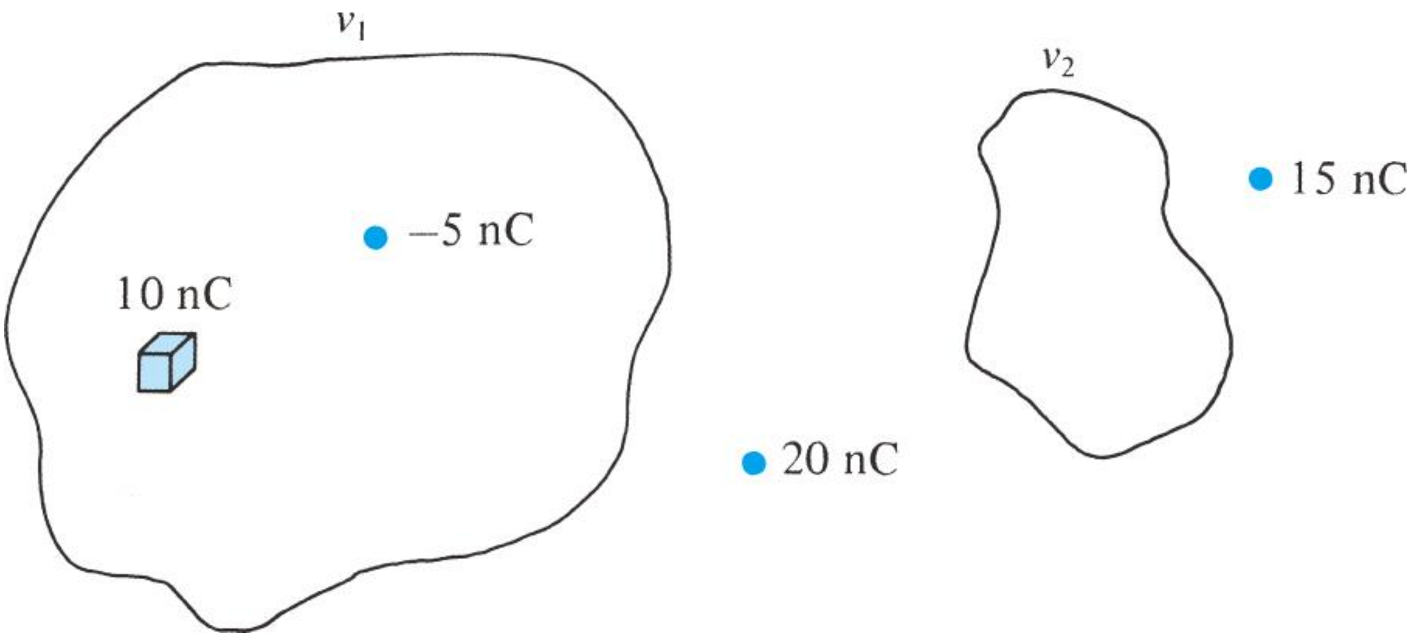
no charge enclosed means no flux

using Divergence theorem, we have

$$\oiint_S \mathbf{D} \cdot d\mathbf{s} = \iiint_v \nabla \bullet \mathbf{D} dv = \iiint_v \rho_v dv$$

$$\nabla \bullet \mathbf{D} = \rho_v \qquad \text{differential form}$$

Cases of Gauss' Law



net electric flux =5.0nC

net electric flux =0

Applications of Gauss Law

Gauss' law makes solutions to problems with planar, cylindrical or spherical symmetry easy

procedure: choose an integration surface so that

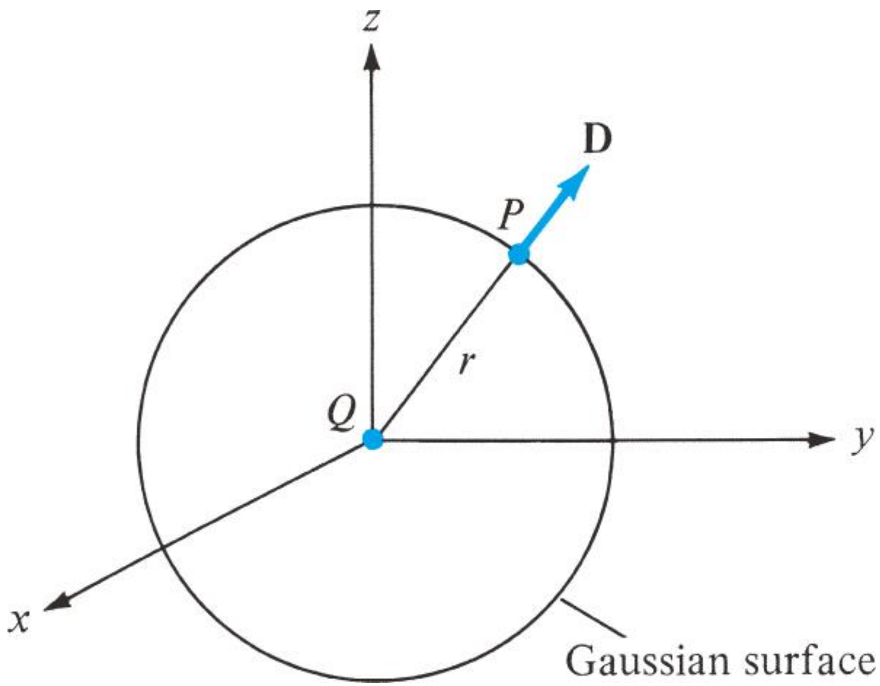
⇨ **D** is everywhere either normal or tangential to surface

$$\text{normal: } \mathbf{D}\cdot d\mathbf{s} = D\,ds; \quad \text{tangential: } \mathbf{D}\cdot d\mathbf{s} = 0$$

⇨ when normal to surface, **D** is also constant on surface

$$\iint_S \mathbf{D}\cdot d\mathbf{s} = \iint_S D\cdot ds = D\cdot S$$

Case Study: A Point Charge



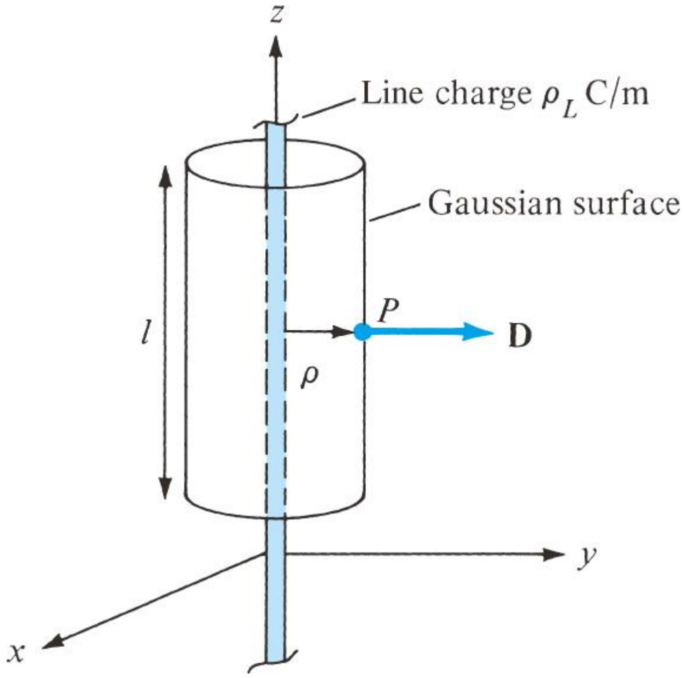
field is in the $\mathbf{a}_r$ direction and depends only on r

select Gaussian surface as a sphere centred at the charge

$$\iint_S \mathbf{D}\cdot d\mathbf{s} = \iint_S D_r \cdot ds = D_r \cdot 4\pi r^2 = Q$$

$$D_r = \frac{Q}{4\pi r^2} \Rightarrow E_r = \frac{Q}{4\pi\varepsilon r^2}$$

Case Study: An Infinite Line Charge



because of symmetry, field is in the $\mathbf{a}_\rho$ Direction and depends only on ρ

choose Gaussian surface as a cylinder

$$\iint_S \mathbf{D}\cdot d\mathbf{s} = \iint_S D_\rho \cdot ds = D_\rho \cdot 2\pi\rho l = \rho_L l$$

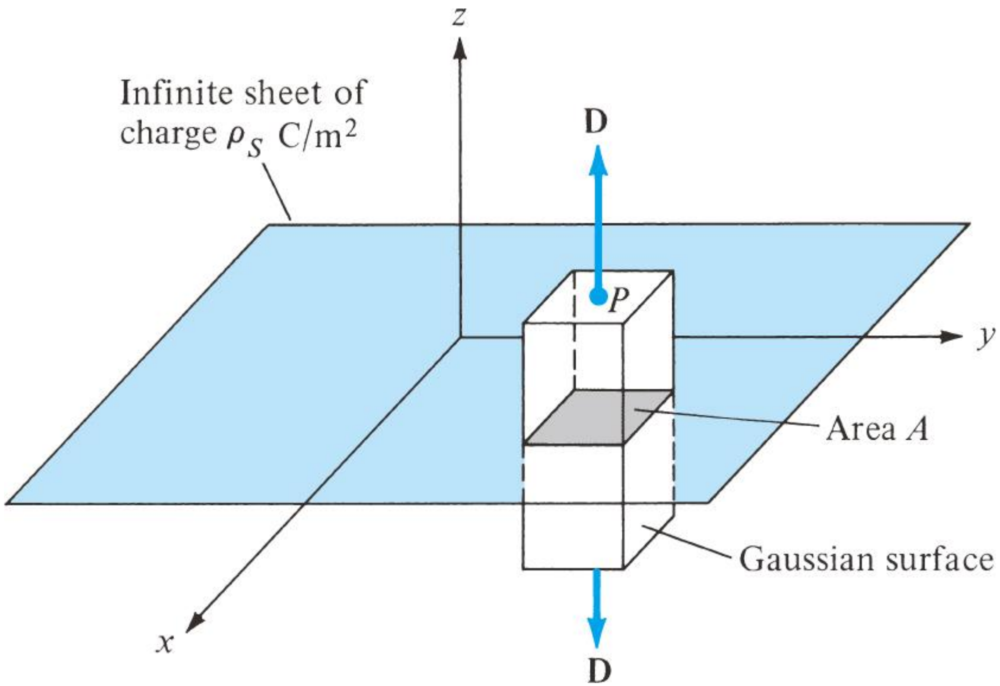
$$D_\rho = \frac{\rho_L}{2\pi\rho} \qquad E_\rho = \frac{\rho_L}{2\pi\varepsilon\rho}$$

due to symmetry, result is obtained in a simple way!

Case Study:

because of symmetry, field is normal to plane and changes only along *z*

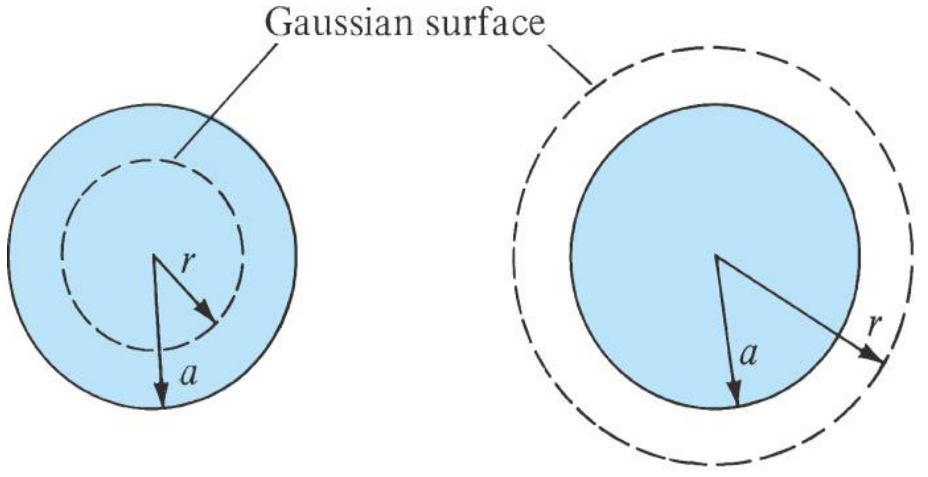
choose Gaussian surface as shown (side integrals cancel)



$$\iint_S \mathbf{D}\cdot d\mathbf{s} = \iint_{top} \mathbf{D}\cdot d\mathbf{s} + \iint_{bottom} \mathbf{D}\cdot d\mathbf{s} = 2\iint_S D_z \cdot ds = 2D_z A = \rho_s A$$

$$D_z = \frac{\rho_s}{2} \Rightarrow E_z = \frac{\rho_s}{2\varepsilon}$$

Case Study: A Uniformly Charged Sphere



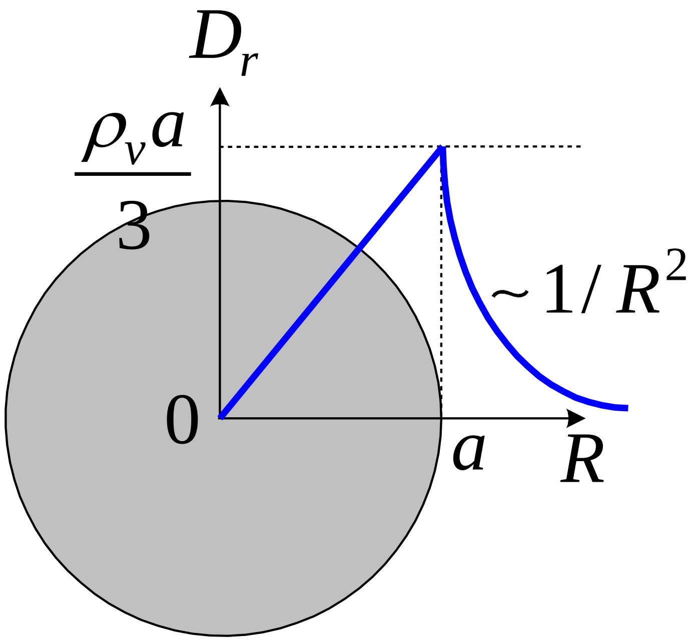
field is in the $\mathbf{a}_r$ direction and depends only on r

select Gaussian surface as a sphere centred at the charge

$$\iint_S \mathbf{D} \cdot d\mathbf{s} = \iint_S D_r \cdot ds = D_r \cdot 4\pi r^2 = \frac{4}{3}\pi r^3 \rho_v, r < a$$

$$D_r = \frac{r}{3}\rho_v, r < a$$

A Uniformly Charged Sphere (Cont'd)



outside the sphere

$$\oiint_{S(r)} \mathbf{D}\cdot d\mathbf{s} = Q = \iiint_{v} \rho_v dv = \rho_v \cdot \frac{4}{3}\pi a^3$$

$$D_r(r)\cdot 4\pi r^2 = Q = \rho_v \frac{4}{3}\pi a^3$$

$$\Rightarrow D_r(r) = \frac{Q}{4\pi r^2} = \frac{\rho_v}{3}\cdot\frac{a^3}{r^2}$$

outside the sphere, the field is identical to a point charge at the centre