1

Question 1

(a) because of symmetry, $\underline{E} = 0$ at the origin

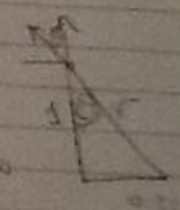
(b) $V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6$

$$= 6V_1 = \frac{6 \times Q}{4\pi\varepsilon_0 r}$$

$$V = \frac{6 \times 2 \times 10^{-9}}{4\pi \times \frac{1}{36\pi} \times 10^{-9} \times 0.25}$$

$$V = 12 \times 9 \times 4 = 432.0\,V$$

(c) $V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6$



$$= \frac{6 \times Q}{4\pi\varepsilon_0 \sqrt{(1)^2 + (0.25)^2}}$$

$$= \frac{6 \cdot 9 \cdot 2}{\sqrt{1^2 + 0.25^2}} = 104.63\,V$$

d) $\theta = \tan^{-1} \frac{0.25}{1} = 14.036$

Because of symmetry only z component of field remain

$$E = \frac{6Q}{4\pi\varepsilon_0 r^2} \cos\theta \; a_z$$

(2)

$$\underline{E} = \frac{6 \times 2.0 \times 10^{-3} \times 0.3702}{4\pi \times \frac{1}{36\pi} \times 10^{-9} \times 1.0625} \, a_z$$

$$\underline{E} = \frac{12 \times 9 \times 0.9702}{1.0625} \, a_z \quad V/m$$

$$\underline{E} = 98.617 \, a_z \, V/m$$

F) $\underline{F} = q\underline{E}$

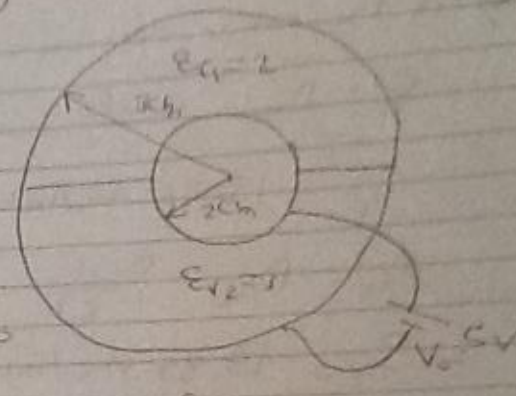
$$= 0.1 \times 10^{-3} \times 98.617$$

$$\underline{F} = 9.8617 \, \hat{a}_z \text{ n Newton}$$

g

③

a) we apply the V-method by solving Laplace equation



$\nabla^2 V = 0$

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$

$\Rightarrow r^2 \frac{\partial V}{\partial r} = C_1 \Rightarrow \frac{\partial V}{\partial r} = \frac{C_1}{r^2}$

$\Rightarrow \frac{\partial V}{\partial r} = -\frac{C_1}{r} + C_2$

at $r = r_1$, $V = V_0$ and at $r = r_2$, $V = 0$

$V_0 = -\frac{C_1}{r_1} + C_2$ — ①

subtracting

$0 = -\frac{C_1}{r_2} + C_2$ — ②

$\Rightarrow C_1\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = V_0 \Rightarrow C_1 = \frac{V_0}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$

$C_1 = \frac{5}{\left(\frac{1}{0.03} - \frac{1}{0.02}\right)} = -0.3$

$C_2 = \frac{C_1}{r_2} = -\frac{0.3}{0.03} = -10$

$\Rightarrow V = \frac{0.3}{r} - 10 \quad 0.02 \le r \le 0.03$

b) $E = -\nabla V = -\frac{\partial V}{\partial r} a_r = \frac{0.3}{r^2} a_r \quad r_1 \le r \le r_2$

(b) $\underline{D}_1 = \varepsilon_1 \underline{E} = 2\varepsilon_0 \underline{E} = \frac{0.6\varepsilon_0}{r^2} a_r \quad r_1 \le r \le r_2, \; 0 \le \theta \le \frac{\pi}{2}$

$\underline{D}_2 = \varepsilon_2 \underline{E} = 3\varepsilon_0 \underline{E} = \frac{0.9\varepsilon_0}{r^2} a_r \quad r_1 \le r \le r_2, \; \frac{\pi}{2} < \theta \le \pi$

line

c) At the inner electrode

$$\rho_{s_1} = D_{n_1} = \frac{0.6\varepsilon_0}{r_1^2} = \frac{0.6 \times \frac{1}{36\pi}\times 10^{-9}}{(0.02)^2} \qquad r = r_1,\ 0 \le \theta \le \frac{\pi}{2}$$

$$\rho_{s_1} = 13.26\ nC/m^2$$

$$\rho_{s_2} = D_{n_2} = \frac{0.9\varepsilon_0}{r_1^2} = \frac{0.9 \times \frac{1}{36\pi}\times 10^{-9}}{(0.02)^2}$$

$$\rho_{s_2} = 19.89\ nC/m^2$$

$$Q = \rho_{s_1} A_1 + \rho_{s_2} A_2 = 2\pi r_1^2 (\rho_{s_1} + \rho_{s_2})$$

$$Q = 2\pi \times (0.02)^2 (13.26 + 19.89) \times 10^{-9}$$

$$Q = 83.315\ pC$$

d) $$C = \frac{Q}{V_0} = \frac{83.315 \times 10^{-12}}{5} = 16.66\ pF$$

e) $$W = \frac{1}{2}\iiint_V \underline{D}\cdot\underline{E}\, dV$$

$$W = \frac{1}{2}\iiint_{V_1} \underline{D}_1 \cdot \underline{E}\, dv + \frac{1}{2}\iiint_{V_2} \underline{D}_1 \cdot \underline{E}\, dv$$

$$W = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi/2}\int_{r_1}^{r_2} \frac{0.18\varepsilon_0}{r^4} \times r^2 \sin\theta\, dr\, d\theta\, d\varphi$$

$$+ \frac{1}{2}\int_0^{2\pi}\int_{\pi/2}^{\pi}\int_{r_1}^{r_2} \frac{0.27\varepsilon_0}{r^4} \times r^2 \sin\theta\, dr\, d\theta\, d\varphi$$

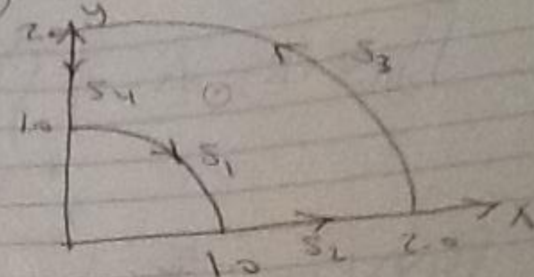
$$W = 0.18\pi\varepsilon_0 \int_{r_1}^{r_2} \frac{1}{r^2}\, dr + 0.27\pi\varepsilon_0 \int_{r_1}^{r_2} \frac{1}{r^2}\, dr$$

$$W = 0.45\pi\varepsilon_0 \left. -\frac{1}{r}\right|_{r_1}^{r_2} = 0.45\pi\varepsilon_0 \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$W = 0.45\pi \times \frac{1}{36\pi} \times 10^{-9} \times 16.667$$

$$W = 0.208\ nJ$$

(5)



Question 3

Stoke's theorem

$$\oint \underline{B} \cdot d\underline{l} = \iint_s (\nabla \times \underline{B}) \cdot d\underline{s}$$

$$\underline{B} = \rho \cos\varphi \, a_\rho + \sin\varphi \, a_\varphi$$

First, we carry out the line integral

$$\oint \underline{B} \cdot d\underline{l} = \int_{S_1} \underline{B} \cdot d\underline{l} + \int_{S_2} \underline{B} \cdot d\underline{l} + \int_{S_3} \underline{B} \cdot d\underline{l} + \int_{S_4} \underline{B} \cdot d\underline{l}$$

over S_1 $\quad d\underline{l} = \rho \, d\varphi \, a_\varphi \; (\rho = 1)$

over S_2 $\quad d\underline{l} = dx \, a_x = d\rho \, a_\rho \; (\varphi = 0)$

over S_3 $\quad d\underline{l} = \rho \, d\varphi \, a_\varphi \; (\rho = 2)$

over S_3 $\quad d\underline{l} = dy \, a_y = d\rho \, a_\rho \; (\varphi = \frac{\pi}{2})$

$$\oint \underline{B} \cdot d\underline{l} = \int_{\pi/2}^{0} \sin\varphi \times 1 \, d\varphi + \int_{1}^{2} \rho \times 1 \times d\rho$$

$$+ \int_{0}^{\pi/2} \sin\varphi \times 2 \, d\varphi + \int_{2}^{0} \rho \times 0 \times d\rho$$

$$\underline{B} \cdot d\underline{l} = -1 + \frac{\rho^2}{2}\Big|_1^2 + 2 = 1 + \frac{3}{2} = \frac{5}{2}$$

$$\nabla \times \underline{B} = \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho B_\varphi) - \frac{1}{\rho} \frac{\partial B_\rho}{\partial \varphi} \right) a_z$$

$$\nabla \times \underline{B} = \left(\frac{\sin\varphi}{\rho} + \sin\varphi \right) a_z$$

⑥

$$\iint(\underline{\nabla}\times\underline{B})\cdot d\underline{s} = \int_{\varphi=0}^{\pi/2}\int_{\rho=1}^{2}\left(\frac{\sin\varphi}{\rho}+\sin\varphi\right)\rho\,d\varphi\,d\rho$$

$$\iint(\underline{\nabla}\times\underline{B})\,d\underline{s} = \frac{5}{2}\times\int_{\varphi=0}^{\pi/2}\sin\varphi\,d\varphi = \ldots$$

$$\iint(\nabla\times\underline{B})\cdot d\underline{s} = \frac{5}{2}$$

$$\Rightarrow \oint \underline{B}\cdot d\underline{l} = \iint_S(\nabla\times\underline{B})\cdot d\underline{s}$$

Question (4) (7)

a) $\underline{D} = 2x^2y\, a_x + 3x^2y^2\, a_y$

$\rho_v = \nabla \cdot \underline{D}$

$= \dfrac{\partial D_x}{\partial x} + \dfrac{\partial D_y}{\partial y}$

$\rho_v = 4xy + 6x^2y \quad C/m^3$

b) $Q = \iiint_V \rho_v \, dv = \int_1^{1.2}\int_1^{1.2}\int_1^{1.2} (4xy + 6x^3y)\, dx\, dy\, dz$

$Q = 0.2 \int_1^{1.2}\int_1^{1.2} (4xy + 6x^2y)\, dx\, dy$

$Q = 0.2 \int_1^{1.2} 2x^2y + 2x^3y \Big|_1^{1.2} dy$

$Q = 0.2 \int_1^{1.2} 2\left((1.2)^2 - (1)^2 + (1.2)^3 - (1)^3\right) y\, dy$

$= 0.2 \times 2 \times 1.168 \int_1^{1.2} y\, dy$

$Q = 0.4672 \times 0.5\left((1.2)^2 - 1\right)$

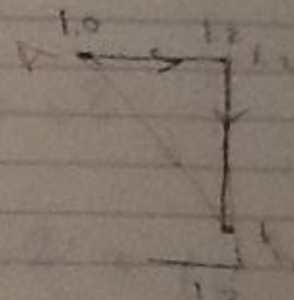
$Q = 0.1027 \; C$

c) $\Psi = Q = 0.1027 \; C$

d) $\underline{E} = \frac{D}{\varepsilon_0} = \frac{2x^2y\,a_x + 3x^2y^2 a_y}{\varepsilon_0}$

$\underline{E} = (72\pi x^2 y\,\underline{a}_x + 108\pi x^2 y^2 \underline{a}_y)\times 10^9$ V/m

e) $V_A - V_B = \int_A^B \underline{E}\cdot \underline{dl}$



$V_A - V_B = \int_A^B E_x\,dx + \int_A^B E_y\,dy$

$V_A - V_B = \int_1^{1.2} 72\pi x^2 (1.2)^2\,dx + \int_{1.2}^{1} 108\pi (1.2)^2 y^2\,dy$

$V_A - V_B = -36\pi \times (1.2)^2 \int_1^{1.2} x^2\,dx$

$V_A - V_B = -12\pi \times (1.2)^2 \left((1.2)^3 - 1\right) = \cancel{}$ -52.68

(9)

$$V = 3x^2 + 6y^2$$

$$\underline{E} = -\nabla V = -\frac{\partial V}{\partial x} a_x - \frac{\partial V}{\partial y} a_y$$

$$\underline{E} = -6x\, a_x - 12y\, a_y \quad \longleftarrow (1)$$

$$\underline{D} = \varepsilon_0 \underline{E} = -6\varepsilon_0 x\, a_x - 6\varepsilon_0 y\, a_y$$

$$W = \frac{1}{2}\iiint (\underline{E}\cdot\underline{D})\, dv$$

$$W = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \varepsilon_0 (36x^2 + 144y^2)\, dx\, dy\, dz$$

$$W = \frac{36}{2}\varepsilon_0 \int_{-1}^{1} x^2\, dx + \frac{144}{2}\varepsilon_0 \int_{-1}^{1} y^2\, dy$$

$$W = 72\,\varepsilon_0 \times 2 \int_{-1}^{1} x^2\, dx$$

$$= 144\,\varepsilon_0 \times \frac{1}{3} x^3 \Big|_{-1}^{1}$$

$$W = 144\,\varepsilon_0 \times \frac{2}{3} = 96\,\varepsilon_0$$

$$= 96 \times \frac{1}{36\pi} \times 10^{-9} = 0.848 \times 10^{-9}\ \text{J}$$