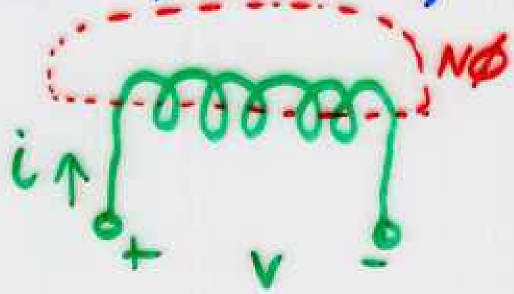
COUPLED INDUCTORS

Recall that inductors store energy in a magnetic field

For example, for a coil of wire with N turns

the total magnetic flux associated with the coil is

$$\phi_t = N\phi = Li \quad , \quad \text{where } i \text{ is the current in the coil}$$



Here we have assumed that the coil has a high permeability core which concentrates the magnetic field

Since

$$N\phi(t) = Li(t)$$

if the current changes, then the flux changes.

but Faraday's Law states that changes in flux induce a voltage (electromagnetic force) which is equal to the rate of change of the total flux.

Hence $$v = \frac{d}{dt}\left(N\phi(t)\right) = L\frac{di}{dt}$$

This concept is called self inductance

* Now we might ask the question: can we generate a magnetic field using current in one circuit, then change that current to produce voltage in a <u>different</u> circuit.

* The answer is Yes! We use coupled inductors, of which the transformer is an example.

* Coupled inductors usually consist of two or more coils wound on a common core

* To explain how they work, we will use the setup in Fig 11.9-1.a)

The voltage source $v_1(t)$ produces a current in loop 1, which is given by

$$i_1(t) = \frac{1}{L_1}\int_{-\infty}^{t} v_1(\lambda)\,d\lambda$$

where L_1 is the self-inductance of the coil in loop 1

The magnetic flux generated in the core by the current in loop 1 is

$$\phi_1(t) = c_1 N_1 i_1(t)$$

where c_1 depends on the geometry + the permeability of the core, and N_1 is the number of turns in the coil

If the coil is tightly wound around the core, and if the permeability of the core is high, then essentially all of the flux remains within the core

$\Rightarrow$ flux in coil 2 = flux in coil 1 = ϕ

If the flux changes, then a voltage is produced across the terminals of the second loop.

$$v_2(t) = N_2 \frac{d\phi}{dt} = c_m N_1 N_2 \frac{di_1}{dt} = M \frac{di_1}{dt}$$

where c_m depends on the magnetic + geometric properties of the core,

$M = c_m N_1 N_2$ is called the Mutual Inductance

it is measured in Henrys

Recall that for the self-inductance case.

$$v_1 = N_1 \frac{d\phi}{dt} \text{ , but } \phi = c_1 N_1 i_1$$

$$\Rightarrow v_1 = c_1 N_1^2 \frac{di_1(t)}{dt}$$

$\Rightarrow$ self-inductance = $c_1 N_1^2$

Note the similarity with mutual inductance

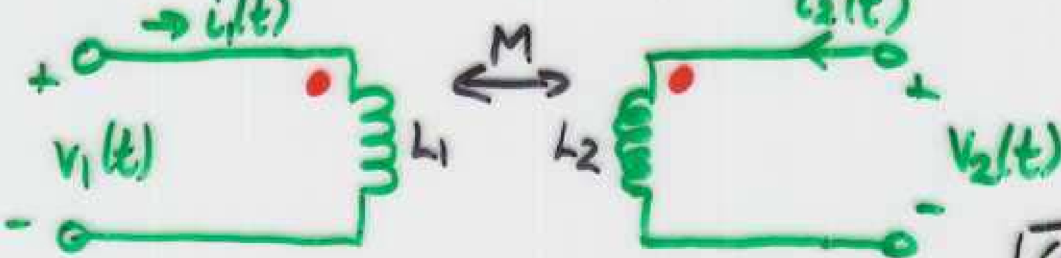
* Hence we have seen that by coupling the flux between two coils, we can generate a voltage in a different circuit by manipulating the voltage in a given circuit.

* What we have not looked at is the sign of the mutually induced voltage

* The sign depends on the direction in which the coils N_1 and N_2 are wound [Fig 11.9-1a, b]

* A dot convention is used to indicate the way in which the coils are wrapped. Terminals with dots are positive at the same time

Consider the following coupled inductor circuit



Coupled Circuit 1

The voltage V_2 = self inductance component + mutual inductance component

$\Rightarrow V_2(t) =$

Similarly $v_1(t) =$

Note that both currents flow in the same direction (ie in or out) of the dotted terminals

For this alternative circuit

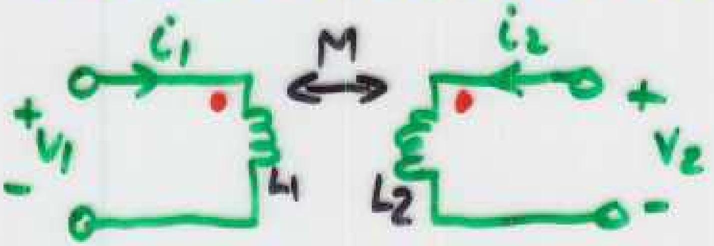
Coupled circuit 2

Current flows into dotted terminal in loop 1, and out of dotted terminal in loop 2.

$\Rightarrow V_2(t) =$

$V_1(t) =$

An alternative model for coupled inductors is



From this model it is clear why coupled inductors are usually limited to non-DC operation

IMPORTANT NOTE

We have assumed here that the magnetic core of the coupler is a linear device. In practice magnetic materials exhibit substantial a non-linearity, commonly known as hysteresis. Hence our models are only valid in limited ranges around a given operating point.