Bode plots (Again).

Most frequency responses are of the form.

$$H(\omega) = \frac{n(\omega)}{d(\omega)}$$

where $n(\omega)$ is a polynomial, so is $d(\omega)$.

Since they are polynomials, they can be written as a product of 1st + 2nd order factors.

$$\text{eg. } n(\omega) = k_n \prod_{i=1}^{L} \left(1 + j\frac{\omega}{\omega_i}\right) \cdot \prod_{i=L+1}^{M} \left(1 + j\alpha_i \frac{\omega}{\omega_i} - \left(\frac{\omega}{\omega_i}\right)^2\right)$$

Do the same for $d(\omega)$, with k_d, ω_{di} and α_{di}

Now the Bode magnitude plot is

$$20\log_{10}(|H(\omega)|) \quad \text{vs} \quad \log_{10}(\omega).$$

But $\log_{10}(|H(\omega)|) = \log_{10}(|n(\omega)|) - \log_{10}(|d(\omega)|)$

this will make things simpler

$$\log_{10}(|n(\omega)|) = \log_{10}(k_n) + \sum_{i=1}^{L} \log_{10}\left(\left|1 + j\frac{\omega}{\omega_i}\right|\right) + \sum_{i=L+1}^{M} \log_{10}\left(\left|1 + j\alpha_i \frac{\omega}{\omega_i} - \left(\frac{\omega}{\omega_i}\right)^2\right|\right)$$

Similarly for $\log_{10}(|d(\omega)|)$

Now let us look at

$$\log_{10}\left(\left|1 + j\frac{\omega}{\omega_i}\right|\right)$$

for $\omega \ll \omega_i$ this is $\approx$

for $\omega \gg \omega_i$ this is $\approx$

Now lets make a big approximation, that.

$$\log_{10}\left(\left|1 + j\frac{\omega}{\omega_i}\right|\right) = \begin{cases} 0 & \text{for } \omega < \omega_i \\ \log_{10}\omega - \log_{10}\omega_i & \text{for } \omega > \omega_i \end{cases}$$

Therefore if

$$20\log_{10}(|n(\omega)|) \approx f(\omega) \quad \text{for} \quad \omega < \omega_i$$

then.

$$20\log_{10}(|n(\omega)|) \approx f(\omega) + 20\log_{10}\omega - 20\log_{10}\omega_i \quad \text{for } \omega > \omega_i$$

Since $20\log_{10}(|H(\omega)|) = 20\log_{10}(|n(\omega)|) - 20\log_{10}(|d(\omega)|)$

Then the zero increases the slope of the Bode plot by 20dB per decade. due to the addition of the $20\log_{10}\omega$ term.

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Now play the same game with a second order term.

$$\log_{10}\left(\left|1 + j\alpha_i\frac{\omega}{\omega_i} - \left(\frac{\omega}{\omega_i}\right)^2\right|\right)$$

a) for $\omega \ll \omega_i$ this is $\approx \log_{10}(1) = 0$

b) for $\omega \gg \omega_i$, this is $\approx \log_{10}\left(\frac{\omega}{\omega_i}\right)^2 = 2\log_{10}\omega - 2\log_{10}\omega_i$

Now if we approximate the actual response by

a) for $\omega < \omega_i$

and b) for $\omega > \omega_i$

~~Therefore~~ if

$$20\log_{10}(|n(\omega)|) \approx f(\omega) \quad \text{for } \omega < \omega_i$$

Then

$$20\log_{10}(|n(\omega)|) \approx f(\omega) + 40\log_{10}\omega - 40\log_{10}(\omega_i) \quad \text{for } \omega > \omega_i$$

Therefore the second-order term increases the slope of $20\log_{10}(|H(\omega)|)$ by 40dB per decade.

The first + second order terms affect the denominator in exactly the same way.

However since

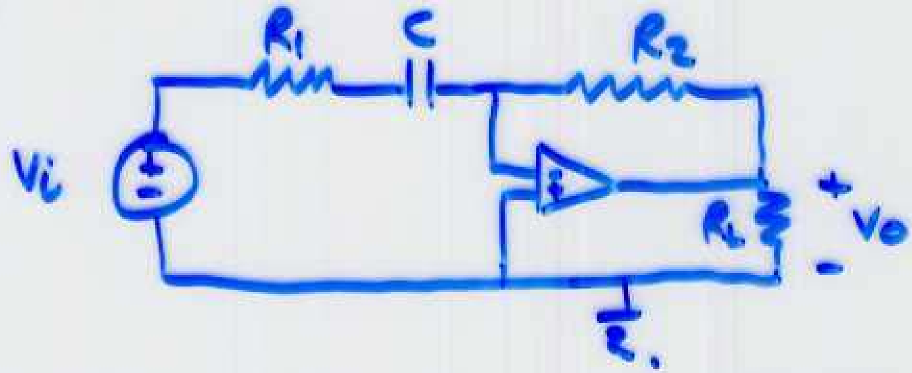
$$20\log_{10}(|H(\omega)|) = 20\log_{10}(|n(\omega)|) - 20\log_{10}(|d(\omega)|)$$

These terms <u>decrease</u> the slope of the bode plot by 20 and 40 dB per decade, respectively #

For previous complicated example.

* Whilst Bode plots simplify analysis, the also lead to simple circuit design methods.

Example.



Design this circuit so that this filter has a Bode plot of the form

Using phasor analysis

$$H(\omega) = \frac{\underline{V_o}}{\underline{V_i}} =$$

Corner freq. (pole) = $\frac{1}{CR_1}$

which we want to equal 500

As ω gets large, $\frac{j\omega}{1+j\omega CR_1} \to \frac{1}{CR_1}$

$\Rightarrow$ we want.

$$20 \log_{10} \left(\frac{R_2}{R_1} \right) = 34.$$

$$\Rightarrow \quad R_2/R_1 = 50$$

Thus our design equations become.

$$\frac{1}{CR_1} = 500$$

$$\frac{R_2}{R_1} = 50$$

More equations than unknowns.

$\Rightarrow$ solution not unique.

If $C = 1\mu F$ is a convenient capacitor then.

$$R_1 = 2k\Omega$$

$$\Rightarrow R_2 = 100k\Omega.$$