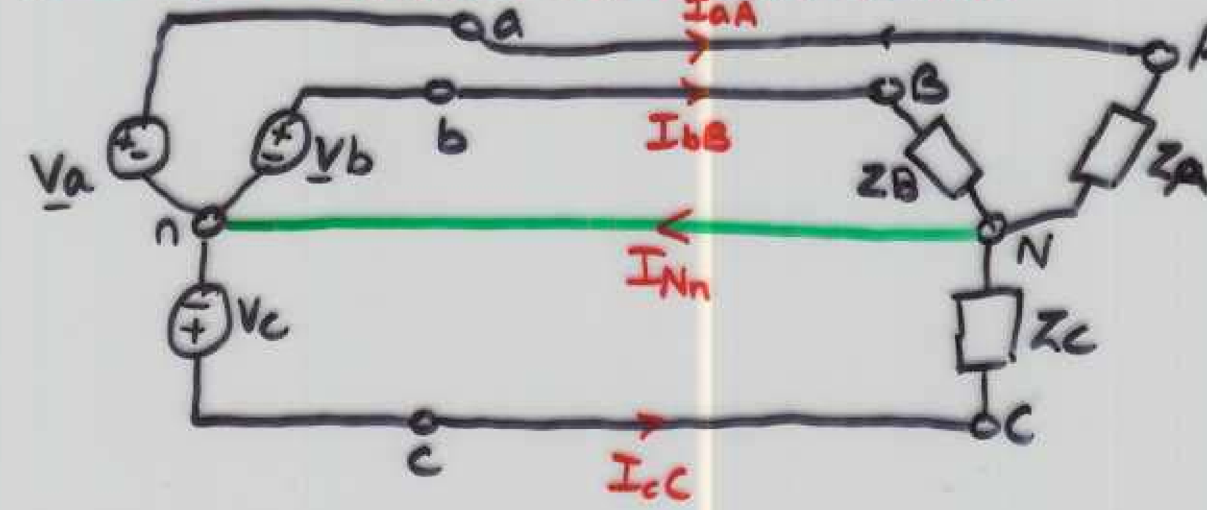
The Y-Y circuit with neutral line



Note - ~~smallcale~~ lowercase for source
upper case for load.

Analyse – using standard phasor analysis.

- each source is connected directly across its load

Hence $I_{aA} = \frac{V_a}{Z_A}$, $I_{bB} = \frac{V_b}{Z_B}$, $I_{cC} = \frac{V_c}{Z_C}$

Now by KCL at Node N

Average power. $P = P_A + P_B + P_C$
is the sum of the powers in each phase

Complex power is calculated the same way.

For example, let $\underline{V_a} = V_p \angle 0$, $\underline{V_b} = V_p \angle -120$
and $\underline{V_c} = V_p \angle -240$

Also, let $\underline{Z_A} = \underline{Z_B} = \underline{Z_C} = Z \angle \theta$

Then.

$$\underline{I_{aA}} = \frac{V_p}{Z} \angle -\theta, \quad \underline{I_{bB}} = \frac{V_p}{Z} \underline{\angle -\theta - 120^\circ}$$

$$\underline{I_{cC}} = \frac{V_p}{Z} \underline{\angle -\theta - 240^\circ}$$

In this case the line currents also have equal magnitudes and are symmetric in phase

* Hence we only have to calculate one of them!
(This is only true for "balanced circuits", with balanced sources and balanced loads)

What is the Neutral current?

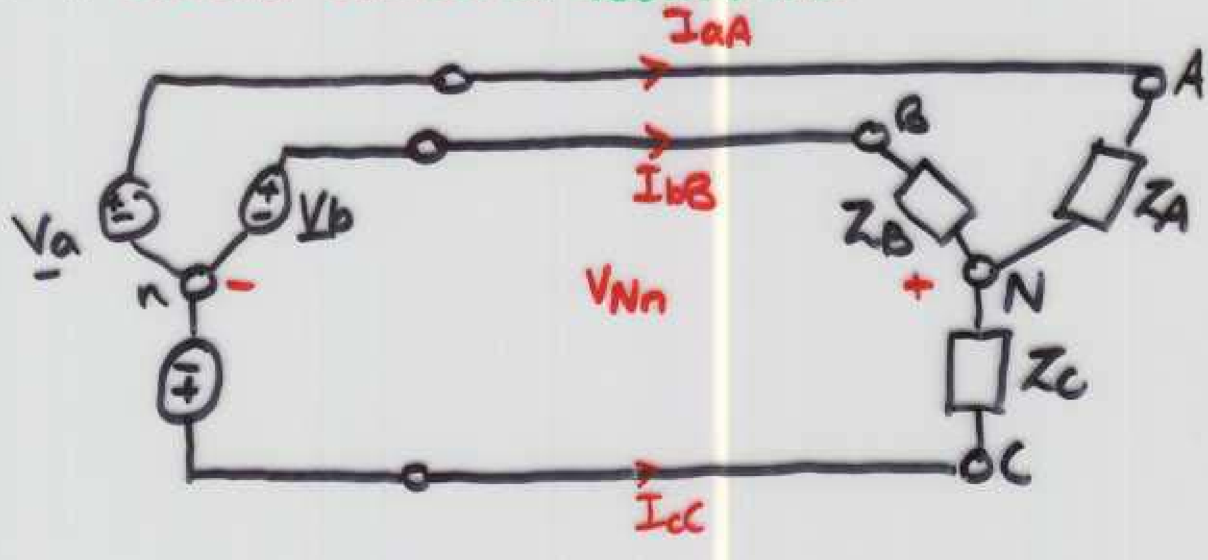
$$I_{Nn} = \frac{V_p}{Z \angle \theta} \left(1 \angle 0^\circ + 1 \angle -120^\circ + 1 \angle -240^\circ \right)$$
$$= 0 \ !$$

Can show that

$$P_A = P_B = P_C = \frac{V_p^2}{2Z} \cos(-\theta)$$

$$\Rightarrow \quad P = \frac{3 V_p^2}{2Z} \cos(-\theta) = \frac{3 V_{A,rms}^2}{Z} \cos(-\theta)$$

Y-Y circuit without a neutral line



We will descibe this circuit in terms of RMS values.

Recall that for a sinusoidal signal of amplitude A, its RMS value is $\frac{A}{\sqrt{2}}$

Let $V_{p,r}$ be the RMS value of the phase voltage

then $\underline{V}_{a,r} = V_{p,r} \angle 0°$, $\underline{V}_{b,r} = V_{p,r} \angle -120°$, $\underline{V}_{c,r} = V_{p,r} \angle -240°$

We will drop the "r" and just assume RMS values.

Note now that $V_N \neq V_n$, thus this circuit is a little more difficult to analyze.
However, principles are the same.

Choose n to be the reference node

KCL at node N

Since $\underline{V}_a = V\angle 0^\circ$, $\underline{V}_b = V\angle -120^\circ$, $\underline{V}_c = V\angle -240^\circ$

$$\underline{V}_{Nn} = \frac{(V\angle -120^\circ) Z_A Z_C + (V\angle 120^\circ) Z_A Z_B + V\angle 0^\circ Z_B Z_C}{Z_A Z_C + Z_A Z_B + Z_B Z_C} \qquad *$$

Once we have $\underline{V}_{Nn}$,

$$\underline{I}_{aA} = \frac{\underline{V}_a - \underline{V}_{Nn}}{Z_A}$$ and similarly for $\underline{I}_{bB}$ and $\underline{I}_{cC}$

Special case: When the load is "balanced" in the sense that

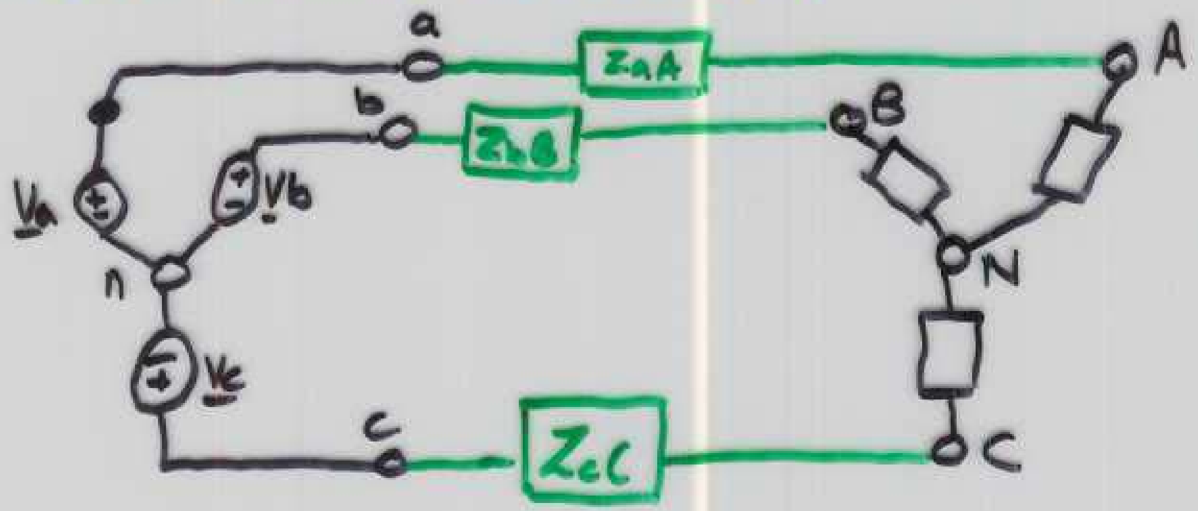
$$Z_A = Z_B = Z_C = Z\angle\theta$$

Substitution into $*$ shows that $\underline{V}_{Nn} = 0$

Hence a balanced Y-Y connection without a neutral line behaves identically to a balanced Y-Y connection with a neutral line

Y-Y circuits with transmission impedances

In the previous examples, the "transmission line" between the load and the source was modelled by ideal conductors. What happens if we want to model them as impedances?



Is this anymore difficult?

No! line impedances are in series with the load. Simply add them!

Note however that differences in Z_{aA}, Z_{bB}, Z_{cC} can make a balanced load appear unbalanced.

Simplifications for balanced circuits

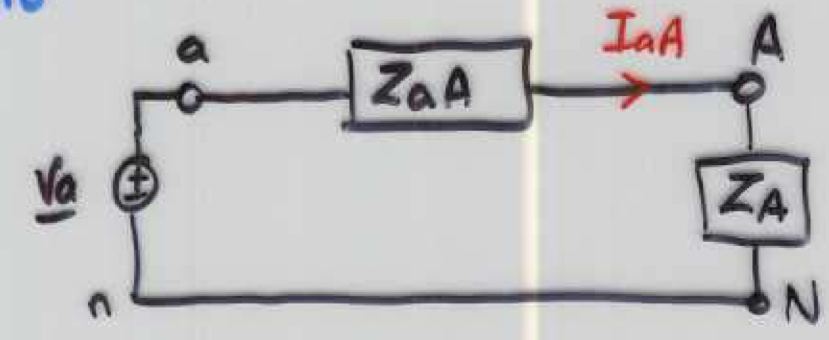
If a Y-Y circuit is "balanced" we have the following simplifications

1. $V_{Nn} = 0$

2. Line currents have same magnitude + phases differing by $120°$

3. Equal power is absorbed by each phase of the load

Hence to solve a balanced Y-Y circuit, we need only consider the following Phase-aA equivalent



EXAMPLES 12.4-1 : 12.4-6

essentially put numbers into the previous analysis

* Remember that if $\underline{V}$ and $\underline{I}$ are the voltage and current phasors for an element, then the complex power is $S = \dfrac{\underline{V}\,\underline{I}^*}{2}$

* If $\underline{V}_{rms}$, $\underline{I}_{rms}$ are the "RMS phasors", which have the same phase as $\underline{V}$ and $\underline{I}$ respectively but have magnitudes reduced by a factor of $\sqrt{2}$ then $S = \underline{V}_{rms}\,\underline{I}^*_{rms}$