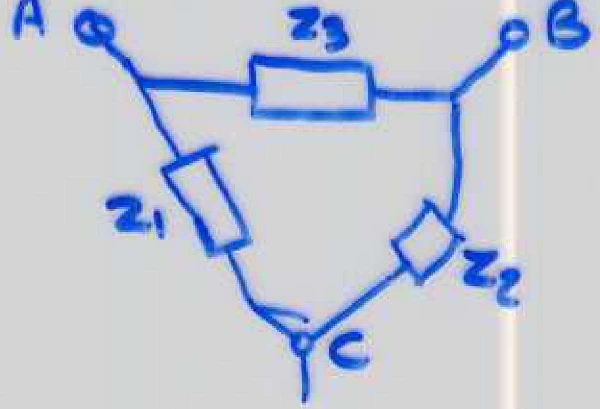
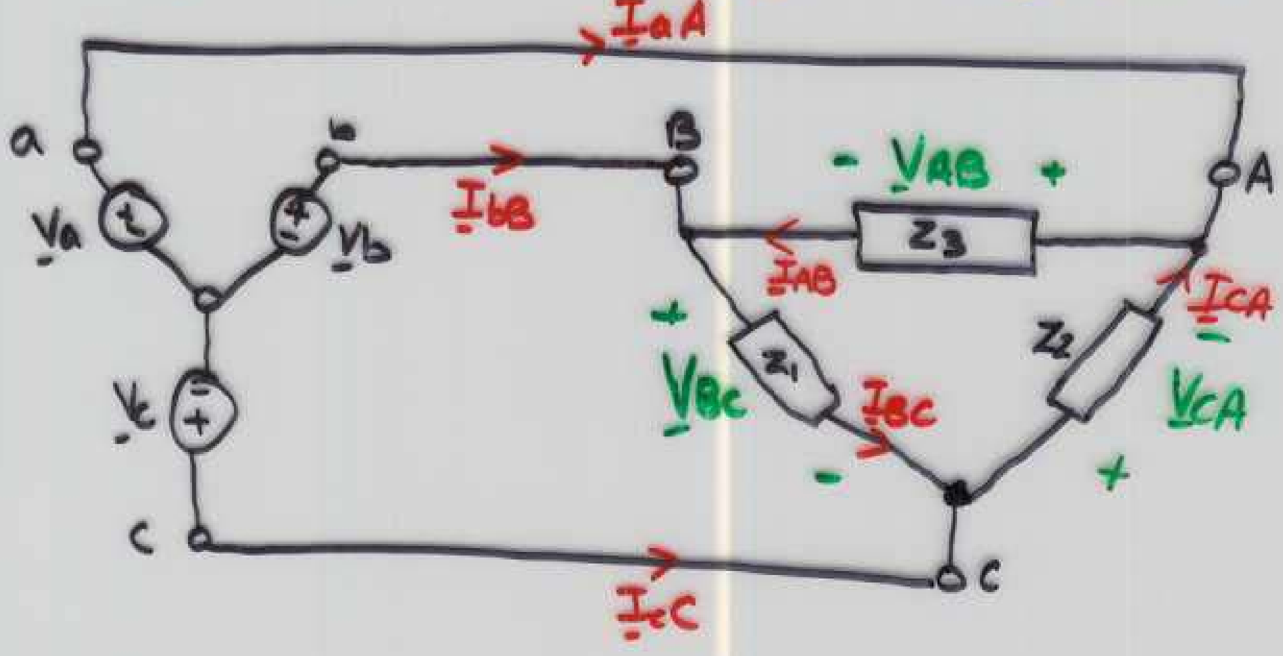
The Y−Δ circuit.

In some applications, the load is more naturally configured as a Δ connection rather than a Y connection.



We can build a "Y-equivalent model" for this circuit, but first we analyze it in general.



KCL at nodes A, B, C.

$$I_{bB} = I_{BC} - I_{AB}$$

$$I_{cC} = I_{CA} - I_{BC}$$

Now $\quad I_{AB} = \dfrac{V_{AB}}{Z_3}$

Also $\quad \underline{V}_{AB} = \underline{V}_a - \underline{V}_b$

If $\quad \underline{V}_a = V_p\angle 0^\circ, \quad \underline{V}_b = V_p\angle -120^\circ$

$$\underline{V}_{AB} = V_p - V_p\cos(-120^\circ) - j\,V_p\sin(-120^\circ)$$

$$= \sqrt{3}\,V_p \angle 30^\circ$$

Now if the load is "balanced", ie $Z_1 = Z_2 = Z_3 = Z\angle\theta$

Then. $\quad \underline{I}_{AB} = \dfrac{\sqrt{3}\,V_p}{Z}\angle 30^\circ - \theta$

Similarly, $\quad \underline{I}_{CA} = \dfrac{\sqrt{3}\,V_p}{Z}\angle 150^\circ - \theta$

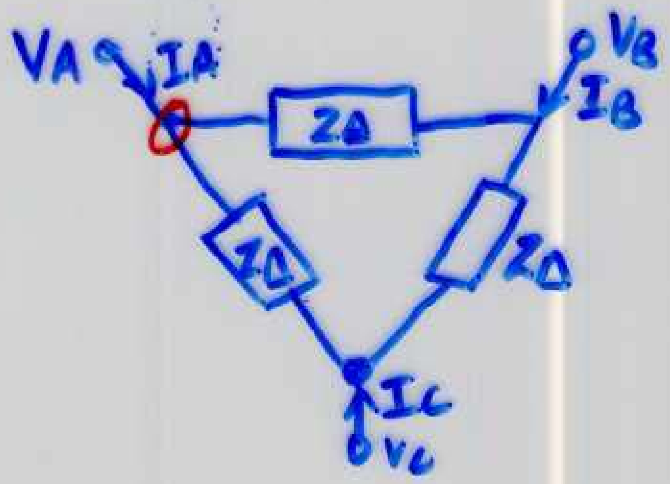
$$\underline{I}_{BC} = \frac{\sqrt{3}\,V_p}{Z}\angle -90^\circ - \theta$$

* We can now find the line currents I_{aA}, etc using the KCL equations at nodes A, B and C

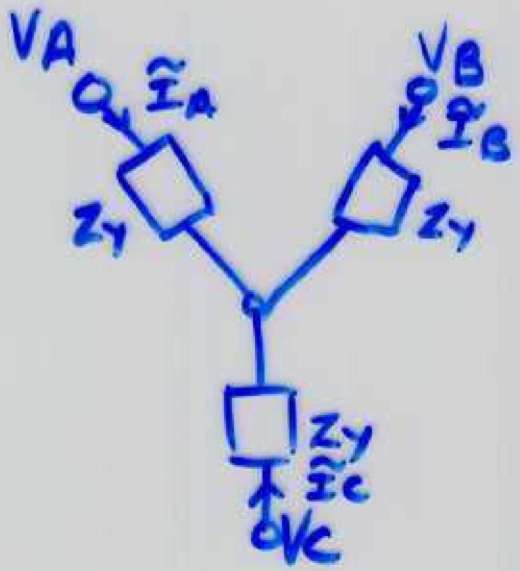
* However, even when the Δ-load is balanced, this is much more difficult than dealing with a Y-Y circuit
* Wouldn't it be nice if we could calculate the line currents by first transforming the Δ-load to an equivalent Y-load, and then using Y-Y analysis.

* This can be done for both balanced and unbalanced loads. We will only look at the balanced case.

* The question is, given



Can we find a Z_Y



such that
$\tilde{I}_A = I_A$
$\tilde{I}_B = I_B$
$\tilde{I}_C = \tilde{I}_C$.

The answer is Yes!
In fact its enough just to consider $\tilde{I}_A$ and I_A.

From Y circuit, $\tilde{I}_A = \dfrac{V_A}{Z_Y}$ ①

From Δ circuit, $I_A = I_{AB} - I_{CA}$ ②

If the voltages are balanced,
ie $\underline{V}_A = V_p \angle 0$, $\underline{V}_B = V_p \angle -120$, $\underline{V}_C = V_p \angle -240$

Then for ① = ② we require

$$\frac{V_A}{Z_Y} = \frac{\underline{V}_A - \underline{V}_B}{Z_\Delta} - \frac{(\underline{V}_C - \underline{V}_A)}{Z_\Delta}$$

$$\Rightarrow \frac{V_A}{Z_Y} = \frac{\sqrt{3}\,V_p \angle 30 - \sqrt{3}\,V_p \angle -210}{Z_\Delta}$$

$$\Rightarrow Z_Y = \frac{V_A Z_\Delta}{\sqrt{3}\,V_p (1\angle 30 - 1\angle -210)}$$

$$= \frac{V_p \angle 0 \, Z_\Delta}{\sqrt{3}\,V_p \left(\cos 30 + j \sin 30 - (\cos(-210) + j \sin(-210))\right)}$$

$$= \frac{Z_\Delta}{\sqrt{3} \cdot 2 \cdot \cos(30^\circ)}$$

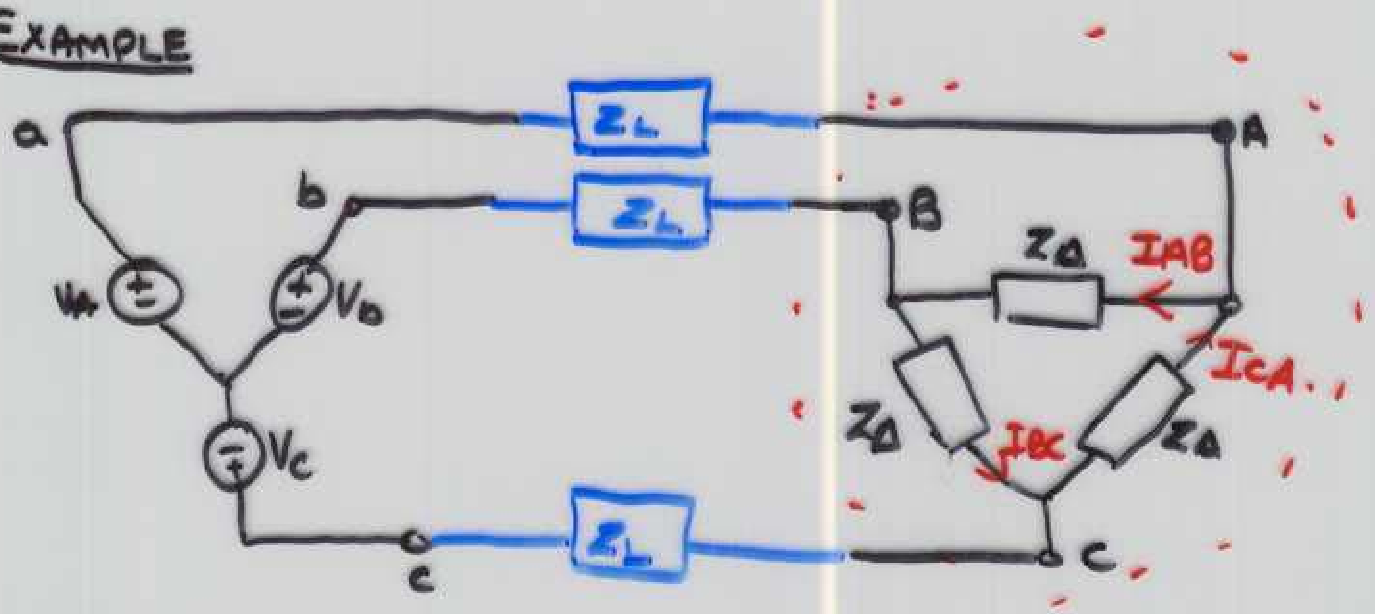
$$= \frac{Z_\Delta}{3}$$

This is the $\Delta \Rightarrow Y$ transformation for balanced loads

This makes it easy to calculate the line currents. I_{aA}, I_{bB}, I_{cC}

To calculate I_{AB}, I_{BC}, I_{CA}, we must then return to the Δ model

EXAMPLE



The RMS values of the source voltages V_a, V_b, V_c are $110\angle 0$, $110\angle -120$ and $110\angle -240$ respectively

If $Z_L = 10 + j5\,\Omega$

and $Z_\Delta = 75 + j\,225\,\Omega$

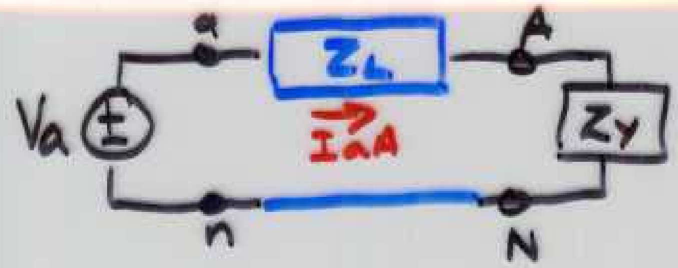
Find I_{AB}, I_{BC} and I_{CA}

SOLUTION

STEP 1 Do A Δ-Y transformation

The equivalent $Z_Y = 25 + j\,75\,\Omega$

STEP 2 PER Phase equivalent circuit for phase aA



$$I_{aA} = \boxed{} = \frac{110\angle 0^\circ}{10+j5+25+j75} = 1.26\angle{-66^\circ} \text{ A rms}$$

The other line currents have the same magnitude but phase differences of 120° and 240°

$$\Rightarrow I_{bB} = 1.26\angle{-186^\circ} \text{ A rms}$$

$$I_{cC} = 1.26\angle 54^\circ \text{ A rms}$$

Hence the load voltage

$$V_{AN} = I_{aA} Z_Y = 99.6\angle 5^\circ \text{ Vrms}$$

Similarly $V_{BN} = 99.6\angle{-115^\circ}$

$$V_{CN} = 99.6\angle 125^\circ$$

Hence the line to line voltages are

$$V_{AB} = V_{AN} - V_{BN} = 172\angle 35^\circ \text{ Vrms}$$

$$V_{BC} = V_{BN} - V_{CN} = 172\angle{-85^\circ} \text{ Vrms}$$

$$V_{CA} = V_{CN} - V_{AN} = 172\angle 155^\circ \text{ Vrms}$$

Therefore the currents in the Δ load are

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = 0.727 \angle -36^\circ \text{ Arms}$$

$$I_{BC} = 0.727 \angle -156^\circ \text{ Arms}$$

$$I_{CA} = 0.727 \angle 84^\circ \text{ Arms.}$$

Now that we have voltages + currents, powers are easy to calculate.

Suggested Problems

12-3: 3

12-4: 1-4, 6

12-6: 1, 2

12-7: 1, 2

12-8: 1, 3, 5, 7