STABILITY

* In most applications it is desirable that the circuit be "stable"

  that is, for every bounded input signal, the output signal is also bounded.

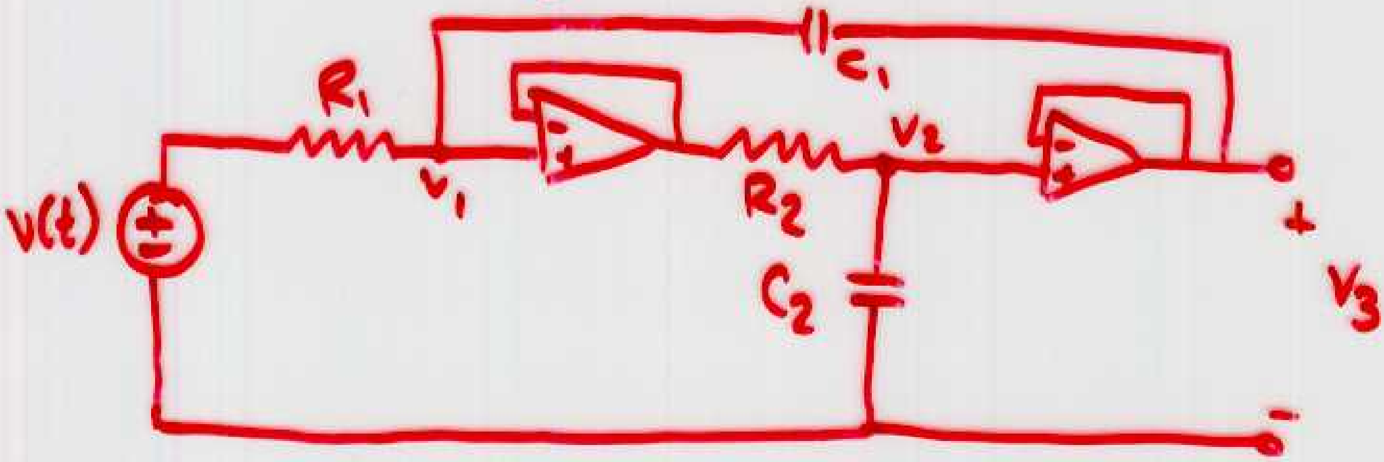
* A nice property of the Laplace Transform is that it immediately tells us whether or not the circuit is stable, we don't have to go away and test every input signal!

Theorem.

1) If the real part of each poles is negative, circuit is stable
2) If the real part of each poles is positive, circuit is unstable
3) Poles on the "$j\omega$" axis are sometimes said to be "quazi-stable" but can be considered unstable in practice

EXAMPLE

Is the following circuit stable?



$$H(s) = \frac{V_3(s)}{V(s)} = \frac{\omega_0^2}{s^2 + \left(\frac{1}{R_1C_1}\right)s + \omega_0^2}$$

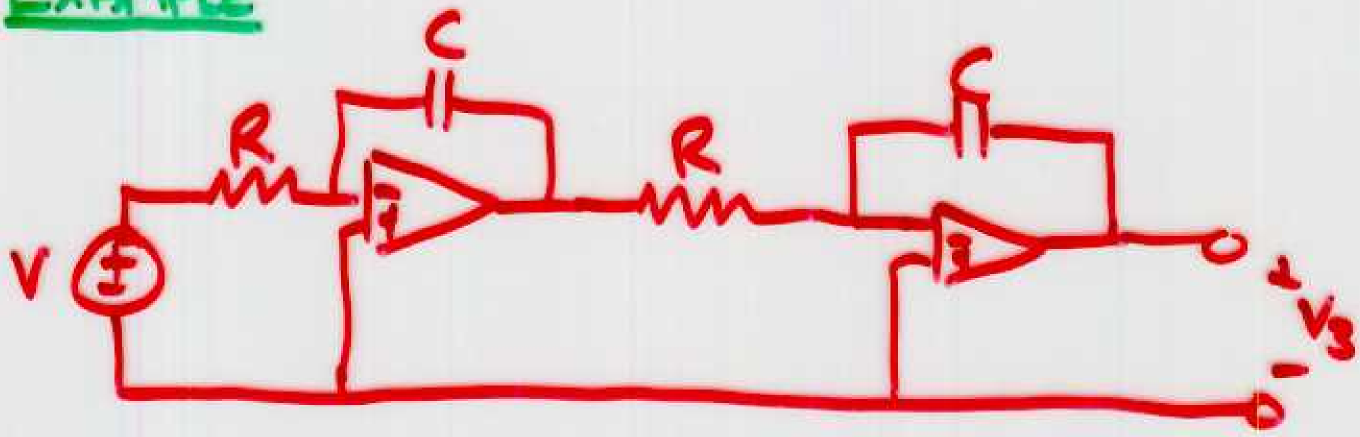
where $\omega_0^2 = \dfrac{1}{R_1R_2C_1C_2}$

Real part of roots $= \dfrac{-1}{2R_1C_1}$

which is always < 0 for positive R_1, C_1.

Hence circuit is stable.

EXAMPLE



$$H(s) = \frac{V_3(s)}{V(s)} = \frac{1}{(RC)^2 s^2}$$

$\Rightarrow$ two poles at origin

$\Rightarrow$ unstable

Example.

$$\text{step response} = \mathcal{L}^{-1}\left\{ \frac{H(s)}{s} \right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{(RC)^2 s^3} \right\}$$

$$= \frac{1}{(RC)^2\, 3!}\, t^3$$

Note that this gets large as $t \to \infty$

IMPORTANT NOTE

Frequency response/phasor analysis only applies to stable circuits, but stability can only be determined from the Laplace Transform.