SECOND-ORDER OSCILLATOR CIRCUITS.

* These circuits are a special sort of quasi-stable circuit which is useful in practice.

* The general form of a second-order transfer function is

$$H(s) = \frac{N(s)}{s^2 + a_1 s + a_0}$$

$$= \frac{N(s)}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$= \frac{N(s)}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

* The poles of this circuit are at.

$$s = -\zeta\omega_0 \pm j\sqrt{1-\zeta^2}\,\omega_0$$

* As $\zeta \to 0$, these poles approach the $j\omega$-axis

* Now look at a special case, where $N(s) = \omega_0^2$.

* The step response of this system is

$$\mathcal{L}^{-1}\left\{ \frac{H(s)}{s} \right\}$$

$$= \mathcal{L}^{-1}\left\{ \frac{1}{s} + \frac{s+d}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \right\}$$

where. $d = \begin{cases} \frac{1}{2\zeta\omega_0}, & \zeta > 0 \\ 0 & \zeta = 0 \end{cases}$

* hence the step response is.

$$\mathcal{L}^{-1}\left\{ \frac{1}{s} + \frac{s+\zeta\omega_0}{(s+\zeta\omega_0)^2 + (1-\zeta^2)\omega_0^2} + \frac{d-\zeta\omega_0}{(s+\zeta\omega_0)^2 + (1-\zeta^2)\omega_0^2} \right\}$$

$$= \Big[1 + e^{-\zeta\omega_0 t}\cos\left(\sqrt{1-\zeta^2}\,\omega_0 t\right) + \frac{d-\zeta\omega_0}{\sqrt{1-\zeta^2}\,\omega_0} e^{-\zeta\omega_0 t} \times \sin\left(\sqrt{1-\zeta^2}\,\omega_0 t\right) \Big] u(t)$$

Writing that again,

$$\text{step response} = u(t) + e^{-\alpha t}\cos(\beta t)\,u(t) + \frac{\alpha - \zeta\omega_0}{\beta} e^{-\alpha t}\sin(\beta t)\,u(t)$$

where $\alpha = \zeta\omega_0$

and $\beta = \sqrt{1-\zeta^2}\,\omega_0$.

So, as $\zeta \to 0$

$$\text{step response} \to [1 + \cos(\omega_0 t)]\,u(t)$$

That is, the circuit generates a cosine wave that does not decay!

In practice there will be some decay, as the term ζ never quite gets to zero, but this is the key to generating sine waves in practice

Questions for Laplace Transform Chapter

14.3 - 4

14.4 - 1, 2

14.5 - 1, 3, 4, 6

14.6 - 2-4

14.7 - 4, 5

14.8 - 3-6

14.9 - 1-7, 10, 12, 13, 17(a, c)

14.11 - 1, 2, 4 (first part)