MAGNETIC FIELDS AND CIRCUITS.

Magnetism : the key to electro-mechanical energy conversion

Mechanical → Electrical = generation

Electrical → Mechanical , eg. motors

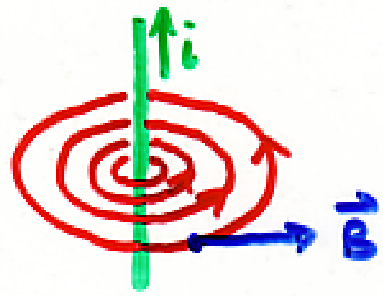
Magnetic Fields

- created by moving charges, eg.
  - current in a wire or coil of wire
  - permanent magnets – electron spin allignment

- Also exert a force on moving charges.

Visualization

- typically via flux lines from Nth seeking to Sth seeking
- spacing reflects intensity of field
- opposites attract



- Algebraically field quantified by a vector $\vec{B}$, called magnetic flux density, in units of webers/m², $Wb\,m^{-2}$
- Direction of $\vec{B}$ is tangent to field
- Magnitude is magnitude of the field strength

Right Hand Rule

- given current in a wire or coil, in which direction is the field?
- use right hand rule.
- for straight wires: thumb in current direction $\Rightarrow$ fingers in field direction
- coils : fingers in current direction $\Rightarrow$ thumb in field direction

vector products.

Dot product

$$\vec{v} \cdot \vec{u} = vu\cos(\theta).$$

where, v, u are lengths of $\vec{v}$ and $\vec{u}$
θ is the angle between $\vec{v}$ and $\vec{u}$
answer is a scalar.

Cross product

$$\vec{v} \times \vec{u} = vu\sin(\theta)\,\vec{p}$$

where $\vec{p}$ is perp. to plane of $\vec{v}$ and $\vec{u}$
direction is via right hand rule.
fingers point from $\vec{v}$ to $\vec{u}$, thumb is direction of $\vec{p}$

Forces on moving charge

A charge q moving with velocity $\vec{u}$ in a magnetic field with flux density $\vec{B}$ experiences a force $\vec{f}$

$$\vec{f} = q\,\vec{u} \times \vec{B}$$

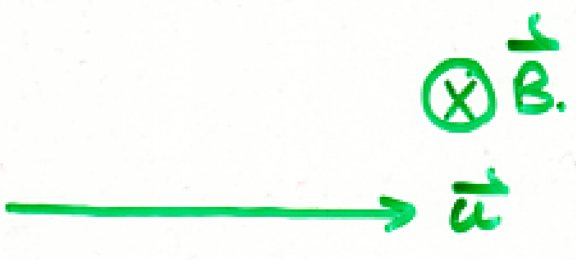
Magnitude of f is $\quad q\,u\,B\sin(\theta)$

Since $\quad u$ ~~sin~~ is in ms^{-1}
$\qquad f$ is in Newtons.
$\qquad q$ is in Coulombs.

$\Rightarrow$ B has units $\dfrac{Ns}{Cm}$. called Tesla.

Example.

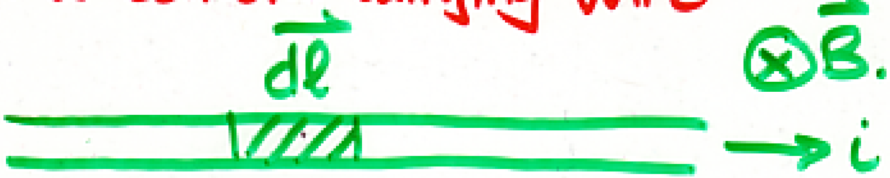
Field into page, charge moving horizontally



In which direction does the charge feel force.

(up the page)

Force on a current carrying wire



Look at a small length, $\vec{d\ell}$

$$d\vec{f} = q\,\vec{u} \times \vec{B}$$

$$q\,\vec{u} = \text{charge} \times \text{velocity} = C \times ms^{-1}$$
$$= Cs^{-1} \times m.$$
$$= i\,\vec{d\ell}$$

$$\Rightarrow \quad d\vec{f} = i\,\vec{d\ell} \times \vec{B}$$

Now integrate to find total $\vec{f}$

This integration is straight forward if the wire is straight and the field constant.

Magnitude: $\quad f = i\,\ell\,B\sin(\theta)$

where ℓ is the total length. (assumed straight)

B is the magnitude of the field (assumed constant)

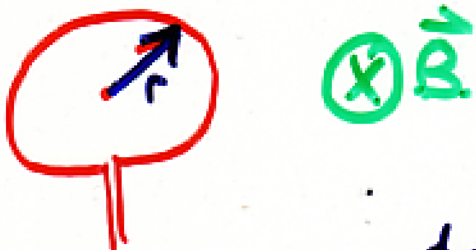
θ is the angle between the field and the wire

Flux Linkage

- $\vec{B}$ is a flux density, what is the total flux?
- integrate over an area.

$$\phi = \int_A \vec{B} \cdot d\vec{A}$$

- Surface integral
- $d\vec{A}$ is an increment of area. direction is normal to the surface.
- If $\vec{B}$ is constant and perp. to ~~area~~ surface, $\phi = B \cdot A$



$$\phi = \pi r^2 B.$$

- what happens if there are N identical loops and flux remains constant?

flux linkage, $\lambda = N\phi$.

- Changes in flux linkage are a key component in the analysis + design of electro-mechanical systems

Faraday's Law.

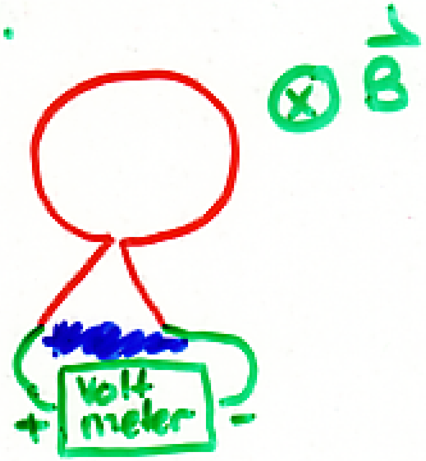
If the flux linkage changes, either because $\vec{B}$ changes or because the coil moves, we get a voltage at the terminals of the coil.

$$e = \frac{d\lambda}{dt}$$

Lenz's Law. (in which direction is e ?).

· the voltage would induce a current in the loop which would generate a magnetic field in the opposite direction to the change

Example.

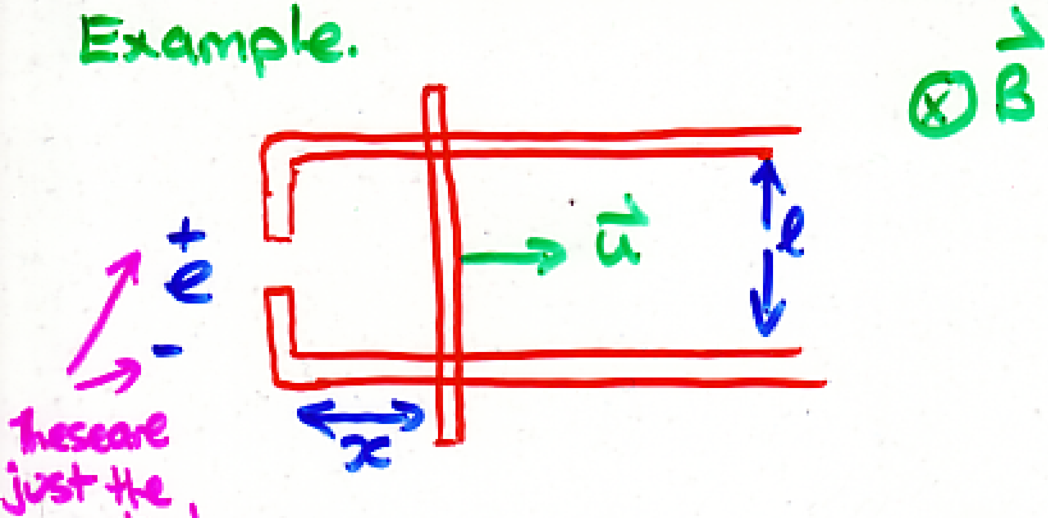


$\vec{B}$ is into page, and increasing

is the reading on the meter positive or negative?

- To oppose the change, induced magnetic field must be out of the page.
- By right hand rule $\Rightarrow$ current is anticlockwise
- $\Rightarrow$ voltage is positive

Example.



Find e.

Flux linkage, $\lambda = \phi = BA$ ($\vec{B}$ and $\vec{u}$ are $\perp$)
$= B\ell x$.

$$|e| = \frac{d\lambda}{dt} = B\ell \frac{dx}{dt} = B\ell u$$

Area is increasing $\Rightarrow$ flux is increasing

$\Rightarrow$ induced magnetic field must oppose change

$\Rightarrow$ must be out of page

$\Rightarrow$ current must be anti clockwise

$\Rightarrow$ e is positive for the given convention

Magnetic Field Intensity

- In many applics. magnetic fields are induced by currents
- In this case it is convenient to introduce $\vec{H}$, the magnetic field intensity
- $\vec{H}$ depends only on currents and wire geometry
- $\vec{B}$ depends on $\vec{H}$ and properties of the material
- If material is homogeneous and linear

$$\vec{B} = \mu \vec{H}$$

μ called the permeability

- In free space, $\mu = 4\pi \times 10^{-7}$ Wb/Am.
- For magnetic materials, such as iron, μ is 1000's of times bigger

EXAMPLE

- A long straight wire carries a current i out of the page
- Find $\vec{H}$ and $\vec{B}$ assuming μ is constant

Solution :

- Fields are constant on circles of radius r.
- Choose to perform $\oint \vec{H} \cdot d\vec{\ell}$ along one of these circles

$\Leftrightarrow \quad H \cdot 2\pi r = i$

$\Rightarrow \quad H = \dfrac{i}{2\pi r}$, Direction is tangent to anticlockwise circle (Right hand rule)

$\Rightarrow \quad B = \dfrac{\mu i}{2\pi r}$

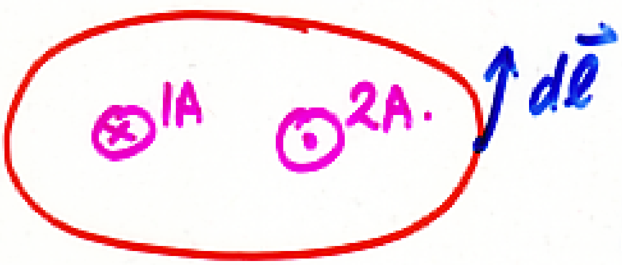
Ampere's Law.

- This is the key to computing $\vec{H}$
- Consider a closed path of incremental length $\vec{dl}$ then.

$$\oint_{path} \vec{H} \cdot \vec{dl} = \underbrace{\sum_k i_k}_{\text{sum of currents flowing through the area.}}$$

- Each current on RHS has a sign with respect to a reference direction
- Ref. direction is induced by right hand rule and direction of path; typically direction is anti-clockwise $\Rightarrow$ ref. dir. is out of page.

Example: Two conductors



$$\oint \vec{H} \cdot \vec{dl} = \sum_k i_k = 2 - 1 = 1$$

NOTE: if $\vec{H}$ can be shown to be constant on the path, then.

$$Hl = \sum_k i_k.$$

EXAMPLE: FLUX DENSITY IN A TORROID

N uniformly spaced turns on a bagel shaped core

- Radius of core centre is R
- Radius of core cross-section is r
- assume $R \gg r$

Solution:

By symmetry H is constant on centre line

$\Rightarrow$ Perform Ampère's law there.

$$\Rightarrow \quad H\,2\pi R = Ni \quad \Rightarrow H = \frac{Ni}{2\pi R}$$

direction is tangent to clockwise traversing of circle

$$\Rightarrow \quad B = \frac{\mu N i}{2\pi R}$$

If $R \gg r$, B is approx constant over whole surface area

$$\Rightarrow \phi = BA = \frac{\mu N i}{2\pi R}\pi r^2 = \frac{\mu N i r^2}{2R}$$

Note that this flux links all turns

$$\Rightarrow \quad \lambda = N\phi = \frac{\mu N^2 i r^2}{2R}$$