EE3CL4 C01: Introduction to Linear Control Systems

Section 2: System Models

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Outline

Differential equation models

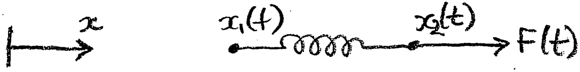
- Most of the systems that we will deal with are dynamic
- Differential equations provide a powerful way to describe dynamic systems
- Will form the basis of our models

- We saw differential equations for inductors and capacitors in 2CI, 2CJ
- What about mechanical systems?
  both translational and rotational

Translational Spring

$F(t)$: resultant force in direction x
Recall free body diagrams and "action and reaction"

- Spring. k: spring constant, L_r: relaxed length of spring

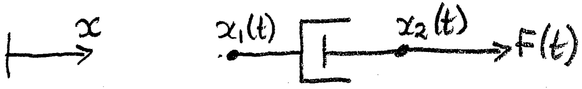


$$F(t) = k\big([x_2(t) - x_1(t)] - L_r\big)$$

Translational Damper

$F(t)$: resultant force in direction x

- Viscous damper. b: viscous friction coefficient

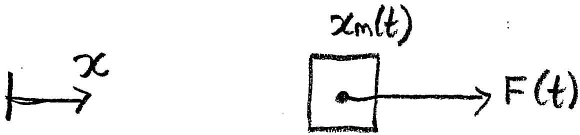


$$F(t) = b\Big(\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt}\Big) = b\big(v_2(t) - v_1(t)\big)$$

Mass

$F(t)$: resultant force in direction x

- Mass: M

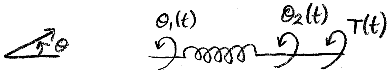


$$F(t) = M\frac{d^2x_m(t)}{dt^2} = M\frac{dv_m(t)}{dt} = Ma_m(t)$$

Rotational spring

$T(t)$: resultant torque in direction θ

- Rotational spring. k: rotational spring constant, ϕ_r: rotation of relaxed spring

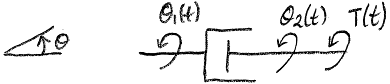


$$T(t) = k\big([\theta_2(t) - \theta_1(t)] - \phi_r\big)$$

Rotational damper

$T(t)$: resultant torque in direction θ

- Rotational viscous damper.
  b: rotational viscous friction coefficient

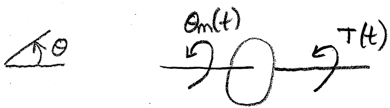


$$T(t) = b\Big(\frac{d\theta_2(t)}{dt} - \frac{d\theta_1(t)}{dt}\Big) = b\big(\omega_2(t) - \omega_1(t)\big)$$

Rotational inertia

$T(t)$: resultant torque in direction θ

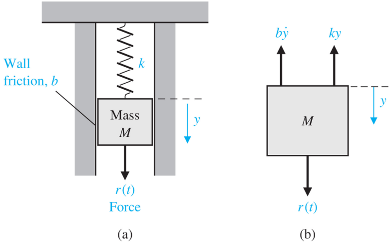
- Rotational inertia: J



$$T(t) = J\frac{d^2\theta_m(t)}{dt^2} = J\frac{d\omega_m(t)}{dt} = J\alpha_m(t)$$

Example system (translational)

Horizontal. Origin for y: $y = 0$ when spring relaxed



- $F = M\frac{dv(t)}{dt}$
- $v(t) = \frac{dy(t)}{dt}$
- $F(t) = r(t) - b\frac{dy(t)}{dt} - ky(t)$

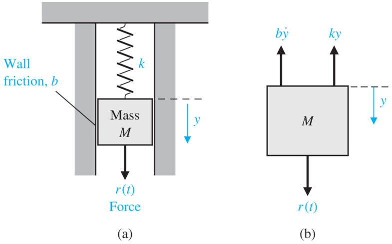
$$M\frac{d^2y(t)}{dt} + b\frac{dy(t)}{dt} + ky(t) = r(t)$$

Example, continued

$$M\frac{d^2y(t)}{dt}+b\frac{dy(t)}{dt}+ky(t)=r(t)$$

Resembles equation for parallel RLC circuit.

Example, continued



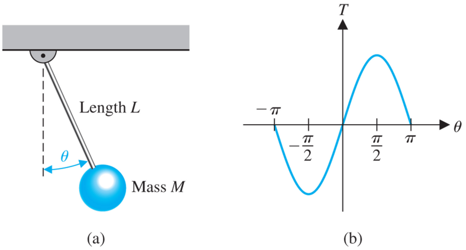
- Stretch the spring a little and hold.
- Assume an under-damped system.
- What happens when we let it go?

Taylor's series

- Nature does not have many linear systems
- However, many systems behave approximately linearly in the neighbourhood of a given point
- Apply first-order Taylor's Series at a given point
- Obtain a locally linear model
- Use this to obtain insight into behaviour of physical system via Laplace Transforms, poles and zeros, etc

- In this course we will focus on the case of a single linearized differential equation model for the system, in which the coefficients are constants
- e.g., in previous examples mass, viscosity and spring constant did not change with time, position, velocity, temperature, etc

Pendulum example



- Assume shaft is light with respect to M, and stiff with respect to gravitational forces
- Torque due to gravity: $T(\theta) = MgL\sin(\theta)$
- Apply Taylor's series around $\theta = 0$:
  $T(\theta) = MgL\Big(\theta - \theta^3/3! + \theta^5/5! - \theta^7/7! + \dots\Big)$
- For small θ around $\theta = 0$ we can build an approximate model that is linear

$$T(\theta) \approx MgL\theta$$

Laplace transform

- Once we have a linearized differential equation (with constant coefficients) we can take Laplace Transforms to obtain the transfer function
- We will consider the "one-sided" Laplace transform, for signals that are zero to the left of the origin.

$$F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\, dt$$

- What does $\int^{\infty}$ mean? $\lim_{T\to\infty} \int^{T}$.
- Does this limit exist?
- If $|f(t)| < Me^{\alpha t}$, then exists for all $\mathrm{Re}(s) > \alpha$. Includes all physically realizable signals
- Note: When multiplying transfer function by Laplace of input, output is only valid for values of s in intersection of regions of convergence

Poles and zeros

- In this course, most Laplace transforms will be rational functions, that is, a ratio of two polynomials in s; i.e.,

$$F(s) = \frac{n_F(s)}{d_F(s)}$$

where $n_F(s)$ and $d_F(s)$ are polynomials

- Definitions:
  - Poles of $F(s)$ are the roots of $d_F(s)$
  - Zeros of $F(s)$ are the roots of $n_F(s)$

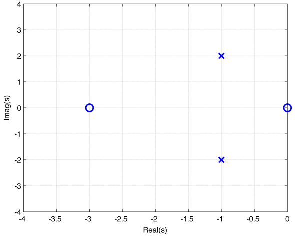
- Hence,

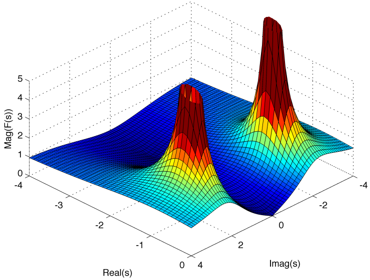
$$F(s) = \frac{K_F \prod_{i=1}^{M}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)} = \left(\frac{K_F \prod_{i=1}^{M} z_i}{\prod_{j=1}^{n} p_j}\right) \frac{\prod_{i=1}^{M}(s/z_i+1)}{\prod_{j=1}^{n}(s/p_j+1)}$$

where $-z_i$ are the zeros and $-p_j$ are the poles

Visualizing poles and zeros

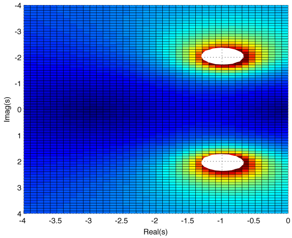
- Consider the simple Laplace transform $F(s) = \frac{s(s+3)}{s^2+2s+5}$.
- zeros: 0, -3; poles: $-1 + j2$, $-1 - j2$
- Pole-zero plot (left) and magnitude of $F(s)$ (right)

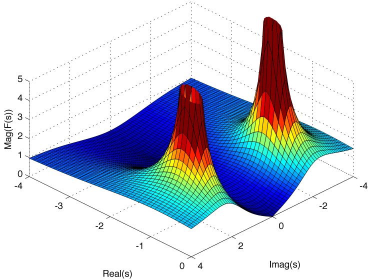




Visualizing poles and zeros

- $F(s) = \frac{s(s+3)}{s^2+2s+5}$; zeros: 0, -3; poles: $-1 + j2$, $-1 - j2$
- $|F(s)|$ from above (left) and prev. view of $|F(s)|$ (right)

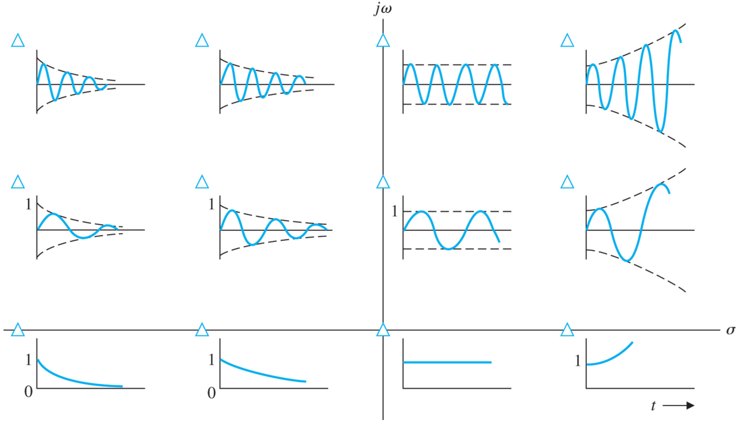




Laplace transform pairs

- Simple ones can be computed analytically;
  often available in tables; see Tab. 2.3 in 12th ed. of text
- For more complicated ones, one can typically obtain the inverse Laplace transform by
  - identifying poles
  - constructing partial fraction expansion
  - using of properties and some simple pairs to invert each component of partial fraction expansion

Some Laplace transform pairs



Recall that complex poles come in conjugate pairs.

Key properties

Linearity

$$\frac{df(t)}{dt} \longleftrightarrow sF(s) - f(0^-)$$

$$\int_{-\infty}^{t} f(x)\,dx \longleftrightarrow \frac{F(s)}{s} + \frac{1}{s}\int_{-\infty}^{0^-} f(x)\,dx$$

Final value theorem

Can we avoid having to do an inverse Laplace transform? Sometimes.

Consider the case when we only want to find the final value of $f(t)$, namely $\lim_{t\to\infty} f(t)$.

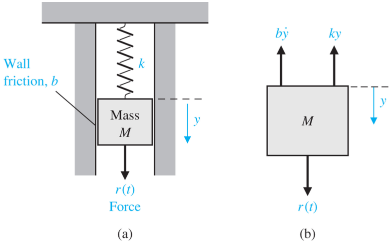
- If $F(s)$ has all its poles in the left half plane, except, perhaps, for a single pole at the origin, then

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Common application: Steady state value of step response

What if there are poles in RHP, or on the $j\omega$-axis and not at the origin?

Mass-spring-damper system



- Horizontal (no gravity)
- Set origin of y where spring is "relaxed"
- $F = M\frac{dv(t)}{dt}$
- $v(t) = \frac{dy(t)}{dt}$
- $F(t) = r(t) - b\frac{dy(t)}{dt} - ky(t)$

$$M\frac{d^2y(t)}{dt} + b\frac{dy(t)}{dt} + ky(t) = r(t)$$

MSD system

$$M\frac{d^2y(t)}{dt} + b\frac{dy(t)}{dt} + ky(t) = r(t)$$

Consider $t \geq 0$ and take Laplace transform

$$M\left(s^2Y(s) - sy(0^-) - \left.\frac{dy(t)}{dt}\right|_{t=0^-}\right) + b\big(sY(s) - y(0^-)\big) + kY(s) = R(s)$$

Hence

$$\begin{aligned} Y(s) = {} & \frac{1/M}{s^2 + (b/M)s + k/M}\, R(s) \\ & + \frac{(s + b/M)}{s^2 + (b/M)s + k/M}\, y(0^-) \\ & + \frac{1}{s^2 + (b/M)s + k/M}\, \left.\frac{dy(t)}{dt}\right|_{t=0^-} \end{aligned}$$

Note that linearity yields superposition

Response to static init. cond.

Spring stretched to a point y_0, held, then let go at time $t = 0$

Hence, $r(t) = 0$ and $\left.\frac{dy(t)}{dt}\right|_{t=0^-} = 0$

Hence,

$$Y(s) = \frac{(s + b/M)}{s^2 + (b/M)s + k/M} \, y_0$$

What can we learn about this response without having to invert $Y(s)$

Standard form

$$Y(s) = \frac{(s + b/M)}{s^2 + (b/M)s + k/M}\, y_0$$

$$= \frac{(s + 2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2}\, y_0$$

where $\omega_n = \sqrt{k/M}$ and $\zeta = \frac{b}{2\sqrt{kM}}$

Poles: $s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

- $\zeta > 1$ (equiv. $b > 2\sqrt{kM}$): distinct real roots, overdamped
- $\zeta = 1$ (equiv. $b = 2\sqrt{kM}$): equal real roots, critically damped
- $\zeta < 1$ (equiv. $b < 2\sqrt{kM}$): complex conj. roots, underdamped

Overdamped case

- $s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$
- Overdamped response: $\zeta > 1$ (equiv. $b > 2\sqrt{kM}$)
- $y(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$
- $y(0) = y_0 \implies c_1 + c_2 = y_0$
- $\left.\frac{dy(t)}{dt}\right|_{t=0} = 0 \implies s_1 c_1 + s_2 c_2 = 0$

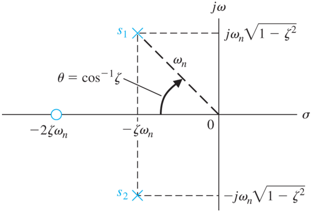
- What does this look like when strongly overdamped
- s_2 is large and negative, s_1 is small and negative
- Hence $e^{s_2 t}$ decays much faster than $e^{s_1 t}$
- Also, $c_2 = -c_1 s_1/s_2$. Hence, small
- Hence $y(t) \approx c_1 e^{s_1 t}$
- Looks like a first order system!

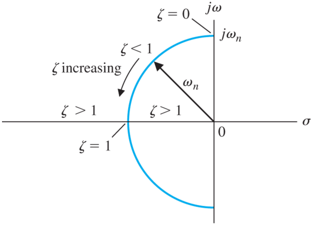
Critically damped case

- $s_1 = s_2 = -\omega_n$
- $y(t) = c_1 e^{-\omega_n t} + c_2 t e^{-\omega_n t}$
- $y(0) = y_0 \implies c_1 = y_0$
- $\left.\frac{dy(t)}{dt}\right|_{t=0} = 0 \implies -c_1\omega_n + c_2 = 0$

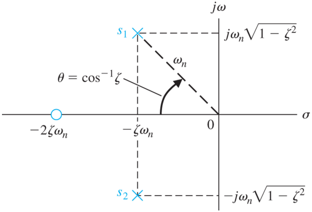
Underdamped case

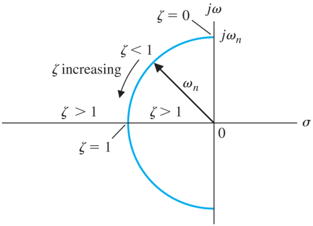
- $s_1, s_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$
- Therefore, $|s_i| = \omega_n$: poles lies on a circle
- Angle to negative real axis is $\cos^{-1}(\zeta)$.





Underdamped case



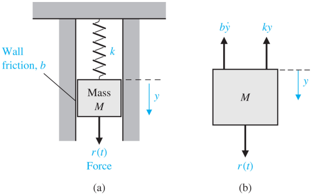


- Define $\sigma = \zeta\omega_n$, $\omega_d = \omega_n\sqrt{1-\zeta^2}$. Response is:

$$
\begin{aligned}
y(t) &= c_1 e^{-\sigma t}\cos(\omega_d t) + c_2 e^{-\sigma t}\sin(\omega_d t) \\
&= A e^{-\sigma t}\cos(\omega_d t + \phi)
\end{aligned}
$$

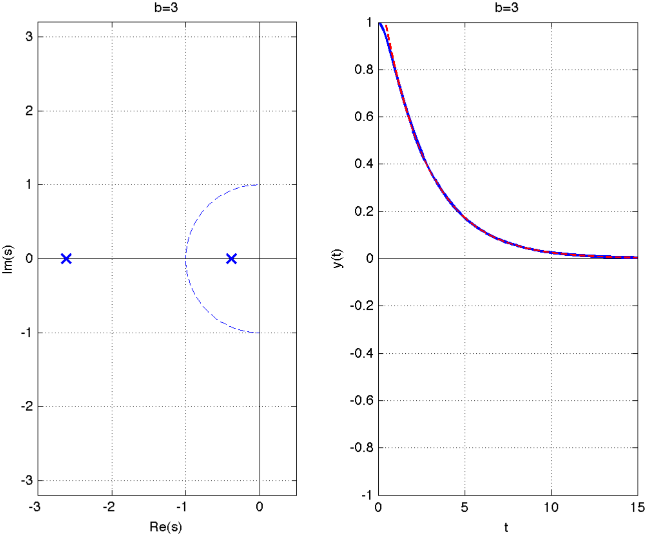
- Homework: Relate A and ϕ to c_1 and c_2.
- Homework: Write the initial conditions $y(0) = y_0$ and $\left.\frac{dy(t)}{dt}\right|_{t=0} = 0$ in terms of c_1 and c_2, and in terms of A and ϕ

Numerical examples

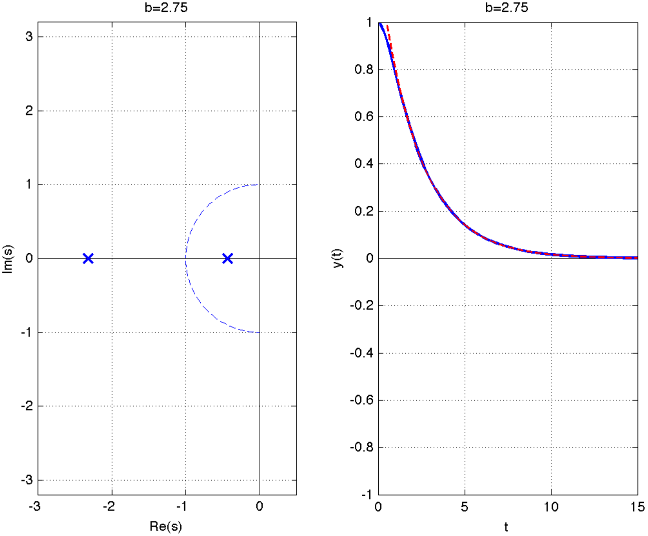


- $Y(s) = \frac{(s+2\zeta\omega_n)}{s^2+2\zeta\omega_n s+\omega_n^2}\, y_0$, where $\omega_n = \sqrt{k/M}$, $\zeta = \frac{b}{2\sqrt{kM}}$
- Poles: $s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$
- $\zeta > 1$: overdamped; $\zeta < 1$: underdamped
- Consider the case of $M = 1$, $k = 1$. Hence, $\omega_n = 1$,
- $b = 3 \to 0$. Hence, $\zeta = 1.5 \to 0$
- Initial conds: $y_0 = 1$, $\left.\frac{dy(t)}{dt}\right|_{t=0} = 0$

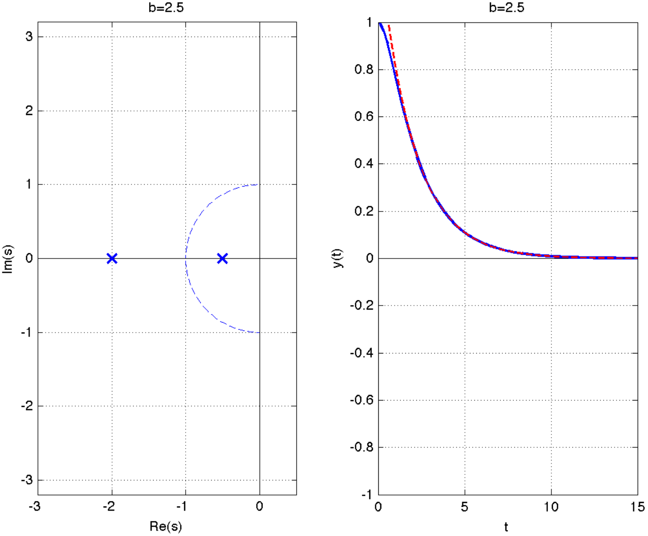
Poles and transient response, $b = 3$



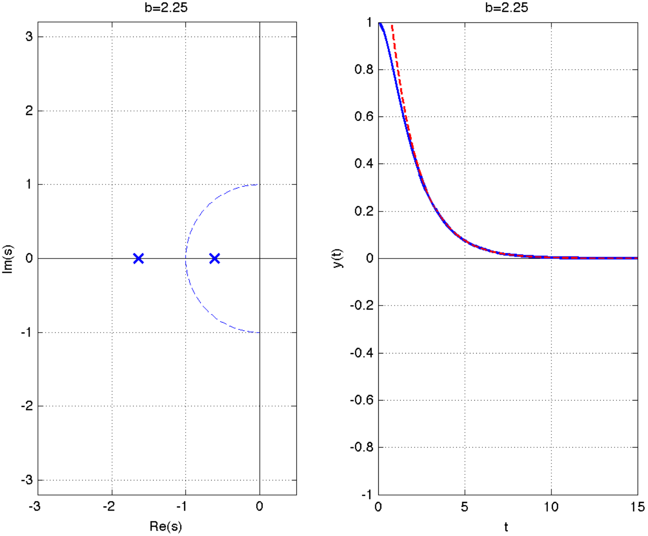
Poles and transient response, $b = 2.75$



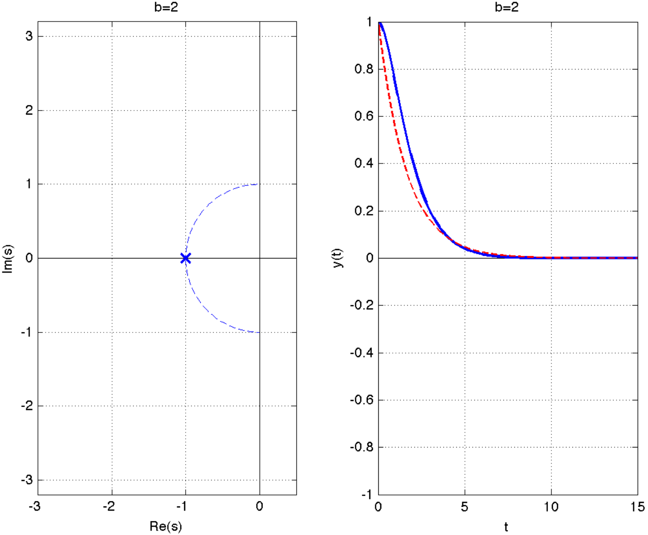
Poles and transient response, $b = 2.5$



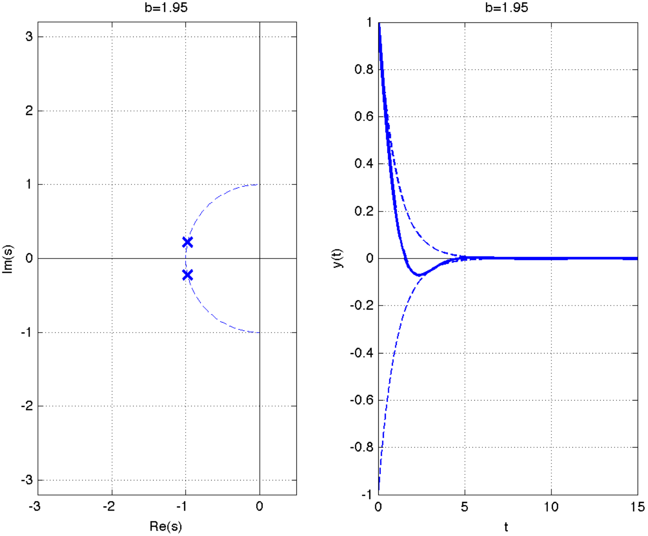
Poles and transient response, $b = 2.25$



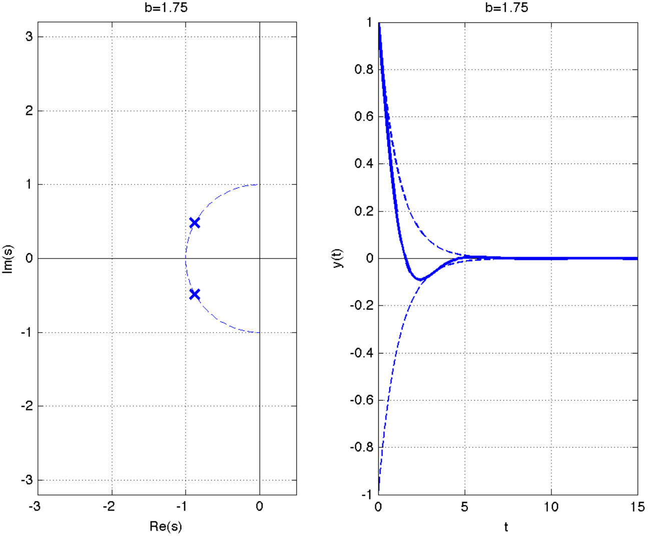
Poles and transient response, $b = 2$



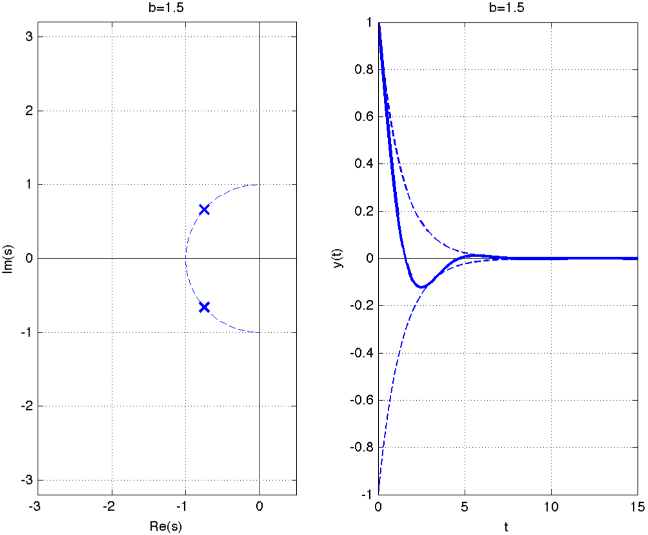
Poles and transient response, $b = 1.95$



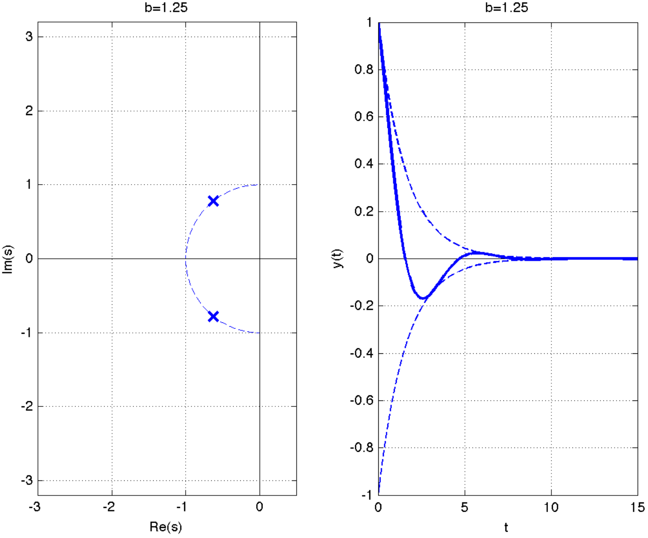
Poles and transient response, $b = 1.75$



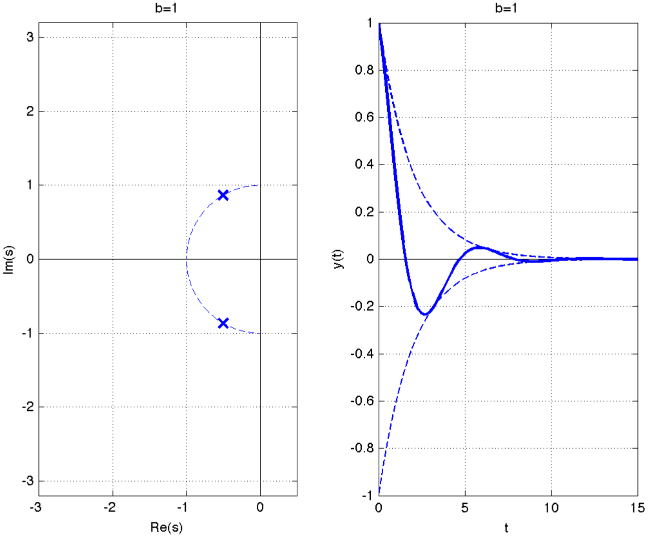
Poles and transient response, $b = 1.5$



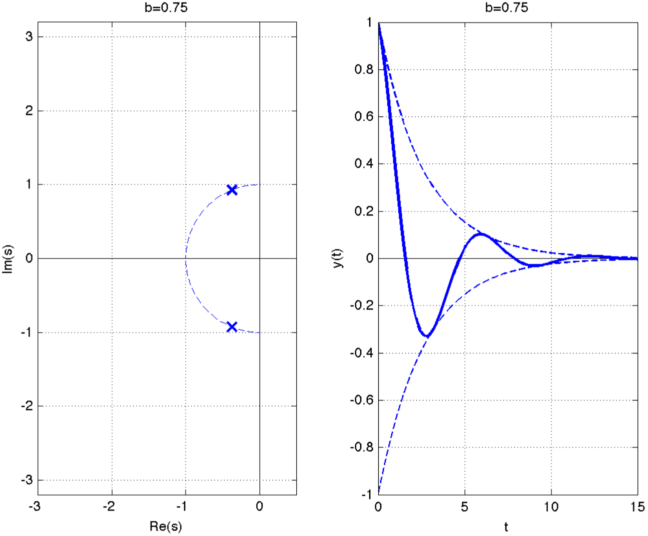
Poles and transient response, $b = 1.25$



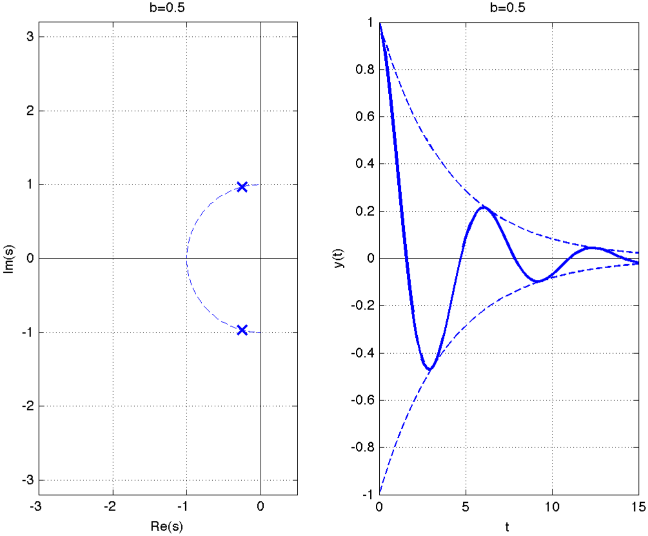
Poles and transient response, $b = 1$



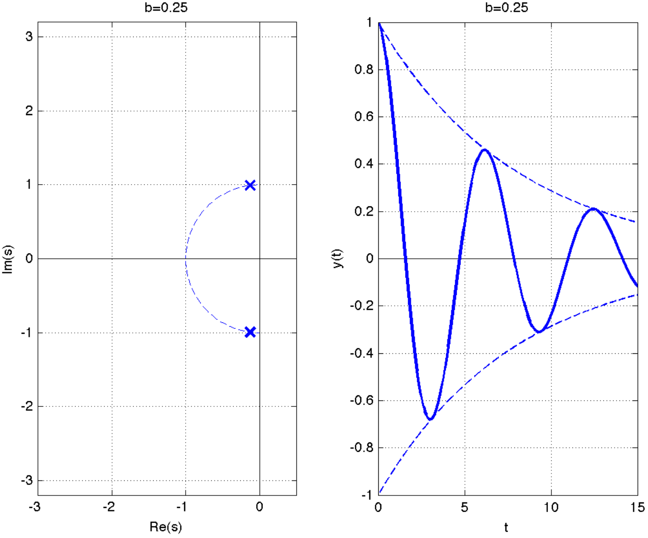
Poles and transient response, $b = 0.75$



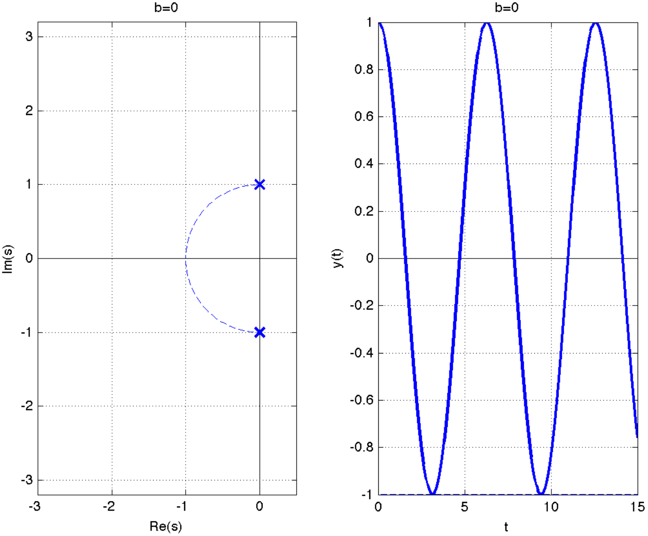
Poles and transient response, $b = 0.5$



Poles and transient response, $b = 0.25$



Poles and transient response, $b = 0$

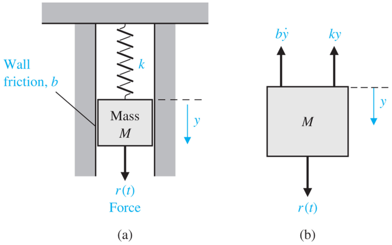


Transfer function

Definition: Laplace transform of output over Laplace transform of input when initial conditions are zero

- Most of the transfer functions in this course will be ratios of polynomials in s.
- Hence, poles and zeros of transfer functions have natural definitions

Example: Recall the mass-spring-damper system,



Transfer function, MSD system

For the mass-spring-damper system,

$$
\begin{aligned}
Y(s) = \frac{1/M}{s^2 + (b/M)s + k/M} R(s) \\
+ \frac{(s + b/M)}{s^2 + (b/M)s + k/M} y(0^-) \\
+ \frac{1}{s^2 + (b/M)s + k/M} \left. \frac{dy(t)}{dt} \right|_{t=0^-}
\end{aligned}
$$

Therefore, transfer function is:

$$
\frac{1/M}{s^2 + (b/M)s + k/M} = \frac{1}{Ms^2 + bs + k}
$$

Step response

- Recall that $u(t) \longleftrightarrow \frac{1}{s}$
- Therefore, for transfer function $G(s)$, the step response is:

$$\mathscr{L}^{-1}\left\{\frac{G(s)}{s}\right\}$$

- For the mass-spring-damper system, step response is

$$\mathscr{L}^{-1}\left\{\frac{1}{s(Ms^2+bs+k)}\right\}$$

- What is the final position for a step input?
  Recall final value theorem. Final position is $1/k$.
- What about the complete step response?

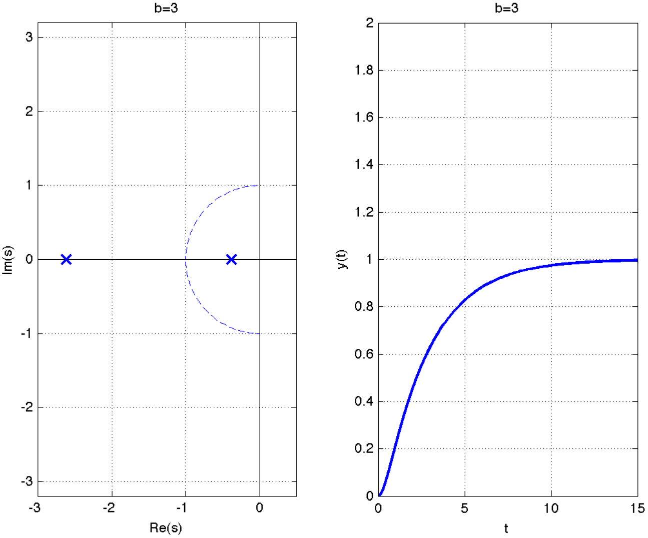
Step response

- Step response: $\mathscr{L}^{-1}\left\{G(s)\frac{1}{s}\right\}$
- Hence poles of Laplace transform of step response are poles of $G(s)$, plus an additional pole at $s = 0$.
- For the mass-spring-damper system, using partial fractions, step response is:

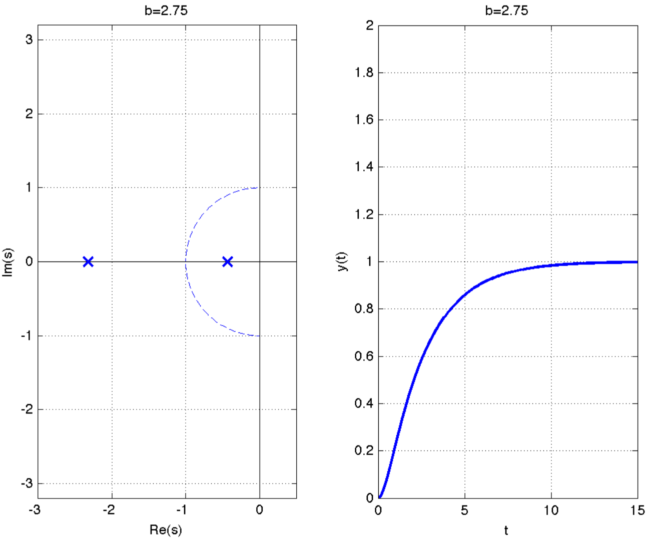
$$\begin{aligned}
&\mathscr{L}^{-1}\left\{\frac{1}{s(Ms^2 + bs + k)}\right\} \\
&\quad = \mathscr{L}^{-1}\left\{\frac{1/k}{s}\right\} - \frac{1}{k}\mathscr{L}^{-1}\left\{\frac{Ms + b}{Ms^2 + bs + k}\right\} \\
&\quad = \frac{1}{k}u(t) - \frac{1}{k}\mathscr{L}^{-1}\left\{\frac{Ms + b}{Ms^2 + bs + k}\right\}
\end{aligned}$$

- Consider again the case of $M = k = 1$, $b = 3 \to 0$. $\omega_n = 1$, $\zeta = 1.5 \to 0$.

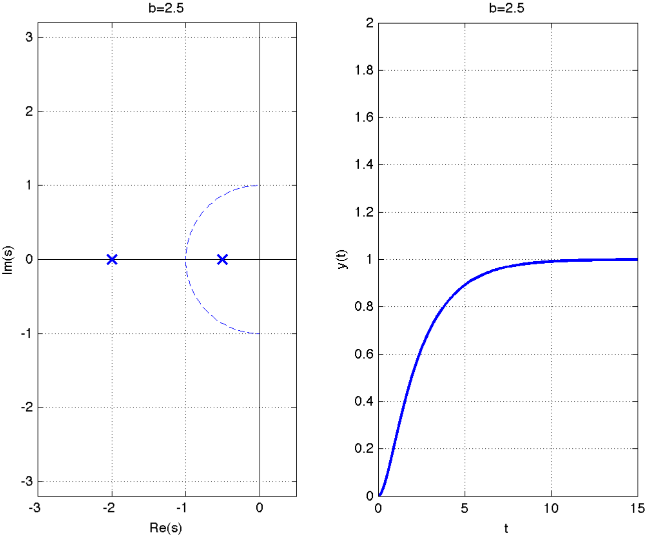
Poles and step response, $b = 3$



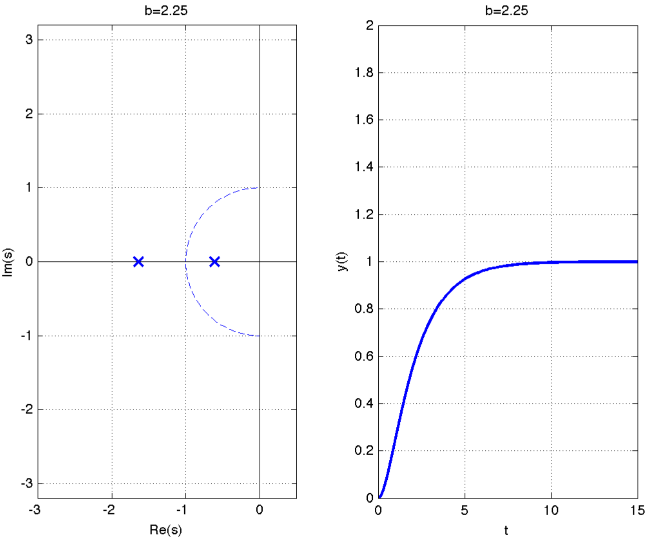
Poles and step resp., $b = 2.75$



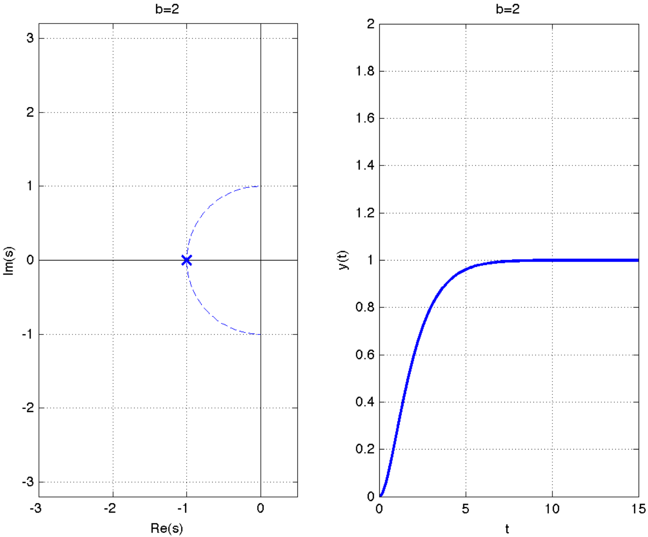
Poles and step resp., $b = 2.5$



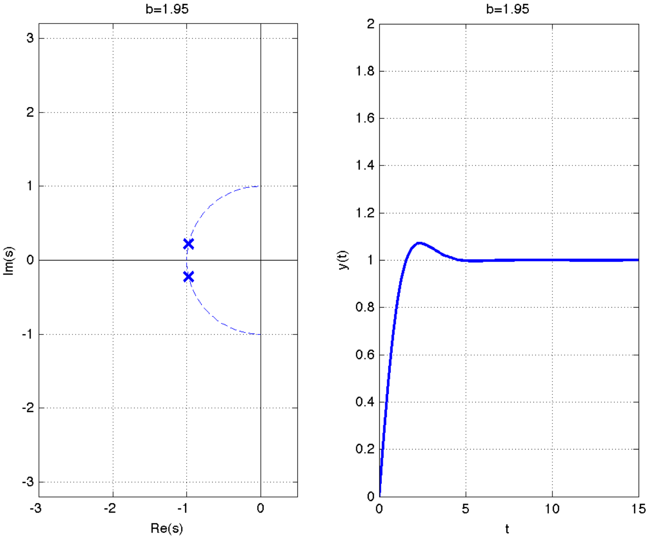
Poles and step resp., $b = 2.25$



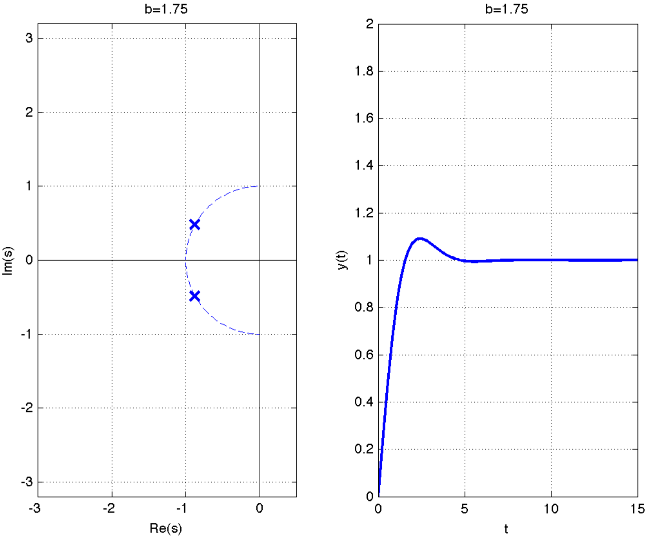
Poles and step resp., $b = 2$



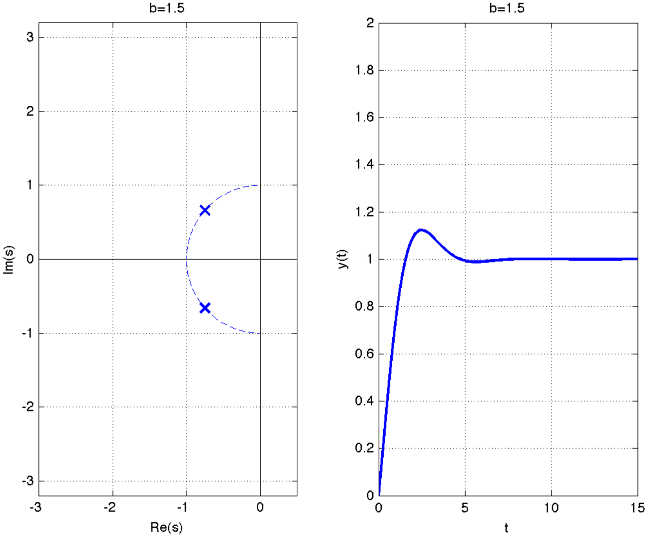
Poles and step resp., $b = 1.95$



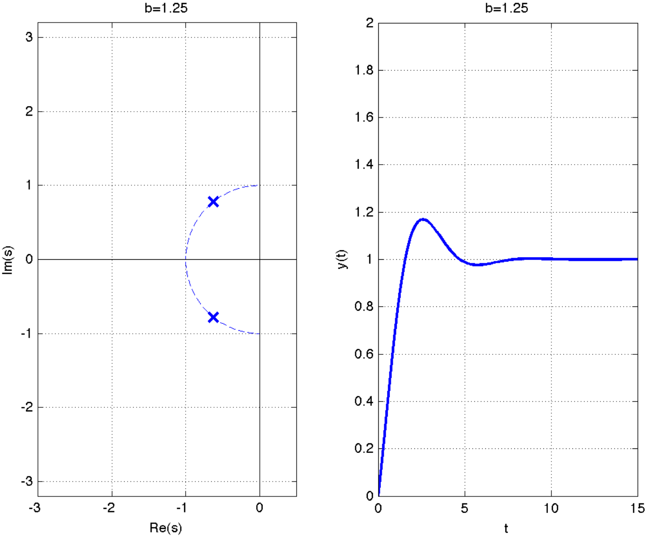
Poles and step resp., $b = 1.75$



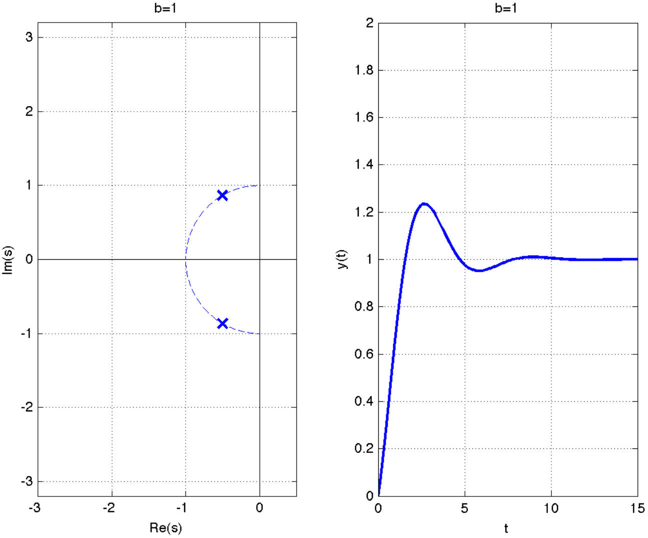
Poles and step resp., $b = 1.5$



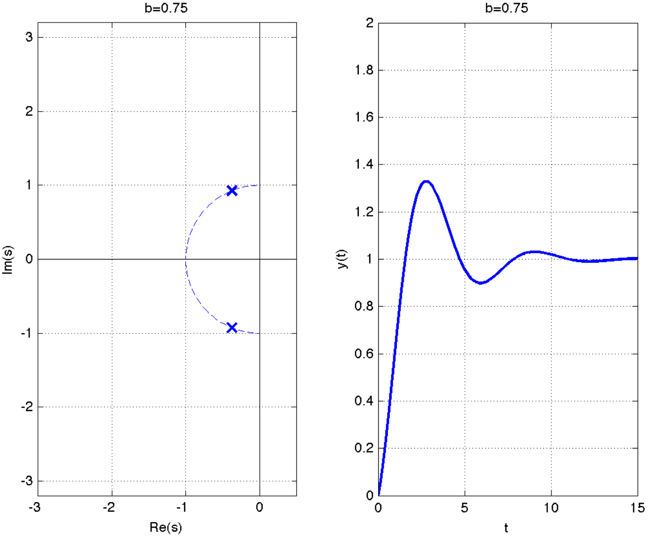
Poles and step resp., $b = 1.25$



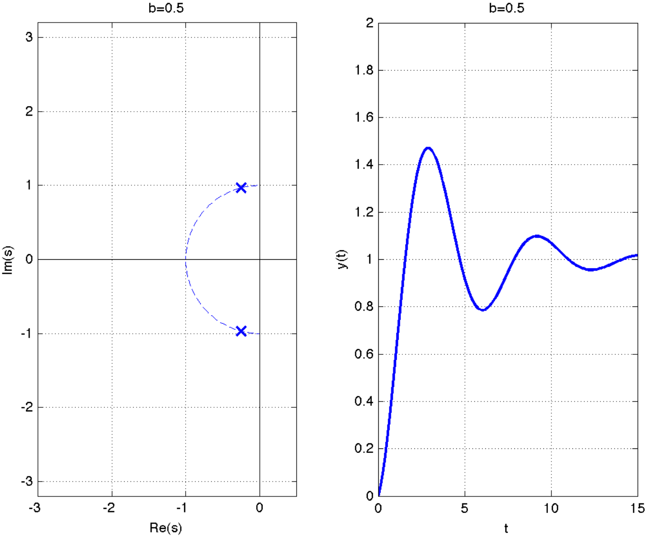
Poles and step resp., $b = 1$



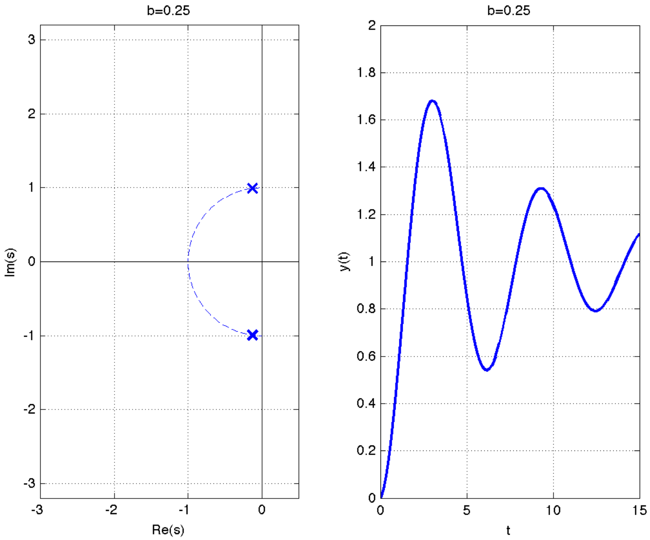
Poles and step resp., $b = 0.75$



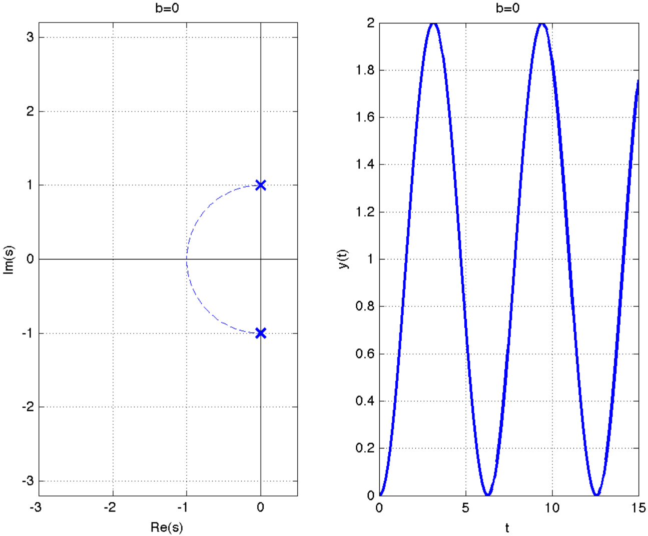
Poles and step resp., $b = 0.5$



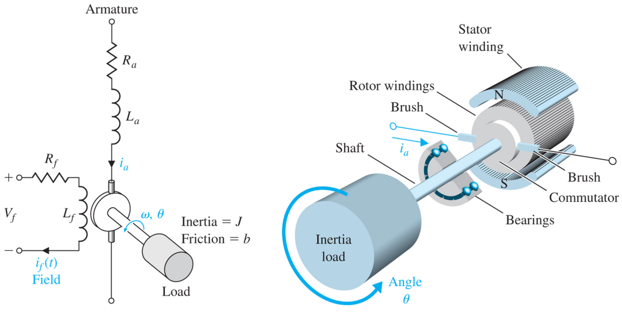
Poles and step resp., $b = 0.25$



Poles and step resp., $b = 0$



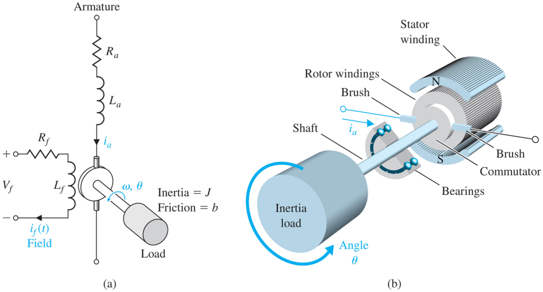
A DC motor



(a) (b)

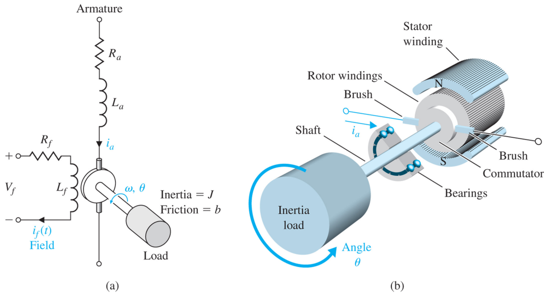
- We will consider linearized model for each component
- Flux in the air gap: $\phi(t) = K_f i_f(t)$ (Magnetic cct, 2CJ4)
- Torque: $T_m(t) = K_1 \phi(t) i_a(t) = K_1 K_f i_f(t) i_a(t)$.
- Is that linear?
- Only if one of $i_f(t)$ or $i_a(t)$ is constant
- We will consider "armature control": $i_f(t)$ constant

Armature controlled DC motor



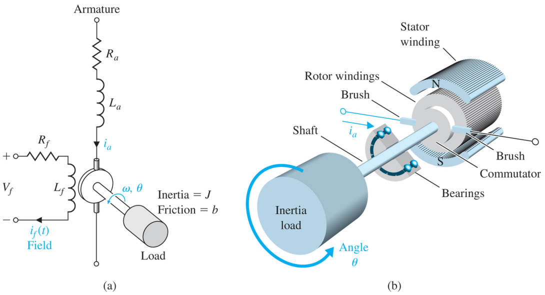
- $i_f(t)$ will be constant (to set up magnetic field), $i_f(t) = I_f$
- Torque: $T_m(t) = K_1 K_f I_f i_a(t) = K_m i_a(t)$
- Will control motor using armature voltage $V_a(t)$
- What is the transfer function from $V_a(s)$ to angular position $\theta(s)$?
- Origin?

Towards transfer function



- $T_m(t) = K_m i_a(t) \longleftrightarrow T_m(s) = K_m I_a(s)$
- KVL: $V_a(s) = (R_a + sL_a)I_a(s) + V_b(s)$
- $V_b(s)$ is back-emf voltage, due to Faraday's Law
- $V_b(s) = K_b\omega(s)$, where $\omega(s) = s\theta(s)$ is rot. velocity
- Remember: transfer function implies zero init. conds

Towards transfer function

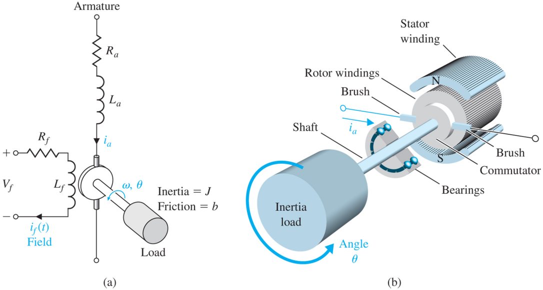


- Torque on load: $T_L(s) = T_m(s) - T_d(s)$
- $T_d(s)$: disturbance. Often small, unknown
- Load torque and load angle (Newton plus friction):

$$T_L(s) = Js^2\theta(s) + bs\theta(s)$$

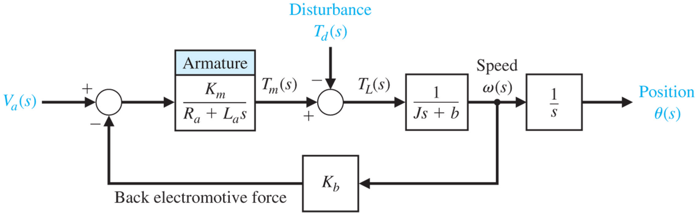
- Now put it all together

Towards transfer function



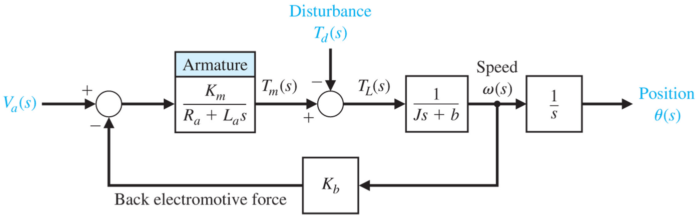
- $T_m(s) = K_m I_a(s) = K_m \left(\frac{V_a(s) - V_b(s)}{R_a + sL_a} \right)$
- $V_b(s) = K_b \omega(s)$
- $T_L(s) = T_m(s) - T_d(s)$
- $T_L(s) = Js^2\theta(s) + bs\theta(s) = Js\omega(s) + b\omega(s)$
- Hence $\omega(s) = \frac{T_L(s)}{Js+b}$
- $\theta(s) = \omega(s)/s$

Block diagram



- $T_m(s) = K_m I_a(s) = K_m \left(\frac{V_a(s) - V_b(s)}{R_a + sL_a} \right)$
- $V_b(s) = K_b \omega(s)$
- $T_L(s) = T_m(s) - T_d(s)$
- $T_L(s) = Js^2\theta(s) + bs\theta(s) = Js\omega(s) + b\omega(s)$
- Hence $\omega(s) = \frac{T_L(s)}{Js+b}$
- $\theta(s) = \omega(s)/s$

Transfer function



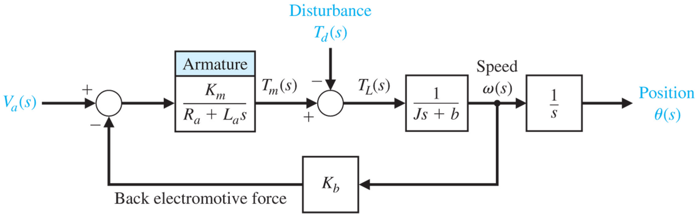
- Set $T_d(s) = 0$ and solve (you MUST do this yourself)

$$G(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{s\big[(R_a + sL_a)(Js + b) + K_b K_m\big]}$$

$$= \frac{K_m}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

- Third order :(

Second-order approximation



$$G(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{s\left[(R_a + sL_a)(Js + b) + K_b K_m\right]}$$

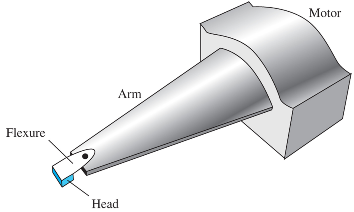
- Sometimes armature time constant, $\tau_a = L_a/R_a$, is negligible
- Hence (you MUST derive this yourself)

$$G(s) \approx \frac{K_m}{s\left[R_a(Js + b) + K_b K_m\right]} = \frac{K_m/(R_a b + K_b K_m)}{s(\tau_1 s + 1)}$$

where $\tau_1 = R_a J/(R_a b + K_b K_m)$

Model for a disk drive read system

(a)



- Uses a permanent magnet DC motor
- Can be modelled using arm. contr. model with $K_b = 0$
- Hence, motor transfer function:

$$G(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{s(R_a + sL_a)(Js + b)}$$

- Assume for now that the arm is stiff

Typical values

$$G(s) = \frac{\theta(s)}{V_a(s)} = \frac{K_m}{s(R_a + sL_a)(Js + b)}$$

Table 2.10 Typical Parameters for Disk Drive Reader

Parameter	Symbol	Typical Value
Inertia of arm and read head	J	1 N m s^2/rad
Friction	b	20 N m s/rad
Amplifier	K_a	10–1000
Armature resistance	R	1 Ω
Motor constant	K_m	5 N m/A
Armature inductance	L	1 mH

$$G(s) = \frac{5000}{s(s+20)(s+1000)}$$

Time constants

- Initial model

$$G(s) = \frac{5000}{s(s+20)(s+1000)}$$

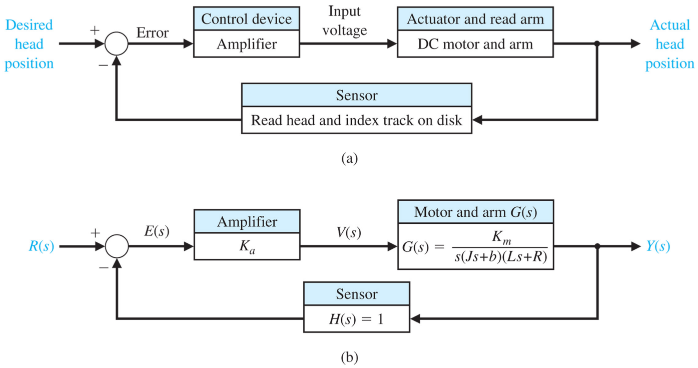
- Motor time constant = 1/20 = 50ms
- Armature time constant = 1/1000 = 1ms
- Hence

$$G(s) \approx \hat{G}(s) = \frac{5}{s(s+20)}$$

A simple feedback controller

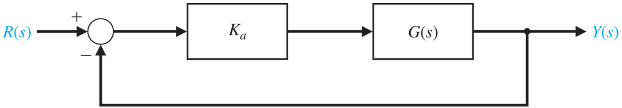
Now that we have a model, how to control?

Simple idea: Apply voltage to motor that is proportional to error between where we are and where we want to be.



Here, $V(s) = V_a(s)$ and $Y(s) = \theta(s)$.

Simplified block diagram



- What is the transfer function from command to position? Derive this yourself

$$\frac{Y(s)}{R(s)} = \frac{K_a G(s)}{1 + K_a G(s)}$$

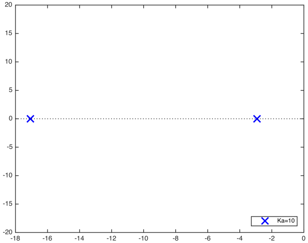
- Using second-order approx. $G(s) \approx \hat{G}(s) = \frac{5}{s(s+20)}$,

$$Y(s) \approx \frac{5K_a}{s^2 + 20s + 5K_a} R(s)$$

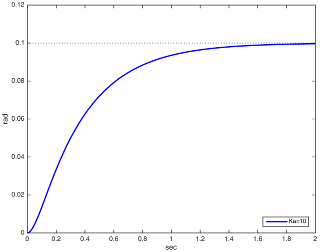
- For $0 < K_a < 20$: overdamped;
  for $K_a > 20$: underdamped

Response to $r(t) = 0.1u(t)$; $K_a = 10$

Poles in s-plane



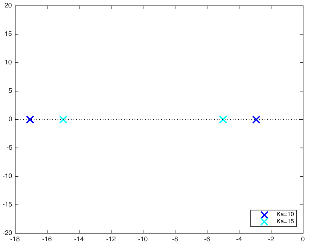
Response



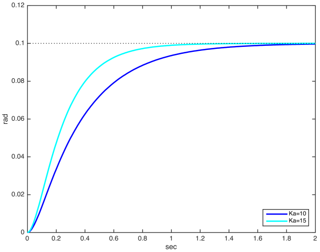
Slow. Slower than IBMs first drive from late 1950's.
Disks in the 1970's had 25ms seek times; now $<$ 10ms
Perhaps increase K_a?
That would result in a "bigger" input to the motor for a given error

Response to $r(t) = 0.1u(t)$; $K_a = 10, 15$

Poles in s-plane



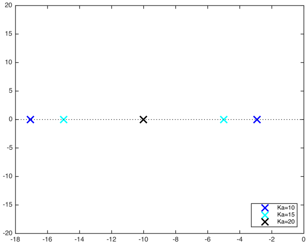
Response



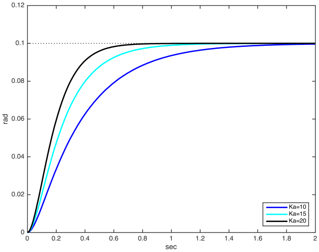
Changing K_a changes the position of the closed-loop poles
Hence, step response changes

Response to $r(t) = 0.1u(t)$; $K_a = 10, 15, 20$

Poles in s-plane



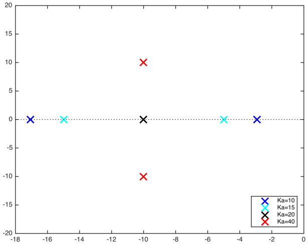
Response



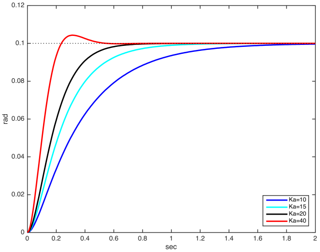
Changing K_a changes the position of the closed-loop poles
Hence, step response changes (now critically damped)

Response to $r(t) = 0.1u(t)$; $K_a = 10, 15, 20, 40$

Poles in s-plane



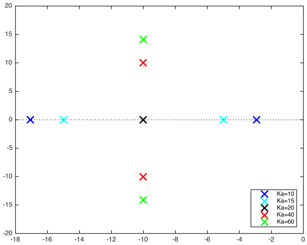
Response



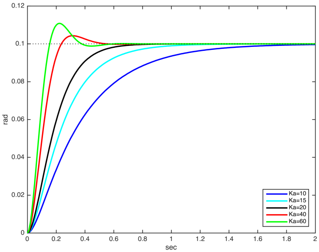
Changing K_a changes the position of the closed-loop poles
Hence, step response changes (now underdamped)

Response to $r(t) = 0.1u(t)$; $K_a = 10, 15, 20, 40, 60$

Poles in s-plane



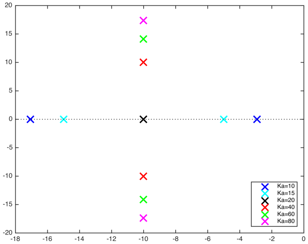
Response



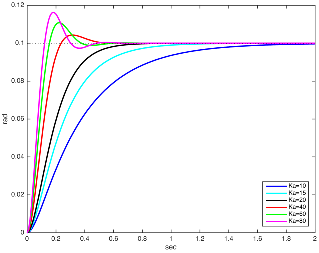
Changing K_a changes the position of the closed-loop poles
Hence, step response changes (now more underdamped)

Response to $r(t) = 0.1u(t)$; $K_a = 10, 15, 20, 40, 60, 80$

Poles in s-plane



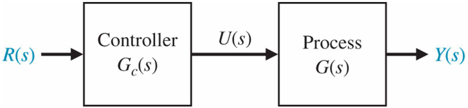
Response



What is happening to the settling time of the underdamped cases?
Only just beats IBM's first drive
What else could we do with the controller? Prediction?

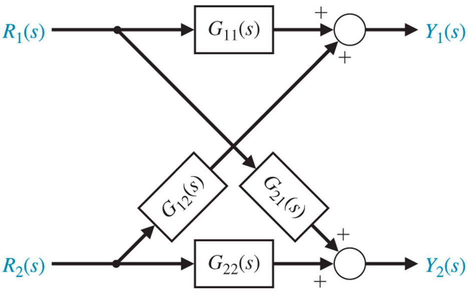
Bock diagram models

- As we have just seen, a convenient way to represent a transfer function is via a block diagram



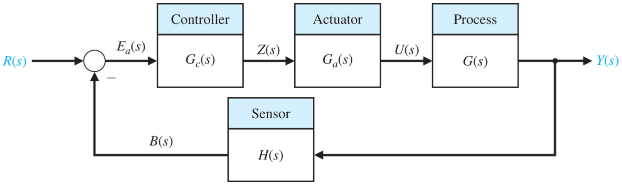
- In this case, $U(s) = G_c(s)R(s)$ and $Y(s) = G(s)U(s)$
- Hence, $Y(s) = G(s)G_c(s)R(s)$
- Consistent with the engineering procedure of breaking things up into little bits, studying the little bits, and then put them together

Simple example



- $Y_1(s) = G_{11}(s)R_1(s) + G_{12}(s)R_2(s)$
- $Y_2(s) = G_{21}(s)R_1(s) + G_{22}(s)R_2(s)$

Example: Loop transfer function



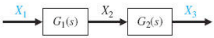
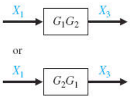
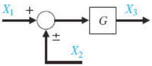
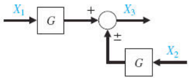
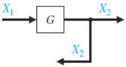
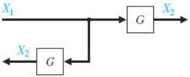
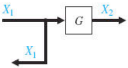
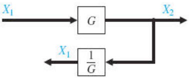
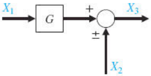
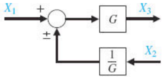
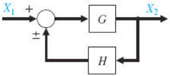
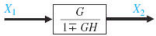
- $E_a(s) = R(s) - B(s) = R(s) - H(s)Y(s)$
- $Y(s) = G(s)U(s) = G(s)G_a(s)Z(s)$
- $Y(s) = G(s)G_a(s)G_c(s)E_a(s)$
- $Y(s) = G(s)G_a(s)G_c(s)\Big(R(s) - H(s)Y(s)\Big)$

$$\frac{Y(s)}{R(s)} = \frac{G(s)G_a(s)G_c(s)}{1 + G(s)G_a(s)G_c(s)H(s)}$$

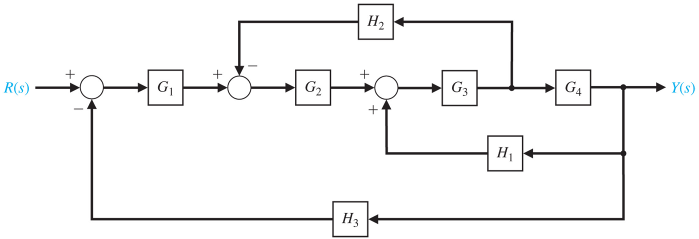
- Each transfer function is a ratio of polynomials in s
- What is $E_a(s)/R(s)$?

Block diagram transformations

Table 2.6 Block Diagram Transformations

Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade		
2. Moving a summing point behind a block		
3. Moving a pickoff point ahead of a block		
4. Moving a pickoff point behind a block		
5. Moving a summing point ahead of a block		
6. Eliminating a feedback loop		

Using block diagram transformations



Using block diagram transformations