FEEDBACK SYSTEMS AND CONTROL

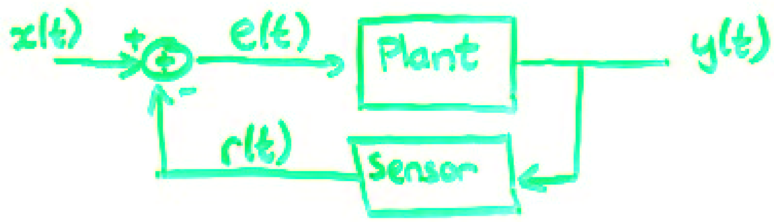
Feedback: return of a fraction of the output of a system to the input

: may occur naturally, or may be enforced for the following reasons

- reduction of sensitivity to parameter variations
- reduction of sensitivity to noise + other disturbances
- improvement of linear behavior.

BASIC CONCEPTS.

Basic feedback system:

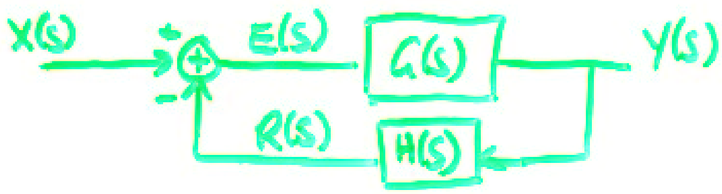


Plant : acts on "error signal" $e(t)$ to produce output $y(t)$

Sensor: measures $y(t)$ and produces feedback signal $r(t)$

Comparator: compares $r(t)$ with "command" signal $x(t)$ to produce "error" signal.

- Such a system is usually difficult to describe, but if the plant and sensor are LTI, then we can describe them via convolution
- However, we will find Laplace Transform descriptions to be far more valuable.
- If plant + senor are LTI



$$E(s) = X(s) - R(s)$$

$$G(s) = \frac{Y(s)}{E(s)} \; ; \qquad H(s) = \frac{R(s)}{Y(s)}$$

$$\Rightarrow \frac{Y(s)}{G(s)} = E(s) = X(s) - R(s) = X(s) - H(s)\,Y(s)$$

$$\Rightarrow T(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)\,H(s)}$$

"closed loop transfer function"

Denominator; $1 + G(s)H(s)$ is a measure of the feedback

$F(s) = 1 + G(s)H(s)$ is called the "return difference"

$L(s) = G(s)H(s)$ is called the "loop transfer function"

- Is the feedback positive or negative?
- Depends on the phase of $L(s)$.

Eg if $\angle L(j\omega) = 0$ feedback is negative

if $\angle L(j\omega) = 180°$ feedback becomes positive

This will be very important in control system design

Reasons for using feedback.

1. Reduction in sensitivity

For the moment, ignore dependence of T on s.

How sensitive is it to changes in G?

$$\Delta T = \frac{\partial T}{\partial G} \cdot \Delta G$$

$$= \frac{1}{(1+GH)^2} \Delta G.$$

Define sensitivity $= \dfrac{\Delta T / T}{\Delta G / G} = \dfrac{\%\text{ change in } T}{\%\text{ change in } G.}$

$$= S_G^T$$

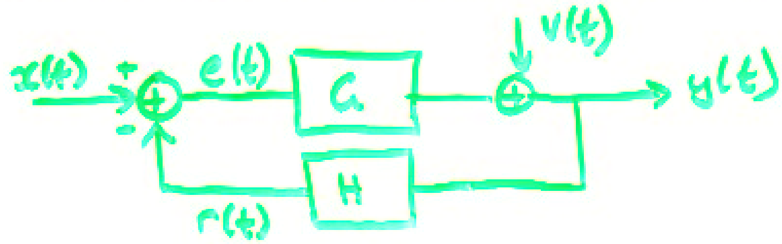
By substituting appropriate formulae.

$$S_G^T = \frac{1}{1+GH} = \frac{1}{F}$$

Typically we want S_G^T to be small!

2. Reduction in sensitivity to noise or disturbances.

Consider feedback system with noisy measurements



Solving this loop, using Laplace Transforms and superposition we have.

$$Y(s) = \underbrace{T(s)\,X(s)}_{\text{Desired}} + \underbrace{\frac{1}{1+G(s)H(s)}\,V(s)}_{\text{must be rejected.}}$$

$\Rightarrow$ want $|1+G(s)H(s)| > 1$ at values of s where $|V(s)|$ is significant

3. Reduction of non-linear effects.

- Feedback can also be used to reduce the sensitivity of a system to non-linear components for

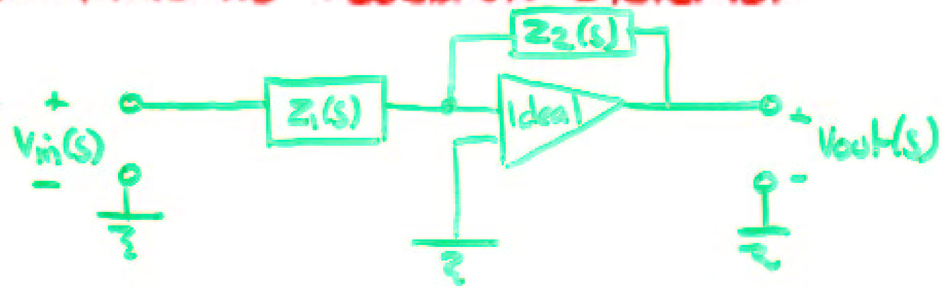
COST OF FEEDBACK.

1. Increased system complexity, both design and implementation
2. Reduced gain

   $$T(s) = \frac{G(s)}{F(s)} \quad \text{rather than just } G(s)$$

3. Possible instability

OP-AMPS AS FEEDBACK SYSTEMS.



Using nodal analysis and properties of ideal op-amp.

$$T(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)}.$$