INPUT-OUTPUT DESCRIPTIONS OF LINEAR TIME-INVARIANT SYSTEMS:

I: TIME DOMAIN DESCRIPTIONS

A: CONVOLUTION

- There are many I-O descriptions of systems: e.g., time domain, frequency-domain, s-domain

- There are even several time domain descriptions e.g, convolution, differential equations...

- We will look at many of these in this course, but will start with convolution.

INTUITION

- We will find that the output of a LTI system, for (essentially) any input signal, can be completely described in terms of its response to a narrow pulse (impulse)

- Quite amazing!
  Why might it be true?

① Each signal can be written as the sum of weighted and time-shifted impulses

② The response to an impulse is called the impulse response

③ Since the system is time invariant, the response to a time shifted impulse is a time-shifted impulse response

④ Since the system is linear the response to a weighted sum of inputs is equal to the weighted sum of the output due to each input

Discrete Time Case

① write a signal as a weighted sum of impulses

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$$

Hence $\quad x[n]\,\delta[n-k] = x[k]\,\delta[n-k]$

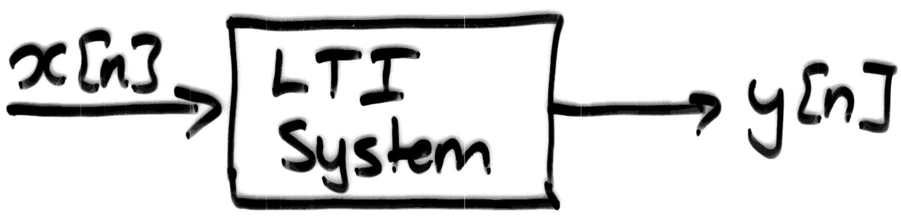
Therefore

$$x[n] = \cdots + x[-1]\,\delta[n+1] + x[0]\,\delta[n] + x[1]\,\delta[n-1] + \cdots$$

$$= \sum_k x[k]\,\delta[n-k]$$

Now what is the system output

$$y[n] = \mathcal{H}\{x[n]\}$$



$\mathcal{H}$ is the operator which describes the system

Since the system is <u>Linear</u>,

$$y[n] = \mathcal{H}\{x[n]\}$$

$$= \mathcal{H}\Big\{\sum_k x[k]\,\delta[n-k]\Big\}$$

$$= \sum_k x[k]\,\mathcal{H}\{\delta[n-k]\}$$

Let $h_k[n] = \mathcal{H}\{\delta[n-k]\}$

This is the response at time n to an impulse at time k.

$$\Rightarrow \quad y[n] = \sum_k x[k]\,h_k[n]$$

Since the system is <u>time invariant</u>,

$$h_k[n] = h_0[n-k]$$

[redacted] $h_0[n]$ is response at time n to an impulse at time zero.

It is called the impulse response and is usually denoted by $h[n]$

Therefore.

$$\boxed{y[n] = \sum_k x[k]\, h[n-k]}$$

$\Rightarrow$ y is a weighted sum of shifted impulse responses

Notation: We sometimes write

$$x[n] * h[n] = \sum_k x[k]\, h[n-k]$$

Note that.

$$\sum_m h[m]\, x[n-m] = \sum_k x[k]\, h[n-k]$$

Now how do we compute this ?