Performing Convolution.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

Simple Procedure for small examples.

① Let $p_k[n] = x[k]\,\delta[n-k]$.
ie a weighted impulse at time k.

② Form $v_k[n] = x[k]\,h[n-k]$
response at all times n, due to implulse at time k.

③ Now
$$y[n] = \sum_k v_k[n].$$

Notes:
- We must do steps ① - ③ for every value of k for which $x[k]$ is non-zero
- Since $x[k]$ can be quite long, this can become a lot of work.

Alternative procedure (recommended)

- Previous approach
  - calculate response at all times n to an input at time k
    then sum over k

- Recommended approach
  - calculate response at time n to inputs at all times k
    redo this for each n.

- The recommended approach saves time when the functional forms stay the same for different n

- What we do here is for a given n_0,
  a) Form $W_{n_0}[k] = x[k]\,h[n_0 - k]$ for all k
  b) Hence $y[n_0] = \sum_k W_{n_0}[k]$
  c) Repeat for next value of n

- Now we know how to form $y[n_0]$ but it still looks like we have to do this for an infinite number of n '

- We simplify this by finding ranges of n for which the functional form of $W_n[k]$ is the same

  Then we only have to look at one point in each range The resulting representation of $y[n]$ is typically of the form

$$y[n] \begin{cases} f_0[n] & n \leq N_0 \\ f_1[n] & N_0 < n \leq N_1 \\ f_2[n] & N_1 < n \leq N_2 \\ \\ f_{m-1}[n] & N_{m-2} < n \leq N_{m-1} \\ f_m[n] & n > N_m. \end{cases}$$

and each $f_i[n] = \sum_k W_n[k]$

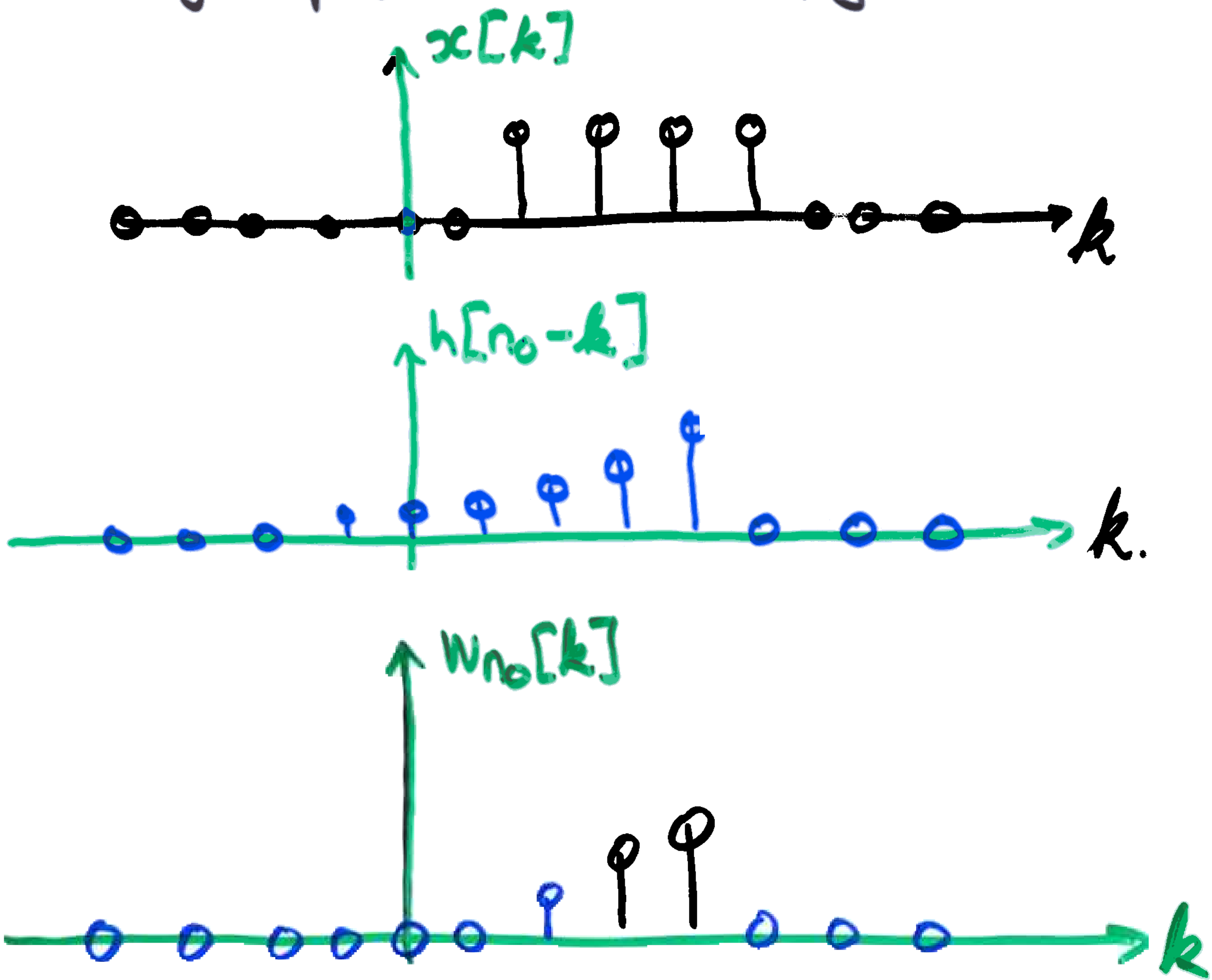
and over each range, the functional form of $W_n[k]$ is the same

- Now we only have to do things m times further advantage of the recommended method that there is a nice graphical procedure for forming $w_{n_0}[k]$.

$$w_{n_0}[k] = x[k]\, h[n_0 - k]$$

k is independent variable
n_0 is a constant

So just plot them and multiply!



Now, $y[n_0] = \sum w_{n_0}[k]$

The thing you may find difficult here is if I give you $h[k]$ what is $h[n_0 - k]$

(This is why we studied shifting and scaling in the introduction)

Lets define $\tilde{u}[k] = h[-k]$ (reflection)

what is $\tilde{u}[k-n_0]$ (shift right by n_0

$$\tilde{u}[k-n_0] = h[-(k-n_0)] = h[n_0-k]$$