Recall that

$$e^{at}u(t) \xleftrightarrow{L} \frac{1}{s-a}, \quad Re\{s\} > a$$

we can also show that

$$e^{(\alpha+j\omega_0)t}u(t) \xleftrightarrow{L} \frac{1}{s-(\alpha+j\omega_0)}, \quad Re\{s\} > \alpha$$

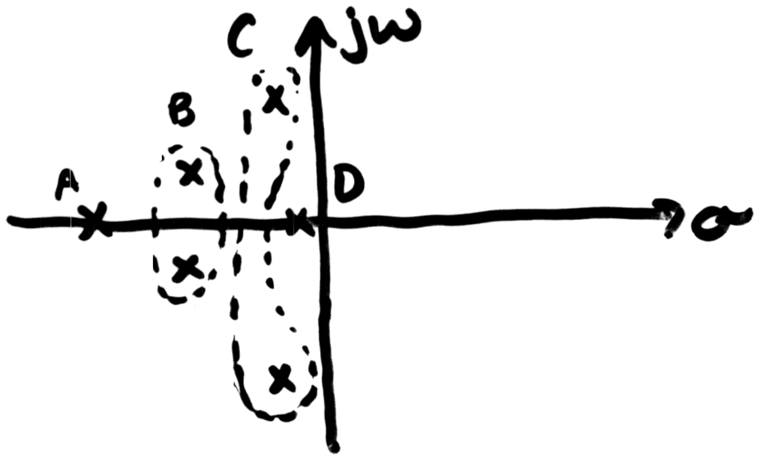
Hence.

$$\left.\begin{matrix} \dfrac{1}{s-(\alpha+j\omega_0)} + \dfrac{1}{s-(\alpha-j\omega_0)} \\ Re\{s\} > \alpha \end{matrix}\right\} \xleftrightarrow{L^{-1}} e^{(\alpha+j\omega_0)t}u(t) + e^{(\alpha-j\omega_0)t}u(t)$$

$$= e^{\alpha t}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]u(t)$$

$$= 2e^{\alpha t}\cos(\omega_0 t)u(t)$$

GETTING A "FEEL" FOR POLES



A causal system has the above poles what does $h(t)$ look like?

- Poles in LHP $\Rightarrow$ system is stable
  $h(t)$ will decay to zero.

Pole A generates a term $e^{-a_A t}u(t)$

a_A is large $\Rightarrow$ fast decay

Pole D generates $e^{-a_D t}u(t)$

a_D is small, $\Rightarrow$ slow decay.

Complex conjugate pair B generates

$e^{\alpha t}\cos(\omega_0 t)u(t)$

α is large (fast decay)

ω_0 is small (slow ripples)

- Complex conjugate pair C generates

$$e^{\alpha_c t}\cos(\omega_{0,c} t)u(t)$$

is small, (slow decay)

ω_{0_c} is large (fast ripples).

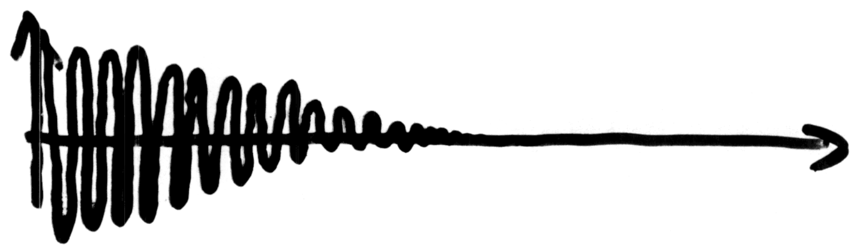
The exact h(t) depends on the coefficients associated with the poles, eg $\frac{B}{s-a_A}$

However, we can make some kind of picture h(t) is the ~~sn~~ sum of the following shapes.

A

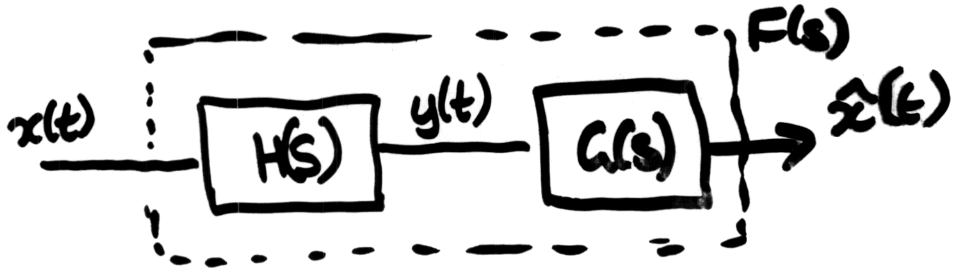
B

C



D

INVERSE SYSTEMS AND EQUALIZERS



- Given $h(t)$, is there a $g(t)$ such that $\hat{x}(t) = x(t)$?
- Look in the Laplace Transform domain

  Given $H(s)$ is there a $G(s)$ such that

  $$F(s) = H(s)G(s) = 1 \quad ?$$

- Of course! $\quad G(s) = \dfrac{1}{H(s)}$

- If $H(s) = \dfrac{b_m/a_n \prod_k (s - c_k)}{\prod_k (s - d_k)}$

  Then $G(s) = \dfrac{\prod_k (s - d_k)}{b_m/a_n \prod_k (s - c_k)}$

  That is zeros of $H(s)$ become poles of $G(s)$.

- Usually $h(t)$ is causal and stable

  $g(t)$ causal and stable ?

Recall that

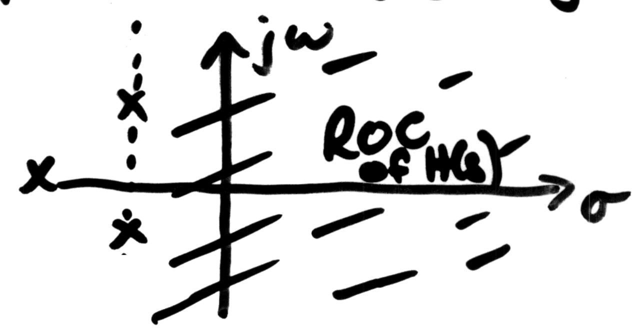
$$h(t) * g(t) \overset{L}{\longleftrightarrow} H(s)G(s)$$, ROC is at least $R_H \cap R_G$

$\Rightarrow$ in order for the choice

$$G(s) = \frac{1}{H(s)}$$

to make sense, R_H and R_G must intersect

If $H(s)$ is causal and stable, then ROC is to the right of the right most pole and includes the $j\omega$-axis



- If $G(s)$ is to be causal, then its ROC must be to the right of its right most pole
  This will intersect with R_H above

- However, poles of $G(s)$ are zeros of $H(s)$
  so for $G(s)$ to be causal, R_G lies to right of right most zero of $H(s)$

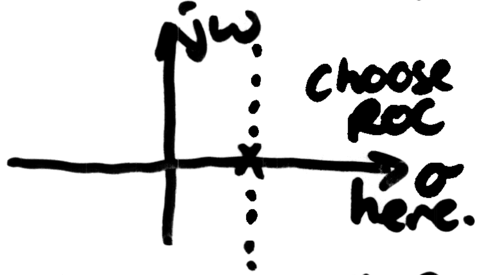
For $G(s)$ to be stable, R_G must include $j\omega$-axis

$\Rightarrow$ for a system to be invertible by a stable and causal $G(s)$, then both poles and zeros of $H(s)$ must be in the left half plane.

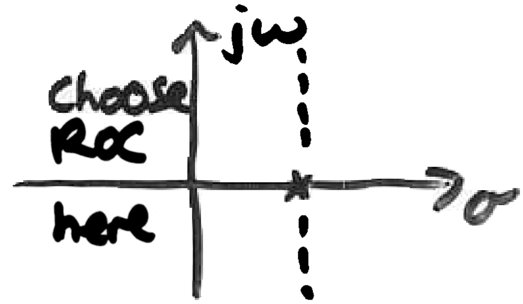
- Such ~~[illegible]~~ systems are said to have minimum phase

Now what if $H(s)$ has a zero in the RHP?

Two choices: s-plane plots for $G(s)$



Intersects with R_H
$\Rightarrow$ $G(s)$ is causal but unstable.



Intersects with R_H
$\Rightarrow$ $G(s)$ is stable but anti-causal