FOURIER REPRESENTATIONS OF SAMPLED SIGNALS.

If.

$$x_c(t) \xleftrightarrow{FT} X_c(j\omega),$$

$$x[n] = x_c(nT) \xleftrightarrow{DTFT} \; ?$$

- The answer to this question will lead us to a key result in sampling theory and practice
- We will first calculate the answer directly, and then the indirect method from the book. The indirect method uses some "non-physical" models, but is generally considered easier to understand.

DIRECT METHOD.

$$X(e^{j\Omega}) = \sum_n x[n] e^{-j\Omega n}.$$

$$= \sum_n x_c(nT)\, e^{-j\Omega n}$$

$$= \sum_n \frac{1}{2\pi} \int X_c(j\omega)\, e^{j\omega nT} d\omega\, e^{-j\Omega n}$$

$$= \frac{1}{2\pi} \int X_c(j\omega) \cdot \underbrace{\sum_n e^{j\omega nT} \cdot e^{-j\Omega n}}_{\text{DTFT of } e^{j\omega nT}} d\omega$$

$$= 2\pi \sum_k \delta(\Omega - \omega T - 2\pi k)$$

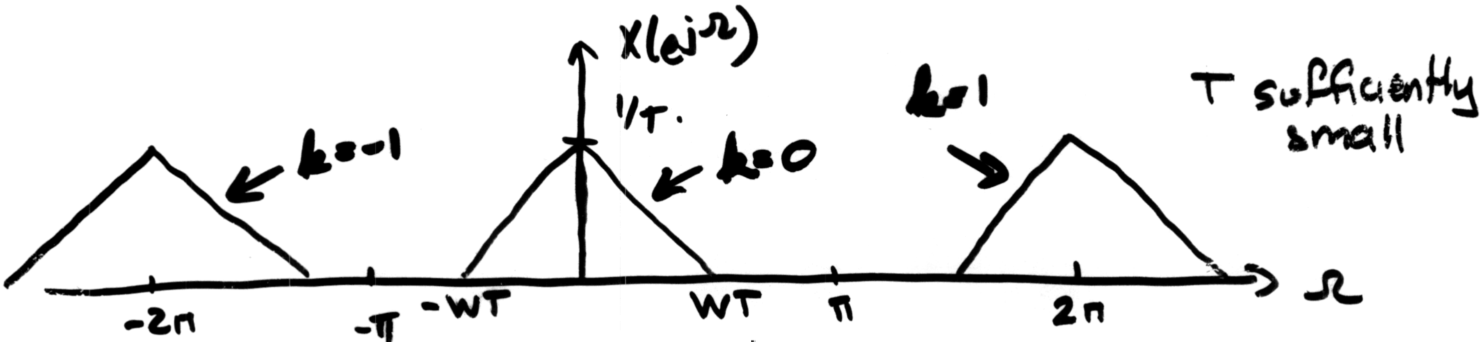
$$\Rightarrow X(e^{j\Omega}) = \sum_k \int X_c(j\omega)\ \delta(\Omega \ \ \omega T - 2\pi k)\ d\omega.$$

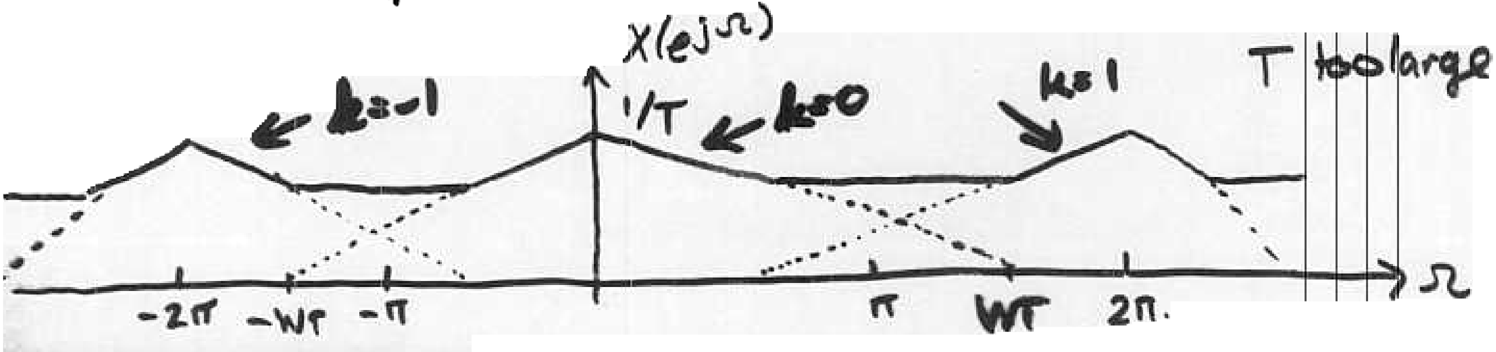
Let $\lambda = +\omega T$

$$= \frac{1}{T} \sum_k \int X_c\left(j \frac{\lambda}{T}\right) \delta(\Omega \ \ \lambda \ \ 2\pi k)\ d\lambda$$

$$\frac{1}{T} \sum_k X_c\left(j \frac{(\Omega - 2\pi k)}{T}\right)$$

$\Rightarrow$ $X(e^{j\Omega})^*$ can be constructed by taking $X(j\omega)$ Scaling it by $1/T$ and then shifting up and down the Ω axis by $2\pi k$





- The model we have used here is direct, and uses physically realizable criteria everywhere.
- However the book takes the following indirect route
- First, we try to represent the discrete-time signal

$x[n]$ in a continuous-time form

What continuous-time form should we choose?

Think of complex exponentials

$x(t) = e^{j\omega t} \quad e^{j\Omega n} = g[n]$

f $\quad g[n] = x(nT)$ then $\Omega = \omega T$

This gives us a hint

More formally we now want to find a continuous time function $x_\delta(t)$ such that.

$$x_\delta(t) \xleftrightarrow{FT} X_\delta(j\omega)$$

$$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$$

and $\quad X_\delta(j\omega) \quad X(e^{j\Omega})\Big|_{\Omega=\omega T}$

what is $x_\delta(t)$?

Well, $$X_\delta(j\omega) = \sum_n x[n]\, e^{-j\omega T n}.$$

but from the time-shift property,

$$\delta(t-nT) \xleftrightarrow{FT} e^{-j\omega T n}$$

$$\Rightarrow \quad x_\delta(t) \quad \sum_n x[n]\,\delta(t-nT) \xleftrightarrow{FT} X_\delta(j\omega) = \sum_n x[n]\, e^{-j\omega T}$$

That is, if we represent a discrete-time signal in continuous time by a weighted sequence of impulses we preserve the Fourier representation.

Fig 4.17

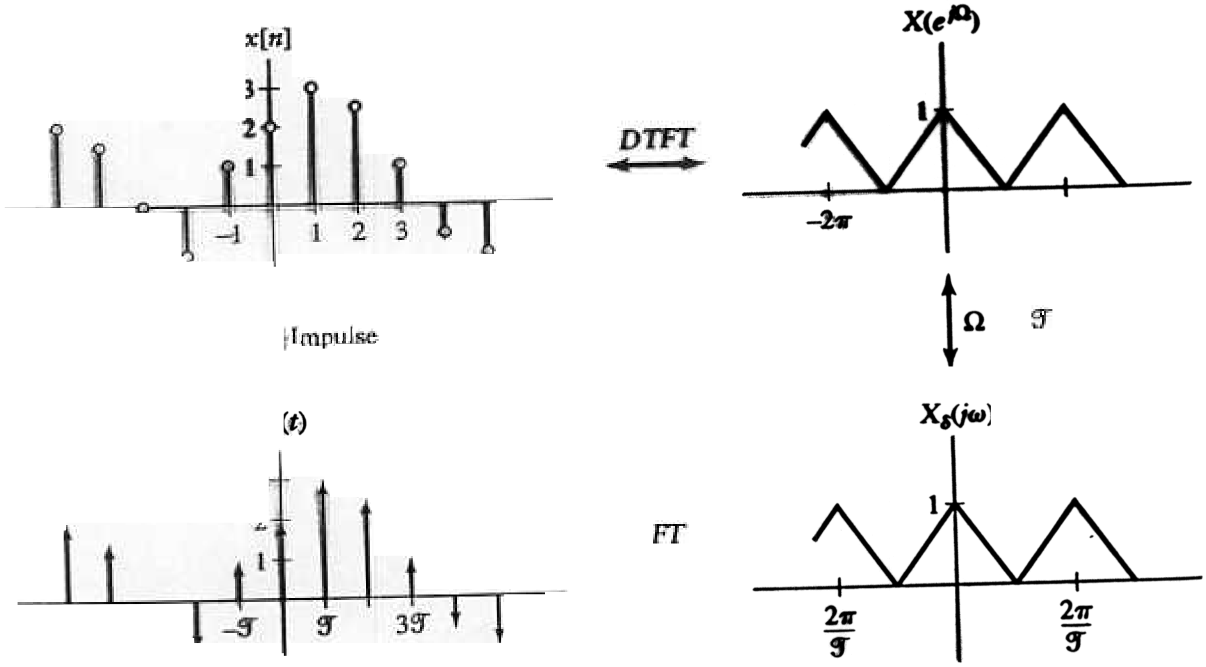


FIGURE 4. Relationship betw FT and DTFT representations of discrete signal.

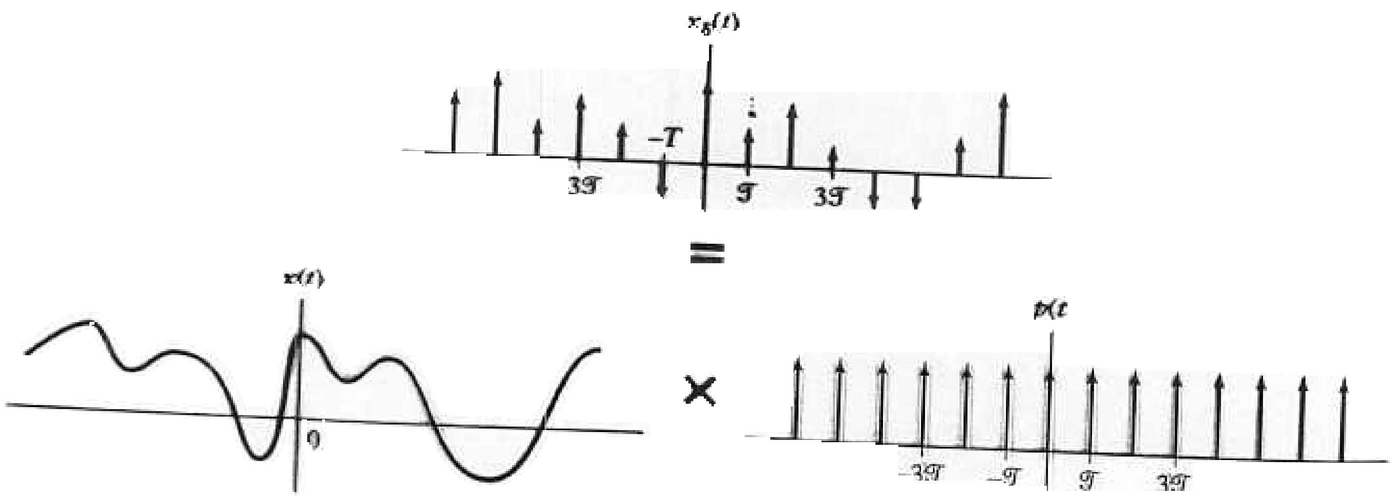


FIGURE 4. 9 Mathematical representat of sampling as the product of given signal an impulse train