Example 4.13

Consider $x(t) = \cos(\pi t)$

$$x(t) \overset{FT}{\longleftrightarrow} X(j\omega) = \pi\,\delta(\omega+\pi) + \pi\,\delta(\omega-\pi)$$

$$x_\delta(t) \overset{FT}{\longleftrightarrow} X_\delta(j\omega) \quad \frac{\pi}{T} \sum_k \delta(\omega+\pi\ k\omega_s) + \delta(\omega-\pi-k\omega_s)$$

$$\omega_s = 2\pi/T$$

For T $1/4$ ω_s 8π, no aliasing

For $T = 3/2$, $\omega_s = 4\pi/3$, sampled signal has taken on the identity of a sinusoid of frequency $\pi/3$

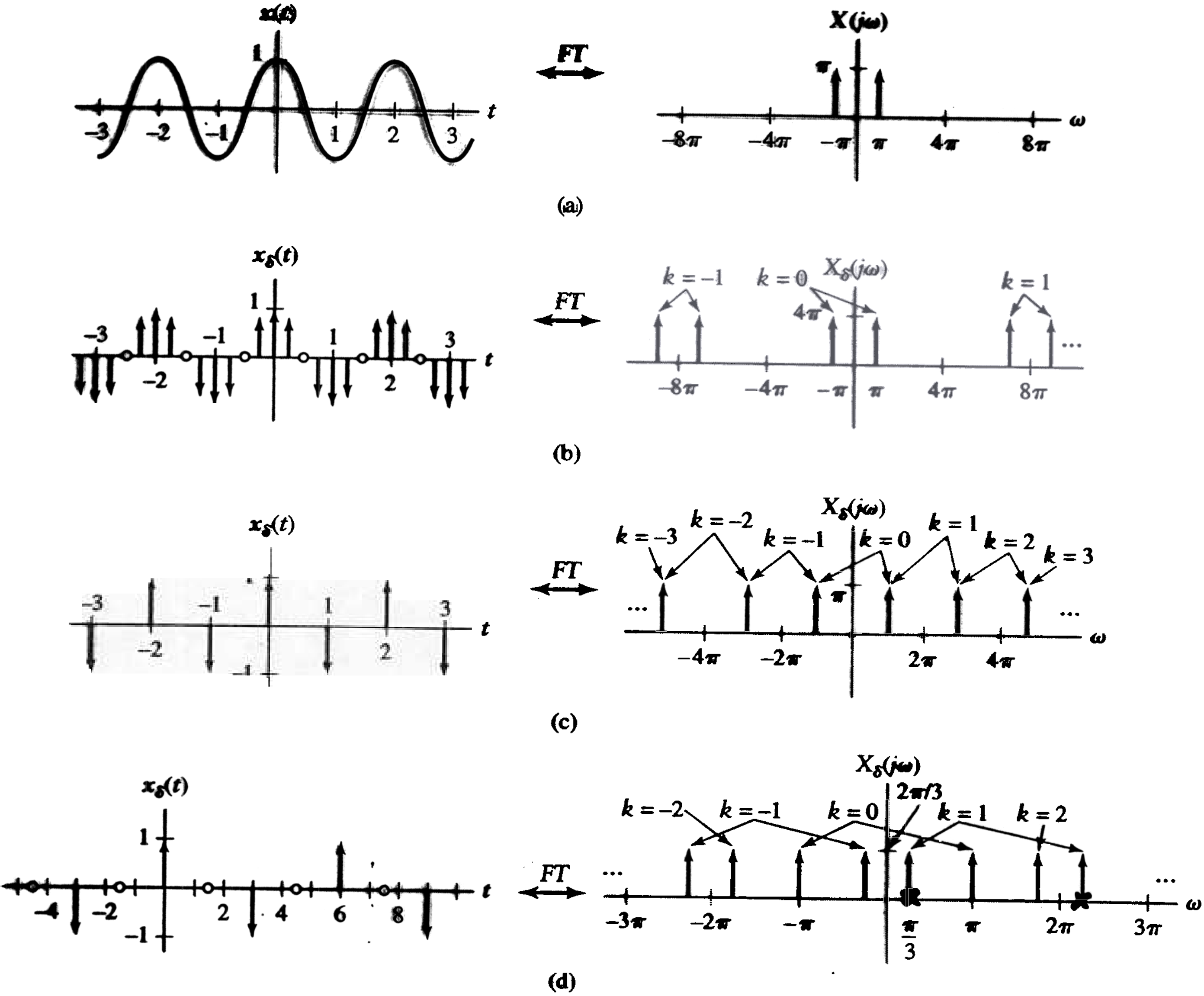

FIGURE 4.22 Effect of sampling a sinusoid at different rates. (a) Original signal and FT. (b) Sampled signal and FT for $\mathcal{T} = \frac{1}{4}$. (c) Sampled signal and FT for $\mathcal{T} = 1$. (d) Sampled signal and FT for $\mathcal{T} = \frac{3}{2}$.

YESTERDAY

Developed a convenient, but non-physical, model of a discrete-time signal

$$x_\delta(t) = \sum_n x[n]\, \delta(t - nT)$$

- This is convenient because

$$X_\delta(j\omega) = \int x_\delta(t)\, e^{-j\omega t}\, dt$$

$$= X(e^{j\Omega})_{\Omega\ \omega T}$$

where $$X(e^{j\Omega}) = \sum_n x[n]\, e^{-j\Omega n}.$$

ie

Continuous-time FT of $x_\delta(t)$

Scaled version of DTFT of $x[n]$