Additional references for
“Enriching the Art of FIR Filter Design . . . ”

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12 August 2010

Due to the strict length requirements for papers to be published in the May 2010 issue of the *IEEE Signal Processing Magazine*, I had to omit a number of important references from the paper

> Timothy N. Davidson,
> “Enriching the Art of FIR Filter Design via Convex Optimization”,
> *IEEE Signal Processing Magazine*,
> vol. 27, no. 3, pp. 89–101, May 2010.

Rather than trying to provide a review of the additional literature that is in any way comprehensive, the goal of this document is simply to draw attention to those references that were omitted due to length constraints in the final stages of editing the paper.

In order to give this document some structure, I will partition it in a similar way to the original paper. References from the original paper will be cited using numbers in brackets, as they were therein, and the additional references provided herein will be cited using an alpa-numeric key based on the authors’ names and the date of publication (the BibTeX ‘alpha’ style).

Opening Remarks / Introduction

- In addition to references [1]–[7], there are a number of other works that provide tutorial overviews of digital filter design, including [Vai93, TVN03, Bur09].
- In the discussion of the PCLS method, it would have been nice to have been able to cite the earlier work of Adams [Ada91], and a later paper by Selesnick *et al* [SLB98].
- In the discussion of linear programming techniques, it would have been nice to have been able to cite the earlier work of Rabiner [Rab72].
- One of the themes of the paper is the importance of design trade-offs. An early discussion on that theme appears in the work of Tufts and Francis [TF70].

Generic Formulation of Filter Design Problems

- In the last paragraph of the left column on page 91, there is a discussion of approaches to solving the formulation in (7) in the general case. Although I only had space to mention branch-and-bound methods and to cite reference [22], there are many approaches to global optimization. A number of those approaches are discussed in [HPT00, HP95, PR02].

  In addition to Nocedal and Wright [23], there are a number of other books on local optimization techniques, including Bertsekas [Ber99], Gill *et al.* [GMW81], Antoniou and Lu [AL07], and the second edition of Nocedal and Wright [NW06]. The book by Antoniou and Lu [AL07] includes a number of filter design examples.

- For the discussion in the next paragraph, some interesting additional references on convex optimization are the seminal work of Nesterov and Nemirovski [NN94], the lectures of Ben-Tal and Nemirovski [BTN01], and the recent work of Bertsekas [Ber09].
- In the discussion on page 92 that precedes equation (8), it would have been nice to have been able to add a reference to the work of Daubechies [Dau92], as the autocorrelation played a key role in her seminal work on wavelets.

Box entitled: Finite Representations of Spectral Mask Constraints

- Some other examples of sophisticated discretization techniques and their application are those in [28], [46] and [TSTN05].
- One example of an exchange algorithm that incorporates different constraints is that for wavelet design in [RD94]. Exchange algorithms also play a key role in many PCLS designs; e.g., [11], [12] and [Lan99].
- The representation of spectral mask constraints using linear matrix inequalities has a far richer history than the references that I was able to cite here ([34]–[39]). One of the key contributions was made by Nesterov [Nes99]. Some other relevant contributions were made by Genin *et al* [GHNV03] and Roh and Vandenberghe [RV06].

Frequency-Domain Criteria

- In the discussion surrounding (16)–(18), there are many citations that could have been added. A few of these are [CP87, KM95, Tse95].
- I did not allocate much space to the discussion of different ways in which magnitude and phase criteria can be combined, as the combination of design criteria was treated generically in the formulation in (7). An elegant overview of some of the possible combinations is available in [42].

Time-Domain Criteria

- Additional references on envelope constrained filter design include [EFC77, VCT97a, VCT97b].

Design Example

- In the opening of this example, it would have been nice to have been able to cite the work of Crochiere and Rabiner [CR83].
- I was unable to find space for a general discussion of interpolated filters, but the basic design principles outlined in the paper also apply to the components of interpolated filters. Some of the key references on interpolated filters are [NCYM84, SNM88, MMS93, Lyo03].
- The idea of interpolated filters was applied to oversampled DFT filter banks by Liu *et al* [LCH00]. The symlet was designed by Daubechies [Dau92].
- The iterative reweighted least squares technique mentioned in the middle of the right hand column on page 98 has been used in many filter design problems. In addition to the work cited in reference [48], some other pertinent references are [PS94, Lan98, LCTR06].

Nonconvex Design Problems

- There has been some recent interest in applying the principles of the "filled function" approach to solving a class of global optimization problems to certain nonconvex filter design problems; e.g., [FT08, WTRD08].
- Once example of a filter design method in which relaxation is employed is that for the design of cosine-modulated filter banks in [KTN09].
- In addition to [61] and [62], some other references on discrete optimization techniques for filters with quantized coefficients are [Kod80, KS80, LP83, LCL93, Kod05, FT08].

Other Filter Architectures

This section was significantly constrained by the limits imposed on the length of the paper.

- Implicit in the earlier discussion was the connection with multi-rate filter banks; e.g., [28]–[31], [54] and [Vai93]. The somewhat different perspective on convexity and filter banks provided by Akkarakaran and Vaidyanathan [AV01] generates considerable insight into filter bank design.
- Some examples of direct design methods for complex-valued FIR filters appear in [47], [PS94, KM95, Tse95, AS97, TCT05]. Furthermore, some problems with nonconvex criteria can be tackled using sequential approximation algorithms in which a convex problem is solved at each step; e.g., [60] and [WBZ$^+$03].
- The tutorial by Van Veen and Buckley [VB88] is one place where the connection between spatial and temporal filtering is made. In addition to [34] and [64], some other papers that make use of this connection are [NZC01, WBZ$^+$03].
- Some other references on "variable" filters with Farrow structure are [LD99, Den01, Tse02, Den04, LJ06, TCT05, Den06, DCTN07, DCNT08, KJ09].
- In the discussion on the design of IIR filters using techniques that involve convex local approximations, some other useful references, in addition to [4], [19], and [67]–[71], are [CP90, LPT98, Lu98, Tse04].
- In addition to [4], some other references on the application of model reduction techniques are [SA92, BKC92, BU01].
- The last paragraph of this section, which considers multi-dimensional filters, is rather brief. The following is a slightly more informative statement.

  > The design of multi-dimensional filters is somewhat more involved than the design of the one-dimensional filters that we have considered herein, but some aspects of our discussion extend to generic design problems for such filters (e.g., [19] and [Dum05, RDV07]), and to their application in wideband antenna arrays; e.g., [SC07].

Conclusion

- In the discussion on embedding in the last paragraph of the conclusion, attention should be drawn to the paper of Mattingley and Boyd [MB10] that appeared in the special issue.

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