An Efficient Design Method for Vector Broadcast Systems with Common Information

R. H. Gohary, T. N. Davidson, and Z.-Q. Luo
Department of Electrical and Computer Engineering
McMaster University
Hamilton, Ontario, Canada

Abstract*— We consider the problem of determining an optimal transmission scheme for broadcasting a common message over vector channels, given (perfect) channel knowledge at both the receive and transmit ends. We provide an efficient method for jointly designing a linear transmitter and and a set of linear receivers so as to minimize a weighted Mean Square Error (WMSE) of the data estimates. The computational efficiency follows from the convex formulations that we develop. These formulations enable utilization of highly efficient interior point methods. For diagonal channel matrices, which appear in multicarrier systems that employ cyclic prefixing, we show that the optimal transmitter is obtained by subcarrier allocation and power loading. The set of minimum MSE transceivers for a vector broadcast system is parametrized by a unitary matrix degree of freedom. For the case of diagonal systems, we show how this unitary matrix can be chosen so that the symbol error rate is minimized (over the given set). This optimal unitary matrix ensures that for each receiver, the subcarrier signal-to-noise ratios (SNRs) are all the same. Simulations indicate that our designs can provide significantly improved performance over standard designs.

I. INTRODUCTION

Several applications require the reliable transmission of a common message from a single transmitter to multiple receivers. Examples include common control signals in cellular communication systems and (subscription based) radio and TV broadcast systems. In such systems, when the channel varies relatively slowly compared to the transmission rate, the receivers are able to estimate a sufficiently accurate model of the channel without substantial sacrifice of the link throughput. If the receivers feed this information back to the transmitter, an optimal transmission scheme with respect to a prescribed measure can be determined for the set of estimated channels.

In this paper, we focus on the design of vector transmission schemes in which data is transmitted on a block-by-block, rather than symbol-by-symbol, basis. Such schemes are often effective for transmission over frequency selective channels; e.g., Discrete MultiTone modulation (DMT) [1], and Orthogonal Frequency Division Multiplexing (OFDM) [2]. However, they also appear naturally in communication systems with multiple antennas at the transmitter and receiver. For single-user vector communication systems, there are several mature approaches to transceiver design (see, for example, [3] for an insightful overview). Typically, the optimal transmission scheme involves a decomposition of the channel into a set of parallel subchannels and the allocation of power and rate to each subchannel. For certain criteria, it may also be

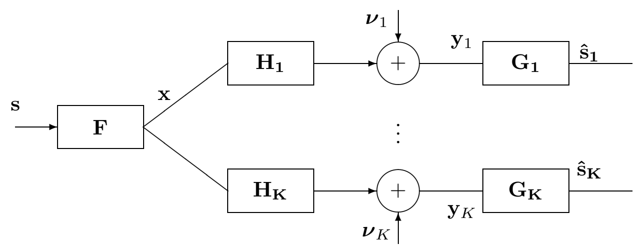

Fig. 1. Block diagram for the broadcasting system under consideration in Section II.

beneficial for the transmitter to linearly combine the symbols intended for transmission over different subchannels (see, for example, [3]–[5]) .

In contrast to the single-user case, effective design methods for vector broadcast systems are just beginning to emerge (e.g., [6]–[9]). The goal of this paper is to contribute to the development of such methods by providing a computationally efficient method for jointly designing a linear transmitter and a set of linear receivers so as to minimize a weighted Mean Square Error (WMSE) measure of the received data estimates. We show that this design problem can be cast as a convex optimization program that can be efficiently solved using Interior Point (IP) methods [10]. Furthermore, when the channel matrices are simultaneously diagonalizable—as they are in MultiCarrier (MC) schemes that employ cyclic prefixing (e.g., DMT/OFDM)—optimal transmission can be obtained by subcarrier allocation and power loading, and the computational effort required to obtain the optimal transmitter can be considerably reduced. We observe that the minimum MSE solution provides a unitary matrix degree of freedom, and then show how to find an optimal rotation that minimizes the average bit error rate for MMSE power-loaded, cyclic prefix based MC broadcast systems.

II. MMSE PROBLEM: PROBLEM STATEMENT AND SOLUTION

A. Formulation

In the broadcasting scheme in Figure 1 a single transmitter sends the same information vector $\mathbf{s}$ to K receivers. Each data block $\mathbf{s}$ is assumed to be zero-mean and white with identity covariance matrix. However, our results carry over

 0-7803-7974-8/03/$17.00

to the colored data case as well, because a whitening matrix $\mathbf{R}_s^{-1/2}$, where $\mathbf{R}_s$ is the covariance matrix of $\mathbf{s}$, can readily be absorbed into the precoder $\mathbf{F}$. Each receiver in Figure 1 has a channel matrix $\mathbf{H}_i$, $i = 1, \ldots, K$ which is of size $q \times n$, where q is the length of the received block, $\mathbf{y}_i$, and n is the length of the transmitted block, $\mathbf{x}$. The design problem is to jointly design the linear transmitter $\mathbf{F}$ and K linear receivers $\mathbf{G}_i$ such that the total (weighted) MSE is minimized and the transmitted power remains below a prescribed level, p. (A related problem is the design of linear transmitters and a single linear receiver in a multiple access scenario [11].)

The received signal vectors $\mathbf{y}_i$, $i = 1, \ldots, K$, in Figure 1 are given by:

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{F} \mathbf{s} + \boldsymbol{\nu}_i,$$

where $\boldsymbol{\nu}_i$ is the zero-mean Gaussian noise associated with the i^{th} receiver, which has a known covariance matrix $\mathbf{R}_i$. The equalizer output is

$$\hat{\mathbf{s}}_i = \mathbf{G}_i \mathbf{H}_i \mathbf{F} \mathbf{s} + \mathbf{G}_i \boldsymbol{\nu}_i, \quad i = 1, \ldots, K. \tag{1}$$

Let $\mathbf{e}_i$ denote the error vector associated with the i^{th} receiver,

$$\mathbf{e}_i = \mathbf{s} - \hat{\mathbf{s}}_i.$$

Then the weighted MSE is given by

$$\mathrm{WMSE} = \sum_{i=1}^{K} \alpha_i \operatorname{Tr}(\mathrm{E}\{\mathbf{e}_i \mathbf{e}_i^H\}), \tag{2}$$

where the α_i's are non-negative weights assigned to different users depending on their relative priorities and $\operatorname{Tr}(\cdot)$ denotes the trace operation. The covariance matrix of $\mathbf{e}_i$ is

$$\mathrm{E}\{\mathbf{e}_i \mathbf{e}_i^H\} = (\mathbf{I} - \mathbf{G}_i \mathbf{H}_i \mathbf{F})(\mathbf{I} - \mathbf{G}_i \mathbf{H}_i \mathbf{F})^H + \mathbf{G}_i \mathbf{R}_i \mathbf{G}_i^H$$

where, as stated earlier, the covariance matrices of the signal and noise are $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$ and $\mathrm{E}\{\boldsymbol{\nu}_i \boldsymbol{\nu}_i^H\} = \mathbf{R}_i$ respectively, and the transmitted signal and receiver noise are uncorrelated; i.e., $\mathrm{E}\{\mathbf{s}\boldsymbol{\nu}_i^H\} = \mathbf{0}$, $i = 1, \ldots, K$.

The problem of designing $\mathbf{F}$ and $\mathbf{G}_i$ so as to minimize the weighted MSE subject to a bound on the transmitted power can be cast as the following optimization problem:

$$\min_{\mathbf{F}, \mathbf{G}_1, \ldots, \mathbf{G}_K} \quad \sum_{i=1}^{K} \alpha_i \operatorname{Tr}(\mathrm{E}\{\mathbf{e}_i \mathbf{e}_i^H\}) \tag{3a}$$

$$\text{subject to} \quad \operatorname{Tr}(\mathbf{F}\mathbf{F}^H) \leq p. \tag{3b}$$

Since the $\mathbf{G}_i$'s are unconstrained variables, they can be eliminated from (3) by first minimizing the weighted MSE with respect to $\mathbf{G}_i$. This results in the MMSE equalizers,

$$\mathbf{G}_i = \mathbf{F}^H \mathbf{H}_i^H \big(\mathbf{H}_i \mathbf{F} \mathbf{F}^H \mathbf{H}_i^H + \mathbf{R}_i\big)^{-1} = \mathbf{F}^H \mathbf{H}_i^H \mathbf{W}_i, \tag{4}$$

where

$$\mathbf{W}_i = \big(\mathbf{H}_i \mathbf{F} \mathbf{F}^H \mathbf{H}_i^H + \mathbf{R}_i\big)^{-1}, \quad i = 1, \ldots, K. \tag{5}$$

Substituting (4) into (3a) yields

$$\begin{aligned}
\mathrm{WMSE} &= n \sum_{i=1}^{K} \alpha_i - \operatorname{Tr}\big(\sum_{i=1}^{K} \alpha_i \mathbf{F}^H \mathbf{H}_i^H \mathbf{W}_i \mathbf{H}_i \mathbf{F}\big) \\
&= n \sum_{i=1}^{K} \alpha_i - \operatorname{Tr}\big(\sum_{i=1}^{K} \alpha_i (\mathbf{W}_i^{-1} - \mathbf{R}_i) \mathbf{W}_i\big) \\
&= \operatorname{Tr}\big(\sum_{i=1}^{K} \alpha_i \mathbf{W}_i \mathbf{R}_i\big).
\end{aligned}$$

Noting that $\mathbf{R}_i$ and $\mathbf{W}_i$ are positive definite for all $i = 1, \ldots, K$, and letting $\mathbf{U} = \mathbf{F}\mathbf{F}^H$, the optimal MMSE transceiver design problem can be cast as:

$$\min_{\mathbf{U}, \mathbf{W}_i,\, i=1,\ldots,K} \quad \operatorname{Tr}\big(\sum_{i=1}^{K} \alpha_i \mathbf{W}_i \mathbf{R}_i\big) \tag{6a}$$

$$\text{subject to} \quad \mathbf{W}_i \succeq (\mathbf{H}_i \mathbf{U} \mathbf{H}_i^H + \mathbf{R}_i)^{-1}, \quad \forall i \tag{6b}$$

$$\operatorname{Tr}(\mathbf{U}) \leq p, \tag{6c}$$

$$\mathbf{U} \succeq \mathbf{0}, \tag{6d}$$

where by $\mathbf{X} \succeq \mathbf{Y}$, we mean that $\mathbf{X} - \mathbf{Y}$ is positive semidefinite. Using the Schur complement [10], the constraint (6b) can be re-written in a linear matrix inequality (LMI) form as:

$$\begin{bmatrix} \mathbf{W}_i & \mathbf{I} \\ \mathbf{I} & \mathbf{H}_i \mathbf{U} \mathbf{H}_i^H + \mathbf{R}_i \end{bmatrix} \succeq \mathbf{0}, \qquad i = 1, \ldots, K. \tag{7}$$

Therefore, the formulation in (6) can be rewritten as:

$$\min_{\mathbf{U}, \mathbf{W}_i} \quad \operatorname{Tr}\big(\sum_{i=1}^{K} \alpha_i \mathbf{W}_i \mathbf{R}_i\big) \tag{8a}$$

$$\text{subject to} \quad \text{(7)}, \operatorname{Tr}(\mathbf{U}) \leq p, \text{ and } \mathbf{U} \succeq \mathbf{0} \tag{8b}$$

This problem is a semidefinite program (SDP) and can be efficiently solved using interior point methods. Several convenient implementations of these methods are available; e.g., [12]. The arithmetic complexity of these methods is at most $O(n^{6.5} \log(1/\epsilon))$, where $\epsilon > 0$ is the solution accuracy. Once an optimal $\mathbf{U}$ is obtained, we need to find an optimal $\mathbf{F}$ such that $\mathbf{F}\mathbf{F}^H = \mathbf{U}$. The set of all WMSE optimal precoders take the form

$$\mathbf{F} = \breve{\mathbf{F}} \mathbf{Q}, \tag{9}$$

where $\breve{\mathbf{F}}$ is the Cholesky factor of the optimal $\mathbf{U}$ and $\mathbf{Q}$ is an arbitrary unitary matrix.

B. Diagonal designs

For a multicarrier system employing cyclic prefixing, the channel matrices $\mathbf{H}_i$ are circulant and hence can be simultaneously diagonalizable using Discrete Fourier Transform (DFT) and Inverse Discrete Fourier Transform (IDFT) matrices. The diagonal elements of the diagonalized channel matrices are given by $\mathbf{H}_i(k, k) = H_i(k)$, where $H_i(k)$ is the frequency response of user i's channel at the kth point on the DFT grid, $\omega_k = 2\pi(k-1)/n$, $k = 1, \ldots, n$. By associating the DFT and IDFT matrices with the equalizer and precoder respectively, we end up with a diagonal channel matrix. We will also assume in this section that the noise covariance matrices are also diagonalized by the DFT matrix; i.e., $\mathbf{R}_i(k,j) = 0,\ j \neq k$ and $\mathbf{R}_i(k,k) = \sigma_i^2(k)$. (This is a common assumption in the design of single-user DMT schemes. See [11] for further details.)

If we assume that the optimal matrices $\mathbf{W}_i$ and $\mathbf{U}$ are diagonal, we can replace them in the formulation by vectors $\mathbf{w}_i$ and $\mathbf{u}$ that represent the diagonal elements of $\mathbf{W}_i$ and $\mathbf{U}$ respectively. (We will argue below that the optimal $\mathbf{W}_i$ and $\mathbf{U}$ are indeed diagonal.) With these new variables, problem (8) can be cast as

$$\min_{\mathbf{u},\mathbf{w}_i} \quad \sum_{i=1}^{K}\sum_{k=1}^{n} \alpha_i \sigma_i^2(k)\mathbf{w}_i(k) \tag{10a}$$

$$\text{subject to} \quad \mathbf{w}_i(k)\big(\mathbf{u}(k)|H_i(k)|^2 + \sigma_i^2(k)\big) \geq 1, \quad \forall i \tag{10b}$$

$$\sum_k \mathbf{u}(k) \leq p, \tag{10c}$$

$$\mathbf{u}(k) \geq 0,\ \ \forall k = 1,\ldots,n. \tag{10d}$$

The constraints in (10b) amount to K sets of n Lorentz cones. The optimization problem (10) is a rotated second order cone program that offers significant computational advantage over the SDP in (8). In particular, the arithmetic complexity of obtaining a solution to the second order cone program is at most $O(n^{3.5}\log(1/\epsilon))$, where $\epsilon > 0$ is the solution accuracy.

In deriving the above formulation (10) we have assumed that the optimal precoder is a diagonal matrix $\mathbf{U}$. We now show that the optimal $\mathbf{U}$ is necessarily diagonal. Suppose that the optimal solution is given by $\mathbf{U}^*$, where $\mathbf{U}^*$ is assumed to be not diagonal. Let $\bar{\mathbf{U}}^*$ be the diagonal part of $\mathbf{U}^*$. Then, $\mathrm{Tr}(\mathbf{U}^*) = \mathrm{Tr}(\bar{\mathbf{U}}^*)$. Therefore, $\bar{\mathbf{U}}^*$ satisfies (6c) and lies in the feasible set of (6). Now, for any positive definite matrix $\mathbf{A}$,

$$\mathrm{Tr}(\mathbf{A}^{-1}) \geq \sum_j \frac{1}{\mathbf{A}_{jj}}\ , \tag{11}$$

with equality holding iff $\mathbf{A}$ is diagonal [13]. For a given $\mathbf{U}$, let $\mathbf{A}_i(\mathbf{U}) = \mathbf{R}_i^{-1/2}(\mathbf{H}_i\mathbf{U}^*\mathbf{H}_i^H + \mathbf{R}_i)\mathbf{R}_i^{-1/2}$, $i = 1,\ldots,K$, where we have assumed that $\mathbf{R}_i \succ \mathbf{0}$. If the equalizers $\mathbf{G}_i$ are chosen as in (4), then for a given $\mathbf{U}$

$$\mathrm{WMSE} = \sum_{i=1}^{K} \alpha_i\, \mathrm{Tr}(\mathbf{A}_i(\mathbf{U})^{-1}). \tag{12}$$

If we define $\mathbf{A}_i^* = \mathbf{A}_i(\mathbf{U}^*)$, and $\bar{\mathbf{A}}_i^* = \mathbf{A}_i(\bar{\mathbf{U}}^*)$, then using (11) we have that

$$\mathrm{Tr}\big((\mathbf{A}_i^*)^{-1}\big) > \mathrm{Tr}\big((\bar{\mathbf{A}}_i^*)^{-1}\big), \qquad i = 1,\ldots,K \tag{13}$$

where the strict inequality holds because $(\mathbf{A}_i^*)^{-1}$ is non-diagonal since $\mathbf{U}^*$ is non-diagonal. Using the strict inequalities in (13) we have that

$$\mathrm{WMSE}\,|_{\mathbf{U}=\mathbf{U}^*} > \mathrm{WMSE}\,|_{\mathbf{U}=\bar{\mathbf{U}}^*}.$$

Thus we have a contradiction to our assumption that $\mathbf{U}^*$ was optimal. Therefore, the optimal precoder $\mathbf{F}$ must be such that $\mathbf{U} = \mathbf{F}\mathbf{F}^H$ is diagonal.

C. Choice of Optimal Rotation

So far, we have shown how to design a linear precoder $\mathbf{F}$ that satisfies an MMSE criterion. From an MMSE perspective, it turned out that the design problem amounts to designing $\mathbf{U} = \mathbf{F}\mathbf{F}^H$. However, as pointed out in (9), this solution offers a unitary matrix degree of freedom. That is, in general we can write $\mathbf{F} = \breve{\mathbf{F}}\mathbf{Q}$, where $\mathbf{Q}$ is an arbitrary unitary matrix to be designed. In this section, we show how $\mathbf{Q}$ can be chosen to essentially minimize the average bit error rate (over the class of MMSE receivers). Our development will involve arguments which parallel those in [4], [5] and [3], where minimum BER transmitter-receiver design for the single-user case was considered.

Consider a complex valued circularly symmetric signal (e.g., M–ary QAM). The i^{th} user equalized output signal block given by (1) can be written as

$$\hat{\mathbf{s}}_i = \mathrm{Diag}(\mathbf{G}_i\mathbf{H}_i\mathbf{F})\mathbf{s} + \mathbf{z}_i \tag{14}$$

where

$$\mathbf{z}_i = \big(\mathbf{G}_i\mathbf{H}_i\mathbf{F} - \mathrm{Diag}(\mathbf{G}_i\mathbf{H}_i\mathbf{F})\big)\mathbf{s} + \mathbf{G}_i\boldsymbol{\nu}_i \tag{15}$$

denotes the interference plus noise term. By modifying the analysis of MMSE multiuser detectors [14], [15], it can be shown [5] that as the block size n increases, the ISI term approaches a Gaussian distributed random variable almost surely. Hence, the i^{th} user's noise plus interference term $\mathbf{z}_i$ can be approximated as a Gaussian process with zero mean and covariance

$$\mathbf{C}_i = \big(\mathbf{G}_i\mathbf{H}_i\mathbf{F} - \mathrm{Diag}(\mathbf{G}_i\mathbf{H}_i\mathbf{F})\big)\big(\mathbf{G}_i\mathbf{H}_i\mathbf{F} - \mathrm{Diag}(\mathbf{G}_i\mathbf{H}_i\mathbf{F})\big)^H + \mathbf{G}_i\mathbf{R}_i\mathbf{G}_i^H.$$

Using this result, we can compute the asymptotic bit error rate as the block size grows for signals drawn from different constellations. In the following, we will only consider QPSK modulation. However similar results can be obtained for higher order circularly symmetric modulation schemes.

For QPSK, the average bit error rate is given by

$$P_e \simeq \frac{1}{2Kn}\sum_{i=1}^{K}\sum_{k=1}^{n} \mathrm{erfc}\left(\frac{[\mathbf{G}_i\mathbf{H}_i\mathbf{F}]_{kk}}{\sqrt{2[\mathbf{C}_i]_{kk}}}\right), \tag{16}$$

where, $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-z^2)dz$. The approximation in the above expression follows from the assumption of Gaussian ISI, and hence the error in the approximation decreases almost surely as the block size, n, grows. Using the fact that for the MMSE equalizers (4),

$$\mathbf{G}_i\big(\mathbf{H}_i\mathbf{F}\mathbf{F}^H\mathbf{H}_i^H + \mathbf{R}_i\big)\mathbf{G}_i^H = \mathbf{G}_i\mathbf{H}_i\mathbf{F},$$

we can show that

$$[\mathbf{C}_i]_{kk} = [\mathbf{G}_i\mathbf{H}_i\mathbf{F}]_{kk} - \big([\mathbf{G}_i\mathbf{H}_i\mathbf{F}]_{kk}\big)^2.$$

Substituting into the right hand side of (16), we get

$$P_e \simeq \frac{1}{2Kn}\sum_{i,k} \operatorname{erfc}\Big(\big(2\big[(\operatorname{Diag}(\mathbf{G}_i\mathbf{H}_i\mathbf{F}))^{-1} - \mathbf{I}\big]_{kk}\big)^{-1/2}\Big) \tag{17}$$

Applying the fact that $\operatorname{erfc}(\frac{1}{\sqrt{2x}})$ is a convex function of x for all $x \leq \frac{1}{3}$ [5], we observe that if

$$[\mathbf{G}_i\mathbf{H}_i\mathbf{F}]_{rr} \geq \frac{3}{4},\ \forall i = 1, \ldots, K \text{ and } \forall\, r = 1, \ldots, n, \tag{18}$$

then the probability of error defined in (17) is a convex function of the diagonal entries of $\mathbf{G}_i\mathbf{H}_i\mathbf{F}$. Hence, a tight lower bound on the average error probability can be obtained by applying Jensen's inequality. That is,

$$\begin{aligned} P_e &\geq \frac{1}{2K}\sum_i \operatorname{erfc}\left(\frac{1}{\sqrt{\frac{2}{n}\sum_r\big[\big(\operatorname{Diag}(\mathbf{G}_i\mathbf{H}_i\mathbf{F})\big)^{-1} - \mathbf{I}\big]_{kk}}}\right) \\ &= \frac{1}{2K}\sum_i \operatorname{erfc}\left(\frac{1}{\sqrt{\frac{2}{n}\operatorname{Tr}\big(\operatorname{Diag}(\mathbf{G}_i\mathbf{H}_i\mathbf{F})\big)^{-1} - 1}}\right). \end{aligned} \tag{19}$$

The lower bound in (19) is achievable if for each $i \in [1, K]$, the following condition holds:

$$[\mathbf{G}_i\mathbf{H}_i\mathbf{F}]_{rr} = [\mathbf{G}_i\mathbf{H}_i\mathbf{F}]_{\ell\ell}, \qquad \forall r,\ \ell \in [1, n]. \tag{20}$$

We now design a precoder that minimizes (19). As in (9), $\mathbf{F} = \breve{\mathbf{F}}\mathbf{Q}$. Hence, we can write

$$\begin{aligned} \mathbf{G}_i\mathbf{H}_i\mathbf{F} &= \mathbf{F}^H\mathbf{H}_i^H\big(\mathbf{H}_i\breve{\mathbf{F}}\breve{\mathbf{F}}^H\mathbf{H}_i^H + \mathbf{R}_i\big)^{-1}\mathbf{H}_i\mathbf{F} \\ &= \mathbf{Q}^H\mathbf{\Gamma}_i\mathbf{Q}, \end{aligned}$$

where $\mathbf{\Gamma}_i = \breve{\mathbf{F}}^H\mathbf{H}_i^H\big(\mathbf{H}_i\breve{\mathbf{F}}\breve{\mathbf{F}}^H\mathbf{H}_i^H + \mathbf{R}_i\big)^{-1}\mathbf{H}_i\breve{\mathbf{F}}$ is positive definite and does not depend on $\mathbf{Q}$. We now restrict our attention to the moderate to high SNR region satisfying (18) and will show how to find a unitary matrix $\mathbf{Q}$ that not only minimizes the lower bound in (19) but also achieves it. We start by observing that $\operatorname{erfc}(\cdot)$ is a monotonically decreasing function of its argument and hence minimizing (19) amounts to solving the following optimization problem,

$$\min_{\mathbf{Q}\mathbf{Q}^H = \mathbf{I}} \operatorname{Tr}\big(\operatorname{Diag}(\mathbf{Q}^H\mathbf{\Gamma}_i\mathbf{Q})\big)^{-1}, \quad \forall i \in [1, K].$$

It can be shown that [5],

$$\operatorname{Tr}\big(\operatorname{Diag}(\mathbf{Q}\mathbf{\Gamma}_i\mathbf{Q}^H)\big)^{-1} \geq \frac{n^2}{\operatorname{Tr}(\mathbf{\Gamma}_i)}, \tag{21}$$

with the lower bound achieved if and only if the diagonal elements of $\mathbf{Q}^H\mathbf{\Gamma}_i\mathbf{Q}$ are all equal. One choice of $\mathbf{Q}$ that satisfies this requirement is $\mathbf{Q} = \mathbf{X}_i\mathbf{V}$, where $\mathbf{X}_i$ is the matrix whose columns are the eigenvectors of $\mathbf{\Gamma}_i$ and $\mathbf{V}$ is the normalized DFT matrix. Since we have a single precoder to optimize with respect to the transmission over K channels, our argument will only hold when $\mathbf{X}_i$ are identical for all $i \in [1, K]$. That is, all $\mathbf{\Gamma}_i$'s share the same eigenvectors. A special case where this happens naturally is in cyclic prefixed multicarrier systems. In that case, the MMSE solution implies diagonal $\mathbf{\Gamma}_i$'s. Hence, $\mathbf{X}_i = \mathbf{I}\ \ \forall i \in [1, K]$ and $\mathbf{Q} = \mathbf{V}$. Therefore, with $\mathbf{Q}$ in (9) chosen to be the DFT matrix, the lower bound in (21), and hence that in (19), are achieved. The resulting minimized bit error rate is given by

$$P_e = \frac{1}{2K}\sum_i \operatorname{erfc}\left(\frac{1}{2\sqrt{n\big[\operatorname{Tr}(\mathbf{\Gamma}_i)\big]^{-1} - 1}}\right). \tag{22}$$

Interestingly, choosing $\mathbf{Q} = \mathbf{V}$ results in each receiver "seeing" identical SNRs on all its subcarriers. Our simulations in the next section indicate that our choice of $\mathbf{Q}$ can generate significant SNR gains.

III. Numerical Results

In the following examples, we consider a two user broadcast system which employs cyclic prefix based multicarrier modulation with 32 subcarriers. The noise at each receiver is assumed to be white with a common variance; i.e., $\mathbf{R}_i = \sigma^2\mathbf{I}$. Hence, each user has the same block SNR, p/σ^2. In each of the following examples the weights α_i in (2) are chosen to be equal.

(i) In this example, each user's channel is a three-tap FIR filter. The frequency responses of these filters are plotted in Figure 2, along with the optimal power allocation. This figure shows that optimal power loading, in the MMSE sense, at an SNR of 10 dB, assigns power in a way similar to the water-filling principle. That is, subcarriers that "see" better channels are assigned higher powers and vice versa.

(ii) In Figure 3, we consider the same channels as in Figure 2. We compare the bit error rate performance when optimal and uniform power loadings are used. First, we remark that simulation results agree with analysis. This supports the fact that the ISI can be modeled accurately by a Gaussian random variable even for the rather small block length $n = 32$. When the unitary matrix degree of freedom is not exploited, we observe that for a bit error rate of 2×10^{-2}, a gain of about 3 dB is obtained via optimal power loading. The potential performance improvement from employing a proper rotation via a DFT matrix at the precoder is also illustrated. The optimal rotation introduces an additional gain of about 8 dB at a bit error rate of 10^{-2}. When the optimal rotation is employed with optimal power loading, a gain of about 3 dB is achieved over the case of optimal rotation and uniform power loading.

IV. Conclusions

In this paper we have addressed the problem of optimal transmitter and receiver designs in a weighted MMSE sense for vector broadcast systems with a common message. A convex formulation was derived for general channel matrices, and an alternative, significantly simplified, formulation was presented for cyclic prefix based multicarrier modulation schemes. For those applications, it was shown that the optimum linear

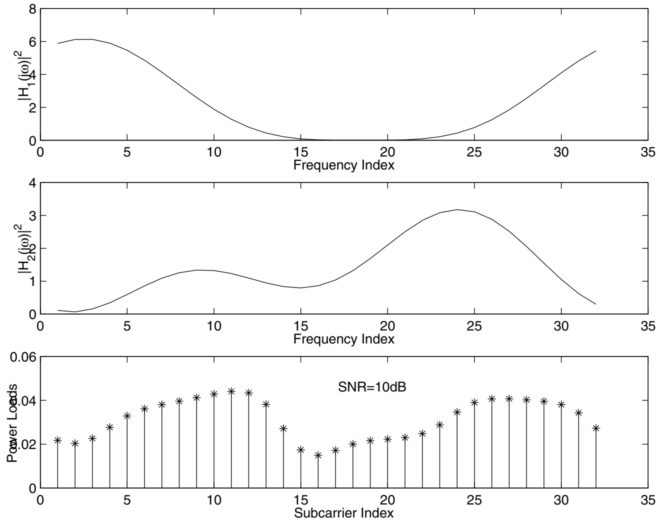

Fig. 2. Optimal power loading in the MMSE sense at a block SNR of 10 dB. The frequency responses of the two users' channels are shown.

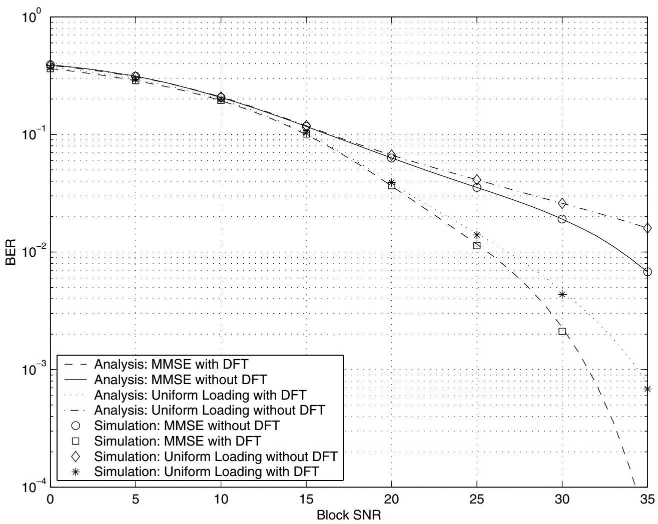

Fig. 3. A comparison of the bit error rate performance of various systems: Analytical and simulation results.

precoder problems performs subcarrier allocation followed by power loading. For moderate to high SNR, we have also shown how to optimally exploit the unitary matrix degree of freedom provided by the MMSE solution.

References

[1] P. S. Chow, J. Cioffi, and J. Bingham, "A practical discrete multitone transceiver loading algorithm for data transmission over spectrally shaped channels," *IEEE Trans. Commun.* , vol. 43, no. 2, pp. 773–775, 1995.

[2] Z. Wang and G. Giannakis, "Wireless multicarrier communications: where Fourier meets Shannon," *IEEE Signal Processing Magazine*, vol. 47, pp. 29–48, May 2000.

[3] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization," *IEEE Trans. Signal Processing*, vol. 51, pp. 2381–2401, Sept. 2003.

[4] Y. Ding, T. N. Davidson, Z.-Q. Luo, and K. M. Wong, "Minimum BER block precoders for zero-forcing equalization," *IEEE Trans. Signal Processing*, vol. 51, pp. 2410–2423, Sept. 2003.

[5] S. S. Chan, T. N. Davidson, and K. M. Wong, "Asymptotically minimum bit error rate block precoders for minimum mean square error equalization," in *Proceedings of the IEEE Signal Processing Workshop on Sensor Array and Multichannel Signal Processing*, (Roslyn, VA, USA), Aug. 2002.

[6] A. J. Goldsmith and M. Effros, "The capacity region of broadcast channels with intersymbol interference and colored Gaussian noise," *IEEE Trans. Informat. Theory*, vol. 47, pp. 219–240, Jan. 2001.

[7] W. Yu and J. Cioffi, "Sum capacity of Gaussian vector broadcast channels," Mar. 2002. Submitted to the *IEEE Transactions on Informat. Theory*. Also available at `http://www.comm.utoronto.ca/~weiyu/publications.html`.

[8] D. N. Tse, "Optimal power allocation over parallel gaussian broadcast channels," in *Proceedings of International Symposium on Information Theory*, (Ulm, Germany), p. 27, June 1997. A longer version is available on line at `http://degas.eecs.berkeley.edu/~dtse/pub.html`.

[9] J. Zhang, A. Sayeed, and B. V. Veen, "Optimal transceiver design for selective wireless broadcast with channel state information," in *Proceedings of the International Conference on Acoustics, Speech and Signal Processing*, (Orlando, FL), pp. 2153–2156, May 2002.

[10] S. Boyd and L. Vandenberghe, "Introduction to convex optimization with engineering applications," 2000. Course Notes, Stanford University.

[11] Z.-Q. Luo, T. N. Davidson, G. B. Giannakis, and K. M. Wong, "Transceiver optimization for multiple access through ISI channels." *IEEE Transactions on Signal Processing*, To appear.

[12] J. F. Strum, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optimzation Methods and Software*, vol. 11–12, pp. 625–653, 1999.

[13] F. A. Graybill, *Matrices with Applications in Statistics*. Belmont, CA: Wadsworth, second ed., 1983.

[14] H. V. Poor and S. Verdu, "Probability of error in MMSE multiuser detection," *IEEE Trans. Informat. Theory*, vol. 43, pp. 858–871, May 1997.

[15] J. Zhang, E. K. P. Chong, and D. N. C. Tse, "Output MAI distributions of linear MMSE multiuser receivers in DS-CDMA systems," *IEEE Trans. Informat. Theory*, vol. 47, pp. 1128–1144, Mar. 2001.