Blind ML Detection of Orthogonal Space-Time Block Codes: Efficient High-Performance Implementations

Wing-Kin Ma, *Member, IEEE*, Ba-Ngu Vo, Timothy N. Davidson, *Member, IEEE*, and Pak-Chung Ching, *Senior Member, IEEE*

***Abstract*—Orthogonal space-time block codes (OSTBCs) have attracted much attention owing to their simple code construction, maximal diversity gain, and low maximum-likelihood (ML) detection complexity when channel state information (CSI) is available at the receiver. This paper addresses the problem of ML OSTBC detection with unknown CSI. Focusing on the binary and quaternary PSK constellations, we show that blind ML OSTBC detection can be simplified to a Boolean quadratic program (BQP). From an optimization viewpoint the BQP is still a computationally hard problem, and we propose two alternatives for dealing with this inherent complexity. First, we consider the semidefinite relaxation (SDR) approach, which leads to a suboptimal, but accurate, blind ML detection algorithm with an affordable worst-case computational cost. We also consider the sphere decoding approach, which leads to an exact blind ML detection algorithm that remains computationally expensive in the worst case, but generally incurs a reasonable average computational cost. For the two algorithms, we study implementation methods that can significantly reduce the computational complexity. Simulation results indicate that the two blind ML detection algorithms are competitive, in that the bit error performance of the two algorithms is almost the same and is noticeably better than that of some other existing blind detectors. Moreover, numerical studies show that the SDR algorithm provides better complexity performance than the sphere decoder in the worst-case sense, and vice versa in the average sense.**

***Index Terms*—Blind and semiblind detection, decoding, lattice decoding, maximum likelihood (ML) detection, relaxation methods, semidefinite programming, space-time block code (STBC), sphere decoding.**

I. Introduction

In multiple-input-multiple-output (MIMO) communications, space-time coding has recently emerged as a promising technique for utilizing multiple transmitter and receiver antennas to improve diversity and achievable rate [1]–[5]. In particular, the space-time block codes (STBCs) based on the theory of orthogonal designs [6]–[10] have received considerable interest. The class of these so-called orthogonal STBCs (OSTBCs) not only maximizes the diversity gain of an MIMO system, but also results in a linear (and, hence, computationally efficient) maximum-likelihood (ML) detection structure when channel state information (CSI) is perfectly known at the receiver. There are other STBC designs [11], [12] that can maximize the diversity gain with higher data rates, but those STBCs usually incur a more complex coherent ML detection structure than the OSTBC scheme.

This paper addresses the problem of OTSBC detection with unknown CSI. Blind ML OSTBC detection has been studied in [13] and [14], where a suboptimal blind ML implementation called the *cyclic ML* was proposed. The idea behind cyclic ML is to decouple the original blind ML problem into simpler subproblems (one of which is the simple coherent ML detection problem) by applying alternating minimization [15], [16]. The cyclic ML detector is not only computationally simple to implement, it can also be extended to other cases such as unknown colored noise covariance and semiblind detection (i.e., detection without CSI but with pilot symbols). Cyclic ML is a method that requires careful initialization of the channel and/or symbols. In particular, a poorly initialized cyclic ML detector is likely to exhibit inferior symbol error performance (compared to the true ML). The cyclic ML method is usually initialized by employing some other suboptimal blind receiver, such as one of the subspace methods [17], [18] (also [13]). Subspace blind detectors are computationally attractive due to their closed-form structures, and they may achieve near ML performance given sufficiently large data samples. Unfortunately, the large data length assumption requires that the channel fading coefficients remain static for a long period of time, which can be violated in certain wireless environments in which the channel coherence time is small.

The goal of this paper is to consider methods that enable optimal or near-optimal implementations of the blind ML OSTBC detector, regardless of the data length. Based on the common assumption of white Gaussian noise contamination, we will show in Section III that the blind ML OSTBC detector with binary or quaternary PSK constellations can be simplified to a Boolean quadratic program (BQP)[1]. This blind

Manuscript received April 29, 2004; revised April 4, 2005. This work was supported in part by the Hong Kong Research Grant Council, the Australian Research Council (ARC) and the Natural Sciences and Engineering Research Council (NSERC) of Canada. The work of T. N. Davidson was supported by the Canada Research Chairs program. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Yucel Altunbasak.

W.-K. Ma is with the Institute of Communications Engineering, National Tsing Hua University, Hsinchu 30013, Taiwan (e-mail: wkma@ieee.org).

B.-N. Vo is with the Department of Electrical and Electronic Engineering, The University of Melbourne, Parkville, Victoria 3010, Australia (e-mail: bv@ee.mu.oz.au).

T. N. Davidson is with the Department of Electrical and Computer Engineering, McMaster University, Hamilton, Ontario, L8S 4K1, Canada (e-mail: davidson@mcmaster.ca).

P.-C. Ching is with the Department of Electronic Engineering, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong (e-mail: pcching@ee.cuhk.edu.hk).

Digital Object Identifier 10.1109/TSP.2005.861885

[1]Our BQP reformulation of the blind ML OSTBC problem was originally proposed in [19]. Around the time of the submission of that work, this reformulation idea was independently reported in a short example in [20, pp. 170-171]. However, there was no detailed discussion in [20] of the implementation of methods to solve the BQP.

ML OSTBC result is attractive because blind ML detection for a generic space-time block coding scheme (e.g., [15]) is considerably more complex to solve than the BQP. However, the BQP is nondeterministic polynomial-time hard (NP-hard) [21], which means that any optimization algorithm capable of determining the globally optimal BQP solution is unlikely to have a polynomial-time complexity in the BQP problem size. In this paper, we propose two alternatives for dealing with this implementation problem, namely *semidefinite relaxation* (SDR) and *sphere decoding*. SDR [22] is a suboptimal BQP solver that guarantees polynomial-time worst-case complexity. SDR has been theoretically shown to provide a number of advantages in terms of approximation accuracy [22]–[27]. In the application of coherent ML multiuser detection for CDMA systems [25], [28]–[30], many simulation results have confirmed that SDR provides quasi-ML performance (also [26], [31], [32] for other applications). On the other hand, sphere decoding [33], [34] is an optimal BQP solver. The NP-hard nature of the BQP implies that the time complexity of sphere decoding, in the worst case, can be expensive. However, extensive results, mostly simulations in the application of coherent MIMO ML detection [35]–[37], have shown that the average complexity performance of sphere decoding is appealing at high SNRs.

This paper is organized as follows. In Section II, we describe the OSTBC scheme and formulate the problem of blind ML detection. We derive the BQP reformulation of that problem in Section III. Sections IV and V, respectively, describe our developments of SDR and sphere decoding algorithms for blind ML OSTBC detection. In particular, in Section IV we will propose a modified version of SDR that can provide substantial computational savings over the original SDR algorithm. The idea behind this modified SDR algorithm is to make use of a computationally cheaper suboptimal blind detector to alleviate the computational burden of the original SDR algorithm. Moreover, some theoretical SDR approximation accuracy advantages in the blind ML OSTBC detection application will be examined in Section IV. As for the sphere decoding technique, our development in Section V will focus on a sphere decoder implementation that is presently believed to be the computationally fastest among various implementations [36], [37]. The SDR and sphere decoding methods can be easily extended to the semi-blind case, as we will illustrate in Section VI. Simulation results in Section VII will show that the bit error performance of the suboptimal blind SDR-ML detector is almost identical to that of the optimal blind ML sphere decoding detector. Moreover, the bit error performance of the two proposed detectors will be shown (by simulations) to be significantly better than that of some other blind detectors, such as the subspace detectors and the cyclic ML detector. Since SDR and sphere decoding offer similar bit error performance, it is interesting to compare the complexity performance of the two methods. This aspect will be numerically studied in Section VII.

II. BACKGROUND

This section reviews the orthogonal space-time coding scheme and the respective blind ML detection problem.

A. Orthogonal Space-Time Block Coding Scheme

We consider a standard space-time block code (STBC) transmission scenario in which the MIMO channel is frequency flat. Let M_t and M_r denote the numbers of transmitter and receiver antennas, respectively. By letting T be the code block length, the received code block of a single STBC can be modeled as

$$\mathbf{Y} = \mathbf{H}\mathbf{C}(\mathbf{s}) + \mathbf{V} \tag{1}$$

where $\mathbf{s} \in \{\pm 1\}^K$ are transmitted bits; $\mathbf{C}(\mathbf{s}) \in \mathbb{C}^{M_t \times T}$ is the STBC function that maps information bits to a code matrix; K is the number of bits per block; $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ is MIMO channel; $\mathbf{Y} \in \mathbb{C}^{M_r \times T}$ is the received code matrix; $\mathbf{V} \in \mathbb{C}^{M_r \times T}$ is the additive white Gaussian noise (AWGN) matrix with zero mean and variance $\mathcal{N}_o$.Orthogonal STBCs (OSTBCs) are a class of codes constructed based on the theory of orthogonal designs [7]–[10]. In the quaternary PSK (QPSK) constellation case, an OSTBC function can be expressed as

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K/2} \mathbf{A}_k s_k + j \sum_{k=1}^{K/2} \mathbf{B}_k s_{k+K/2} \tag{2}$$

where $\mathbf{A}_k, \mathbf{B}_k \in \mathbb{R}^{M_t \times T}$ are the constituent matrices of the code, $s_k \in \{\pm 1\}$ is the kth element of $\mathbf{s}$, and $j = \sqrt{-1}$. In the binary PSK (BPSK) constellation case, the matrices $\mathbf{B}_k$ are absent from (2). Both the QPSK and BPSK OSTBCs can be alternatively represented by a single formula

$$\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K} \mathbf{X}_k s_k \tag{3}$$

where $\mathbf{X}_k \in \mathbb{C}^{M_t \times T}$ are given by $\{\mathbf{X}_k\}_{k=1}^K = \{\mathbf{A}_1, \ldots, \mathbf{A}_{K/2}, j\mathbf{B}_1, \ldots, j\mathbf{B}_{K/2}\}$ for the QPSK case, and $\{\mathbf{X}_k\}_{k=1}^K = \{\mathbf{A}_1, \ldots, \mathbf{A}_K\}$ for the BPSK case. The matrices $\mathbf{X}_k$ are specially designed such that for any $\mathbf{s} \in \{\pm 1\}^K$,

$$\mathbf{C}(\mathbf{s})\mathbf{C}^H(\mathbf{s}) = \|\mathbf{s}\|_2^2 \mathbf{I} = K\mathbf{I} \tag{4}$$

where $\|\cdot\|_2$ denotes the 2-norm. The semiorthogonal code property in (4) has been shown to lead to the maximum spatial diversity gain [7]. Another benefit of using the OSTBC scheme lies in the computational efficiency in its coherent ML detection. When channel state information (CSI) is available, the maximum-likelihood (ML) detection of $\mathbf{s}$ is given by

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \{\pm 1\}^K} \|\mathbf{Y} - \mathbf{H}\mathbf{C}(\mathbf{s})\|_F^2 \tag{5}$$

where $\|\cdot\|_F$ denotes the Frobenius norm. Supposing that $\mathbf{C}(\cdot)$ is a generic space-time block code mapping function, solving (5) can be computationally challenging for large K [35], [37]. For the orthogonal space-time block coding scheme, it can shown using (3) and (4) that the coherent ML solution in (5) is given by $\hat{s}_k = \mathrm{sign}(\mathrm{Re}\{\mathrm{tr}\{\mathbf{Y}\mathbf{X}_k^H\mathbf{H}^H\}\})$, where $\mathrm{sign}: \mathbb{R} \rightarrow \{\pm 1\}$ is the threshold decision function. Clearly, the coherent ML OSTBC detector is computationally simple to implement.

B. Formulation of Blind ML Detection

When the CSI is unknown at the receiver, the ML detector structure depends on the model of the multi-antenna channel $\mathbf{H}$. Here we apply the two usual assumptions that lead to the so-called *deterministic blind ML* detector [13], [14], [38]. First, we assume that the channel fading effects are slowly time varying such that $\mathbf{H}$ remains static over P consecutive code blocks. In this case, it is appropriate to add a block index p to the OSTBC signal model in (1)

$$\mathbf{Y}_p = \mathbf{H}\mathbf{C}(\mathbf{s}_p) + \mathbf{V}_p, \quad p = 1, \ldots, P \tag{6}$$

where $\mathbf{Y}_p$ is the pth received signal block, $\mathbf{s}_p \in \{\pm 1\}^K$ is the pth bit symbol block, and $\mathbf{V}_p$ contains the AWGN samples. Second, $\mathbf{H}$ is assumed to be a deterministic unknown. For notational convenience, we collect all bit symbol blocks to form a single bit vector

$$\mathbf{s}_{1:P} = [\mathbf{s}_1^T, \ldots, \mathbf{s}_P^T]^T \in \{\pm 1\}^{KP}. \tag{7}$$

With the two channel assumptions, the ML detector for the received signal frame $[\mathbf{Y}_1, \ldots, \mathbf{Y}_P]$ is the detection method that maximizes the log likelihood function over both the channel and the bit symbols

$$\{\hat{\mathbf{H}}, \hat{\mathbf{s}}_{1:P}\} = \arg \max_{\substack{\mathbf{H} \in \mathbb{C}^{M_r \times M_t} \\ \mathbf{s}_{1:P} \in \{\pm 1\}^{KP}}} \ln p(\mathbf{Y}_1, \ldots, \mathbf{Y}_P \,|\, \mathbf{H}, \mathbf{s}_{1:P}) \tag{8}$$

where $p(\mathbf{Y}_1, \ldots, \mathbf{Y}_P \,|\, \mathbf{H}, \mathbf{s}_{1:P})$ is the p.d.f. of the received signal frame conditioned on the channel and the bit symbols. Problem (8) can be shown to be equivalent to [15], [38]

$$\{\hat{\mathbf{H}}, \hat{\mathbf{s}}_{1:P}\} = \arg \min_{\substack{\mathbf{H} \in \mathbb{C}^{M_r \times M_t} \\ \mathbf{s}_{1:P} \in \{\pm 1\}^{KP}}} \sum_{p=1}^{P} \|\mathbf{Y}_p - \mathbf{H}\mathbf{C}(\mathbf{s}_p)\|_F^2. \tag{9}$$

Unlike the known CSI case where $\mathbf{s}_p$ is detected from its respective received code block $\mathbf{Y}_p$ [shown in (5)], ML detection in the unknown CSI case detects $\{\mathbf{s}_1, \ldots, \mathbf{s}_P\}$ jointly from the whole signal frame $[\mathbf{Y}_1, \ldots, \mathbf{Y}_P]$.

Given a generic linear code function $\mathbf{C} : \{\pm 1\}^K \rightarrow \mathbb{C}^{M_r \times M_t}$, the problem of minimizing the blind ML objective function value in (9) over both $\mathbf{H}$ and $\mathbf{s}_{1:P}$ is challenging. A popular suboptimal technique for dealing with this problem is to first obtain an initial estimate of either $\mathbf{H}$ or $\mathbf{s}_{1:P}$, and then to alternately minimize the blind ML objective function over $\mathbf{H}$ and $\mathbf{s}_{1:P}$ in a cyclic fashion [13]–[15]. The performance of this cyclic blind ML method depends much on the initialization of $\mathbf{H}$ and $\mathbf{s}_{1:P}$. In particular, poor initializations are likely to result in significantly degraded performance of cyclic ML.

In the following sections, we will investigate how blind ML OSTBC detection can be implemented more effectively.

III. Simplification of Blind ML OSTBC Detection

Blind ML detection for the OSTBC scheme can be simplified by exploiting the special structures of OSTBCs. To illustrate this, define frame matrices

$$\mathcal{Y} = [\mathbf{Y}_1, \ldots, \mathbf{Y}_P], \tag{10}$$
$$\mathcal{C}(\mathbf{s}_{1:P}) = [\mathbf{C}(\mathbf{s}_1), \ldots, \mathbf{C}(\mathbf{s}_P)]. \tag{11}$$

The blind ML detection problem for the OSTBC scheme, given by (9), can be reexpressed as

$$\{\hat{\mathbf{H}}, \hat{\mathbf{s}}_{1:P}\} = \arg \min_{\substack{\mathbf{H} \in \mathbb{C}^{M_r \times M_t} \\ \mathbf{s}_{1:P} \in \{\pm 1\}^{KP}}} \|\mathcal{Y} - \mathbf{H}\mathcal{C}(\mathbf{s}_{1:P})\|_F^2. \tag{12}$$

If we are only interested in the blind ML detected bits $\hat{\mathbf{s}}_{1:P}$, then we can rewrite (12) as (see, e.g., [16], [20], [38])

$$\hat{\mathbf{s}}_{1:P} = \arg \min_{\mathbf{s}_{1:P} \in \{\pm 1\}^{KP}} \left\{ \min_{\mathbf{H} \in \mathbb{C}^{M_r \times M_t}} \|\mathcal{Y} - \mathbf{H}\mathcal{C}(\mathbf{s}_{1:P})\|_F^2 \right\}. \tag{13}$$

Let us consider the alternate ML detection formulation in (13); once $\hat{\mathbf{s}}_{1:P}$ is found, the ML channel estimate $\hat{\mathbf{H}}$ can be obtained in a straightforward manner [38]. The inner minimization term in (13) is a least squares problem given $\mathcal{C}(\mathbf{s}_{1:P})$, and it yields a closed form [20], [38]

$$\min_{\mathbf{H} \in \mathbb{C}^{M_r \times M_t}} \|\mathcal{Y} - \mathbf{H}\mathcal{C}(\mathbf{s}_{1:P})\|_F^2 = \|\mathcal{Y}[\mathbf{I} - \mathbf{\Pi}(\mathbf{s}_{1:P})]\|_F^2 \tag{14}$$

where

$$\mathbf{\Pi}(\mathbf{s}_{1:P}) = \mathcal{C}^H(\mathbf{s}_{1:P})[\mathcal{C}(\mathbf{s}_{1:P})\mathcal{C}^H(\mathbf{s}_{1:P})]^{-1}\mathcal{C}(\mathbf{s}_{1:P}) \tag{15}$$

denotes the orthogonal projector of $\mathcal{C}^H(\mathbf{s}_{1:P})$. Owing to the semi-orthogonal code property $\mathbf{C}(\mathbf{s})\mathbf{C}^H(\mathbf{s}) = K\mathbf{I}$ for any $\mathbf{s} \in \{\pm 1\}^K$ [cf., (4)], (15) can be reduced to

$$\mathbf{\Pi}(\mathbf{s}_{1:P}) = \frac{1}{KP}\mathcal{C}^H(\mathbf{s}_{1:P})\mathcal{C}(\mathbf{s}_{1:P}). \tag{16}$$

Substituting (16) and (14) into (13), we show that the blind ML detector can be reduced to

$$\begin{aligned}
\hat{\mathbf{s}}_{1:P} &= \arg \min_{\mathbf{s}_{1:P} \in \{\pm 1\}^{KP}} \|\mathcal{Y}\|_F^2 - \frac{1}{KP}\|\mathcal{Y}\mathcal{C}^H(\mathbf{s}_{1:P})\|_F^2 \\
&\equiv \arg \max_{\mathbf{s}_{1:P} \in \{\pm 1\}^{KP}} \operatorname{tr}\{\mathcal{Y}\mathcal{C}^H(\mathbf{s}_{1:P})\mathcal{C}(\mathbf{s}_{1:P})\mathcal{Y}^H\} \\
&= \arg \max_{\mathbf{s}_{1:P} \in \{\pm 1\}^{KP}} \sum_{p=1}^{P}\sum_{q=1}^{P} \operatorname{tr}\left\{\mathbf{Y}_p\mathbf{C}^H(\mathbf{s}_p)\mathbf{C}(\mathbf{s}_q)\mathbf{Y}_q^H\right\}.
\end{aligned} \tag{17}$$

The problem in (17) can be further simplified by exploiting the linear code structure $\mathbf{C}(\mathbf{s}) = \sum_{k=1}^{K}\mathbf{X}_k s_k$ [cf., (3)]. Define matrices $\mathbf{G}_{\mathcal{Y},pq} \in \mathbb{R}^{K \times K}$ with (k, ℓ)th entry

$$[\mathbf{G}_{\mathcal{Y},pq}]_{k\ell} = \operatorname{Re}\left\{\operatorname{tr}\left\{\mathbf{Y}_p\mathbf{X}_k^H\mathbf{X}_\ell\mathbf{Y}_q^H\right\}\right\} \tag{18}$$

for $p, q = 1, \ldots, P$. Using the linear code structure, the blind ML detector in (17) can be reexpressed as

$$\begin{aligned}
\hat{\mathbf{s}}_{1:P} &= \arg \max_{\mathbf{s}_{1:P} \in \{\pm 1\}^{KP}} \sum_{p=1}^{P}\sum_{q=1}^{P} \mathbf{s}_p^T\mathbf{G}_{\mathcal{Y},pq}\mathbf{s}_q \\
&= \arg \max_{\mathbf{s}_{1:P} \in \{\pm 1\}^{KP}} \mathbf{s}_{1:P}^T\mathbf{G}_{\mathcal{Y}}\mathbf{s}_{1:P}
\end{aligned} \tag{19}$$

where

$$\mathbf{G}_{\mathcal{Y}} = \begin{bmatrix} \mathbf{G}_{\mathcal{Y},11} & \mathbf{G}_{\mathcal{Y},12} & \cdots & \mathbf{G}_{\mathcal{Y},1P} \\ \mathbf{G}_{\mathcal{Y},21} & \mathbf{G}_{\mathcal{Y},22} & & \vdots \\ \vdots & & \ddots & \vdots \\ \mathbf{G}_{\mathcal{Y},P1} & & & \mathbf{G}_{\mathcal{Y},PP} \end{bmatrix}. \tag{20}$$

The following property is noted.

Property 1: The matrix $\mathbf{G}_{\mathcal{Y}}$ is positive semidefinite (PSD).

Proof: Define $\mathbf{F}_p = [\mathrm{vec}\{\mathbf{Y}_p\mathbf{X}_1^H\},\ldots,\mathrm{vec}\{\mathbf{Y}_p\mathbf{X}_K^H\}]$ for $p=1,\ldots,P$, and $\mathbf{F}=[\mathbf{F}_1,\ldots,\mathbf{F}_P]$. It can be verified that $\mathbf{G}_{\mathcal{Y}} = \mathrm{Re}\{\mathbf{F}^H\mathbf{F}\}$, and that $\mathrm{Re}\{\mathbf{F}^H\mathbf{F}\}$ is PSD. ■

The blind ML detection problem in (19) is a Boolean quadratic program (BQP), whose optimal solution can be very expensive to compute. The optimal BQP solution can be obtained by evaluating the objective function for all points in $\{\pm 1\}^{KP}$, but this "brute-force" point search process requires $\mathcal{O}(2^{KP})$ operations which is prohibitive for large KP. In Section V we will consider a point search algorithm that can have a significantly lower operational cost than the "brute-force" method, namely *sphere decoding* [35]–[37]. A drawback of sphere decoding is that its computational cost in the worst case can be as expensive as that of the "brute-force" point search. For this reason, it is also interesting to examine suboptimal BQP solvers that provide computational efficiency even in the worst case. A rather simple suboptimal BQP solver is to consider the following simple optimization problem

$$\max_{\|\mathbf{s}_{1:P}\|_2^2 = KP} \mathbf{s}_{1:P}^T \mathbf{G}_{\mathcal{Y}} \mathbf{s}_{1:P}. \tag{21}$$

Problem (21) is a relaxation of the BQP in which the original feasible set $\{\pm 1\}^{KP}$ is replaced by the bigger set $\{\mathbf{s}_{1:P} \in \mathbb{R}^{KP} \mid \|\mathbf{s}_{1:P}\|_2^2 = KP\}$. Problem (21) reduces to the problem of finding the principal eigenvector of $\mathbf{G}_{\mathcal{Y}}$, which is much easier to solve than the BQP. To approximate the BQP solution, we can round the solution of (21) to the nearest point in $\{\pm 1\}^{KP}$. In this paper, this approximate BQP method is referred to as *norm relaxation*. It is interesting to note that norm relaxation turns out to be equivalent to the singular value decomposition (SVD) relaxation method [13], and the subspace blind channel estimator in [18].[2] In the subsequent section we will consider another relaxation-based approximate BQP method, namely *semidefinite relaxation* [22]–[25], [28], which can be shown to provide more promising approximation accuracy than the norm relaxation method.

IV. Semidefinite Relaxation

This section considers the application of the semidefinite relaxation (SDR) algorithm to the blind ML OSTBC detection problem. In Section IV-A, a brief review of the SDR algorithm will be given. In Section IV-B, we will study the theoretical approximation accuracy promised by SDR, and in Section IV-C, we will propose a modified SDR algorithm that can offer substantial computational savings compared to its predecessor.

A. Review of SDR and Its Implementation Issues

For notational simplicity, the blind ML detection problem in (19) is rewritten as

$$\begin{aligned} \mathcal{L}_{\mathrm{ML}} &= \max \mathbf{s}^T\mathbf{G}\mathbf{s} \\ \text{s.t. } & s_i^2 = 1, \quad i=1,\ldots,KP \end{aligned} \tag{22}$$

in which some subscripts in the original problem are dropped. Here, $\mathcal{L}_{\mathrm{ML}}$ denotes the maximal objective function value achieved by the blind ML detector. From a nonlinear programming viewpoint, (22) is hard to solve because the constraints $s_i^2 = 1$ are nonconvex. In norm relaxation, those unit magnitude constraints are relaxed to obtain a simpler optimization problem. SDR considers a different form of relaxation in which the unit magnitude constraints are maintained. The semidefinite relaxation problem for (22) is given by

$$\mathcal{L}_{\mathrm{SDR}} = \max_{\mathbf{S}} \mathrm{tr}\{\mathbf{S}\mathbf{G}\} \tag{23a}$$
$$\text{s.t.} S_{ii} = 1, \quad i=1,\ldots,KP \tag{23b}$$
$$\mathbf{S} \succeq \mathbf{0} \tag{23c}$$

where $\mathbf{S} \succeq \mathbf{0}$ means that $\mathbf{S}$ is positive semidefinite, and S_{ii} denotes the (i,i)th element of $\mathbf{S}$. Problem (23) is a relaxation of (22) because any $\mathbf{S} = \mathbf{s}\mathbf{s}^T, \mathbf{s} \in \{\pm 1\}^{KP}$, is a feasible point of (23). Problem (23) is a convex semidefinite program, the globally optimal solution of which can be efficiently computed by readily available optimization algorithms [39]–[41]. The approximation of the BQP solution from SDR can be done in several ways. Since SDR is based on the idea of ignoring the rank-1 constraint $\mathbf{S} = \mathbf{s}\mathbf{s}^T$, an obvious BQP solution approximation is to apply rank-1 approximations to the SDR solution [28]–[30]. For example, if $\hat{\mathbf{S}}$ denotes the SDR solution in (23) and $\mathcal{P}\{\hat{\mathbf{S}}\}$ denotes the principal eigenvector of $\hat{\mathbf{S}}$, an approximate BQP solution can be obtained by

$$\hat{\mathbf{s}}_{\mathrm{SDR}} = \mathrm{sign}(\mathcal{P}\{\hat{\mathbf{S}}\}) \tag{24}$$

where $\mathrm{sign}: \mathbb{R}^n \to \{\pm 1\}^n$ is an element-wise threshold decision function. Another possible BQP solution approximation, which was found to be very effective in coherent ML detection applications [25], is the Goemans-Williamson randomized method [22]. A summary highlighting the implementation of the randomized algorithm as well as the SDR method has been presented in [25].

The process central to SDR is that of finding the solution to (23). This step usually dominates the complexity of the whole SDR algorithm. A common approach for solving the SDR problem is the application of interior-point algorithms [39]–[41]. These algorithms provide good control over the numerical precision of the SDR solution obtained. An interior-point algorithm, such as that in [40], can find a near-optimal SDR solution with an operational cost of

$$\mathcal{O}((KP)^{3.5}\log \epsilon^{-1}) \tag{25}$$

[2]The connections of SVD relaxation, norm relaxation, and the subspace channel estimator in [18] are as follows. Let $\bar{\mathbf{F}} = [\mathrm{Re}\{\mathbf{F}\}^T \mathrm{Im}\{\mathbf{F}\}^T]^T$, where $\mathbf{F}$ is defined in the Proof of Property 1. The SVD relaxation method uses the dominant left singular vector of $\bar{\mathbf{F}}$ as the channel estimate, and the dominant right singular vector of $\bar{\mathbf{F}}$ as the symbol estimate. It can be shown that the dominant right singular vector of $\bar{\mathbf{F}}$ is also the norm relaxation solution [in (21)] up to a scalar, and that the dominant left singular vector of $\bar{\mathbf{F}}$ is the subspace channel estimate up to a scalar.

where ϵ is a given parameter governing the required numerical precision of the solution; see [39], [40] for more information about this tolerance parameter. Equation (25) shows that there is a tradeoff between SDR solution precision and computational cost. Our experience with simulation results is that high solution precision may not be necessary. The reason is as follows: Since the procedures of mapping the SDR solution to an approximate BQP solution, such as that in (24), usually carry out some forms of rounding, they may be insensitive to small errors in the SDR solution.

B. Worst-Case Approximation Accuracy

The SDR method offers certain guarantees on the quality of the approximation it provides. To illustrate this, we compare the maximal objective function values before and after relaxation is applied; i.e.,

$$|\mathcal{L}_{\rm SDR} - \mathcal{L}_{\rm ML}| \tag{26}$$

where $\mathcal{L}_{\rm ML}$ and $\mathcal{L}_{\rm SDR}$ are defined in (22) and (23), respectively. It is usually found that a relaxation algorithm yields a better approximation if the gap in (26) is tighter (e.g., the multiuser ML detection application [25]). It has been shown [22] that

$$|\mathcal{L}_{\rm SDR} - \mathcal{L}_{\rm ML}| \leq \left(\frac{\pi}{2} - 1\right) \mathcal{L}_{\rm ML} \simeq 0.57\mathcal{L}_{\rm ML} \tag{27}$$

for any $\mathbf{G} \succeq \mathbf{0}$. (Note that in our application we always have $\mathbf{G} \succeq \mathbf{0}$; cf., Property 1.) The bound in (27) represents the worst-case approximation accuracy of SDR. In practice, one may find that the SDR approximation accuracy on average is much better than the worst-case. This expectation will be found to be true in our blind OSTBC ML detection application, as the simulation results in Section VII will show.

On the other hand, the SDR method can be shown to provide a better approximation accuracy than the norm relaxation method mentioned in the last section. This is described in the following theorem.

Theorem 1: Let

$$\mathcal{L}_{\rm NR} = \max_{\|\mathbf{s}\|_2^2 = KP} \mathbf{s}^T\mathbf{G}\mathbf{s} \tag{28}$$

denote the maximal objective function value of the norm relaxation method. For any $\mathbf{G}$

$$|\mathcal{L}_{\rm SDR} - \mathcal{L}_{\rm ML}| \leq |\mathcal{L}_{\rm NR} - \mathcal{L}_{\rm ML}| \tag{29}$$

where $\mathcal{L}_{\rm ML}$ and $\mathcal{L}_{\rm SDR}$ are, respectively, the maximal objective function values of the ML and SDR methods in (22) and (23).

Theorem 1 can be obtained by applying the results in [42], in which a broader and more sophisticated aspect in optimization is addressed. In Appendix I, we provide an alternative proof using a different idea.

C. Modified SDR for Complexity Reduction

When compared to some simple closed-form based blind detectors (such as norm relaxation or its equivalent counterparts), the SDR method is computationally more expensive due to the computational overhead for the interior-point optimization algorithm to find the SDR solution. Here we propose a method that sometimes enables us to obtain the SDR solution without running the interior-point algorithm. The idea is to make use of any other computationally cheaper suboptimal blind detector. The following theorem provides the framework of the proposed method.

Theorem 2: If there is a vector $\tilde{\mathbf{s}} \in \{\pm 1\}^{KP}$ that satisfies the condition

$$\mathrm{Diag}(\tilde{\mathbf{s}} \odot (\mathbf{G}\tilde{\mathbf{s}})) - \mathbf{G} \succeq \mathbf{0} \tag{30}$$

where $\odot$ is the Hadamard product (element-wise product) and $\mathrm{Diag}(\mathbf{a})$ is a diagonal matrix with ith diagonal given by a_i, then

$$\mathbf{S} = \tilde{\mathbf{s}}\tilde{\mathbf{s}}^T \tag{31}$$

is an optimal SDR solution in (23). Such an $\tilde{\mathbf{s}}$ is also an optimal blind ML solution (or optimal BQP solution).

The Proof of Theorem 2 is detailed in Appendix II. We should stress that the ideas behind proving Theorem 2 are closely related to those in [26], in which a different aspect of coherent ML performance analysis is considered. From an implementation viewpoint, Theorem 2 offers an opportunity for complexity reduction: Suppose that a suboptimal blind symbol decision, denoted by $\hat{\mathbf{s}}_{\rm subopt} \in \{\pm 1\}^{KP}$, can be obtained with low computational cost. Sometimes $\hat{\mathbf{s}}_{\rm subopt}$ will coincide with the blind SDR-ML decision. Using the optimality condition in (30), we can inspect whether $\hat{\mathbf{s}}_{\rm subopt}$ is capable of forming the SDR solution. By doing so, the SDR interior-point optimization process is only necessary when $\hat{\mathbf{s}}_{\rm subopt}$ fails to satisfy the optimality condition in (30). This idea leads to the *modified SDR* algorithm, as shown in Table I.

TABLE I
MODIFIED SDR ALGORITHM

Modified SDR Implementation
Given $\mathbf{G} \in \mathbb{R}^{KP\times KP}$, and a suboptimal decision $\hat{\mathbf{s}}_{subopt} \in \{\pm 1\}^{KP}$.
if $\mathrm{Diag}(\hat{\mathbf{s}}_{subopt} \odot (\mathbf{G}\hat{\mathbf{s}}_{subopt})) - \mathbf{G} \succeq \mathbf{0}$
output $\hat{\mathbf{s}}_{subopt}$ as the blind SDR-ML solution,
else
run the original SDR algorithm to compute the blind SDR-ML solution.

It is clear from the above procedure that the original and modified SDR algorithms produce the same decision. However, the average complexity of the modified SDR algorithm can be substantially lower than that of the original SDR algorithm if the chosen suboptimal detector manages to achieve the optimality condition in (30) with reasonable probability. This complexity advantage will be numerically confirmed in Section VII. As an aside, we should mention that the modified SDR idea can be applied to other applications, such as coherent ML MIMO detection [43].

V. Sphere Decoding

In this section, we show how sphere decoding can be used to exactly implement the blind ML OSTBC detector with BPSK or QPSK constellations. There are several variants for the sphere decoder implementation [33]–[37], and here we will consider a computationally fast sphere decoder implementation described in [36], [37]. The suggested sphere decoder, which we call the *Boolean Schnorr–Euchner (SE) sphere decoder*, is customized such that its operations are more efficient for the Boolean quadratic maximization problem of blind ML OSTBC detection. To keep the paper self-contained, we will describe some important ideas that constitute the Boolean SE sphere decoder in Section V-A. Then, blind ML OSTBC detection using sphere decoding will be presented in Section V-B. The pseudocode of the Boolean SE sphere decoder is given in Table IV.

A. Sphere Decoding Algorithm

Consider the following integer least squares (ILS) problem

$$\min_{\mathbf{s}\in\mathcal{A}^m} \mathcal{L}(\mathbf{s}) \tag{32}$$

where $\mathcal{A} \subseteq \mathbb{Z}$ is a set of integers, $\mathbf{s} = [s_1, \ldots, s_m]^T$, and

$$\mathcal{L}(\mathbf{s}) = \|\mathbf{b} - \mathbf{R}\mathbf{s}\|_2^2 \tag{33}$$

is the ILS objective function with $\mathbf{b} \in \mathbb{R}^m$ and $\mathbf{R} \in \mathbb{R}^{m\times m}$. Without loss of generality for the blind detection application here, the matrix $\mathbf{R}$ is assumed to have an upper triangular matrix structure with diagonals $R_{ii} > 0$ for all i [33]. To illustrate the sphere decoding principle, define a subset

$$\mathcal{S}(d) = \{\mathbf{s} \in \mathcal{A}^m | \mathcal{L}(\mathbf{s}) \le d\}. \tag{34}$$

Now suppose that we are given a squared radius, denoted by d_0, such that the optimal ILS solution lies in $\mathcal{S}(d_0)$. In practice, such a d_0 can be determined by some heuristic means [34], [44]; e.g., if a suboptimal ILS solution, denoted by $\hat{\mathbf{s}}_{\rm subopt} \in \mathcal{A}^m$, can be obtained with low computational cost, we can set $d_0 = \mathcal{L}(\hat{\mathbf{s}}_{\rm subopt})$. Hence, solving the ILS problem is equivalent to solving the following sphere constrained ILS problem

$$\min_{\mathbf{s}\in\mathcal{S}(d_0)} \mathcal{L}(\mathbf{s}). \tag{35}$$

Sphere decoding algorithms are point search methods particularly designed to solve (35). An advantage of sphere decoding is that if a large number of points in $\mathcal{A}^m$ are excluded from $\mathcal{S}(d_0)$, then sphere decoding will be much more efficient than a complete point search for (32). However, in a worst-case situation, such as when d_0 is poorly initialized, sphere decoding can be as expensive as the complete point search.

The first idea that leads to the suggested sphere decoder is the Viterbo-Boutros (VB) radius contraction concept [34], given as shown in Table II.

Essentially, the VB approach attempts to accelerate the point search process by iteratively contracting the search radius.

TABLE II
VB Radius Contraction

VB Radius Contraction Concept	
Given	$\mathbf{b}$, $\mathbf{R}$, and an initial squared radius d_0.
Step 1.	Set the search squared radius $d := d_0$.
Step 2.	Apply enumeration to find a point $\mathbf{s} \in \mathcal{A}^m$ that satisfies $\mathcal{L}(\mathbf{s}) < d$; i.e., finding a lattice point strictly inside the search sphere.
Step 3.	If $\mathbf{s}$ exists, modify $d := \mathcal{L}(\mathbf{s})$, save $\hat{\mathbf{s}} := \mathbf{s}$, and repeat Step 2; otherwise terminate the algorithm and output $\hat{\mathbf{s}}$.

TABLE III
SE Enumeration for $\{\pm 1\}$ Alphabets

Schnorr-Euchner Enumeration for ± 1 Alphabets	
Given	$\mathbf{b}$, $\mathbf{R}$, and a squared search radius d.
Step 1.	Set the level index $i := m$.
Step 2.	Set s_i by minimizing $$\min_{s_i\in\{\pm 1\}} \mathcal{L}_i([\ s_i\ \mathbf{s}_{(i+1):m}^T\ ]^T)$$
Step 3.	If $\mathcal{L}_i(\mathbf{s}_{i:m}) < d$ If $i > 1$ set $i := i-1$; otherwise output $\mathbf{s}$ and terminate. else Any point that ends with $\mathbf{s}_{(i+1):m}$ will lie outside the search sphere. Modify $\mathbf{s}_{(i+1):m}$ such that $\mathbf{s}_{(i+1):m}$ is a previously unvisited segment. To do so we may increase i.
Step 4.	Repeat Step 2.

The second key idea is the enumeration strategy (in Step 2 of the VB sphere decoding concept). We employ the Schnorr-Euchner (SE) enumeration [36], [37]. The SE enumeration is particularly simple to apply in the case of $\mathcal{A} = \{\pm 1\}$, which is the focus of this paper. To illustrate this, we use the upper triangular structure of $\mathbf{R}$ to decompose the ILS objective function into

$$\mathcal{L}(\mathbf{s}) = \sum_{j=1}^{m} \ell_j(\mathbf{s}_{j:m}) \tag{36}$$

where $\mathbf{s}_{j:m} = [s_j, \ldots, s_m]^T$ is a segment of $\mathbf{s}$, and

$$\ell_j(\mathbf{s}_{j:m}) = \left| b_j - R_{jj}s_j - \sum_{k=j+1}^{m} R_{jk}s_k \right|^2. \tag{37}$$

Define a partial ILS objective function

$$\mathcal{L}_i(\mathbf{s}_{i:m}) = \sum_{j=i}^{m} \ell_j(\mathbf{s}_{j:m}) = \ell_i(\mathbf{s}_{i:m}) + \mathcal{L}_{i+1}(\mathbf{s}_{(i+1):m}). \tag{38}$$

The concept of the SE enumeration is illustrated as shown in Table III.

What makes the SE enumeration different from the other enumerations is Step 2: When exploring a new level denoted by the index i, the preference of deciding the value of s_i is based on the minimum partial ILS objective function value given $\mathbf{s}_{(i+1):m}$ (In conventional sphere decoder implementations such as the VB sphere decoder [34], the preference is always given to $s_i := -1$). Hence, the SE enumeration itself can be regarded

TABLE IV
PSEUDO CODE OF THE BOOLEAN SE SPHERE DECODING ALGORITHM

Given	$\mathbf{b}$, $\mathbf{R}$, and an initial squared radius d_0.
Step 1.	(Initialization) Set $i := m$, $d := d_0$, $\mathcal{L}_{m+1} := 0$, and $\xi_m := 0$.
Step 2.	Set $s_i := \mathrm{sign}(b_i - \xi_i)$ and $s_{next,i} := -s_i$.
Step 3.	if $\lvert b_i - \xi_i - R_{ii}s_i\rvert^2 + \mathcal{L}_{i+1} \geq d$ (outside sphere) go to Step 4, else (inside sphere) go to Step 6.
Step 4.	if $i < m$ (level down) $i := i+1$ and go to Step 5, else terminate the algorithm and output $\hat{\mathbf{s}}$.
Step 5.	(enumeration) if $s_{next,i} = \pm 1$ set $s_i := s_{next,i}$, $s_{next,i} := 0$, and go to Step 3; else (i.e., given $\mathbf{s}_{i+1}^m$, all possible s_i have been visited) go to Step 4.
Step 6.	if $i > 1$ (level up) $\mathcal{L}_i := \lvert b_i - \xi_i - R_{ii}s_i\rvert^2 + \mathcal{L}_{i+1}$, $\xi_{i-1} := \sum_{k=i}^m R_{i-1,k}s_k$, $i := i-1$, and then go to Step 2; else go to Step 7.
Step 7.	(A valid point has been found; apply VB search radius contraction) Update $d := \mathcal{L}_1 = \lvert b_1 - \xi_1 - R_{11}s_1\rvert^2 + \mathcal{L}_2$ and save $\hat{\mathbf{s}} := \mathbf{s}$. Then, set $i := i+1$ and go to Step 5.

a suboptimal minimizer of the ILS objective function. From (37) and (38), it can be shown that the equation in Step 2 of SE enumeration is simply

$$\arg\min_{s_i \in \{\pm 1\}} \mathcal{L}_i\left(\left[s_i \ \mathbf{s}_{(i+1):m}^T\right]^T\right) = \mathrm{sign}\left(b_i - \sum_{k=i+1}^m R_{ik}s_k\right). \quad (39)$$

Note that (39) is reminiscent of a decision-feedback detection process; see [37] for more discussions on the connection between sphere decoding and decision feedback.

Combining the ideas of VB radius contraction and the ± 1 SE enumeration, we arrive at the sphere decoding algorithm in Table IV. We call the suggested algorithm the Boolean SE sphere decoder.

B. Application of Sphere Decoding to Blind ML Detection

To apply sphere decoding to blind ML detection, the following idea [45] can be used. The blind ML BQP problem in (19) can be alternatively written as

$$\min_{\mathbf{s} \in \{\pm 1\}^{KP}} \mathbf{s}^T(\rho\mathbf{I} - \mathbf{G})\mathbf{s} \quad (40)$$

for some real constant ρ, $|\rho| < \infty$. Let us choose ρ to be greater than the largest eigenvalue of $\mathbf{G}$, such that $\rho\mathbf{I} - \mathbf{G}$ is positive definite. Then we can perform Cholesky factorization $\mathbf{R}^T\mathbf{R} = \rho\mathbf{I} - \mathbf{G}$, where $\mathbf{R} \in \mathbb{R}^{KP \times KP}$ is the upper triangular Cholesky factor of $\rho\mathbf{I} - \mathbf{G}$. Thus, (40) is equivalent to an ILS

$$\min_{\mathbf{s} \in \{\pm 1\}^{KP}} \|\mathbf{R}\mathbf{s}\|_2^2 \quad (41)$$

to which sphere decoding algorithms can be applied.

VI. EXTENSION TO SEMIBLIND ML DETECTION

The blind ML detection techniques developed in the previous sections can be extended to semiblind detection. In the semiblind detection scenario, we assume that the first L symbol blocks in $\mathbf{s}_{1:P} = [\mathbf{s}_1^T, \ldots, \mathbf{s}_P^T]^T$ are pilot symbol blocks while the remaining symbol blocks contain data. Let

$$\mathbf{s}_t = \mathbf{s}_{1:L} = [\mathbf{s}_1^T, \ldots, \mathbf{s}_L^T]^T \quad (42)$$

$$\mathbf{s}_d = \mathbf{s}_{(L+1):P} = [\mathbf{s}_{L+1}^T, \ldots, \mathbf{s}_P^T]^T \quad (43)$$

denote the training symbol block and the data symbol block, respectively. Following the development in the previous sections, the semiblind (deterministic) ML detector is given by

$$\hat{\mathbf{s}}_d = \arg\max_{\mathbf{s}_d \in \{\pm 1\}^{K(P-L)}} [\mathbf{s}_t^T \mathbf{s}_d^T]\mathbf{G}_{\mathcal{Y}}\begin{bmatrix}\mathbf{s}_t \\ \mathbf{s}_d\end{bmatrix} \quad (44)$$

where $\mathbf{G}_{\mathcal{Y}}$ is defined in (20). By partitioning

$$\mathbf{G}_{\mathcal{Y}} = \begin{bmatrix}\mathbf{W}_{\mathcal{Y}} & \mathbf{Z}_{\mathcal{Y}} \\ \mathbf{Z}_{\mathcal{Y}}^T & \mathbf{\Gamma}_{\mathcal{Y}}\end{bmatrix} \quad (45)$$

where $\mathbf{W}_{\mathcal{Y}} \in \mathbb{R}^{KL\times KL}$, $\mathbf{Z}_{\mathcal{Y}} \in \mathbb{R}^{KL\times K(P-L)}$, and $\mathbf{\Gamma}_{\mathcal{Y}} \in \mathbb{R}^{K(P-L)\times K(P-L)}$, Problem (44) can be reexpressed as

$$\max_{\mathbf{s}_d \in \{\pm 1\}^{K(P-L)}} \mathbf{s}_d^T\mathbf{\Gamma}_{\mathcal{Y}}\mathbf{s}_d + 2\mathbf{s}_d^T\left(\mathbf{Z}_{\mathcal{Y}}^T\mathbf{s}_t\right) + \mathbf{s}_t^T\mathbf{W}_{\mathcal{Y}}\mathbf{s}_t. \quad (46)$$

To apply the SDR technique to semiblind ML detection, Problem (46) has to be reformulated to a form reminiscent of the homogenous BQP in (22). By introducing an extra variable $\alpha \in \{\pm 1\}$, (46) can be shown to be equivalent to the following homogenous BQP [25], [28]

$$\max_{\substack{\mathbf{s}_d \in \{\pm 1\}^{K(P-L)} \\ \alpha \in \{\pm 1\}}} \left[\mathbf{s}_d^T \ \alpha\right]\begin{bmatrix}\mathbf{\Gamma}_{\mathcal{Y}} & \mathbf{Z}_{\mathcal{Y}}^T\mathbf{s}_t \\ \mathbf{s}_t^T\mathbf{Z}_{\mathcal{Y}} & \mathbf{s}_t^T\mathbf{W}_{\mathcal{Y}}\mathbf{s}_t\end{bmatrix}\begin{bmatrix}\mathbf{s}_d \\ \alpha\end{bmatrix}. \quad (47)$$

It is easy to verify that if $(\mathbf{s}_d^\star, \alpha^\star)$ denotes the optimal solution of (47), then the semiblind ML solution is $\hat{\mathbf{s}}_d = \alpha^\star\mathbf{s}_d^\star$. Hence, we can use the SDR algorithm (either original or modified) to approximate (47), followed by using the above relation to obtain an approximate semiblind ML solution.

To use sphere decoding to implement semiblind ML detection, we note that (46) can be reformulated as

$$\hat{\mathbf{s}}_d = \arg\min_{\mathbf{s}_d \in \{\pm 1\}^{P-L+1}} \mathbf{s}_d^T(\rho\mathbf{I} - \mathbf{\Gamma}_{\mathcal{Y}})\mathbf{s}_d - 2\mathbf{s}_d^T(\mathbf{Z}_{\mathcal{Y}}^T\mathbf{s}_t) \quad (48)$$

where $\rho > 0$ is a constant chosen to be greater than the largest eigenvalue of $\mathbf{\Gamma}_{\mathcal{Y}}$. By defining $\mathbf{R}_{\mathcal{Y}} \in \mathbb{R}^{K(P-L)\times K(P-L)}$ to be the upper triangular Cholesky factor of $\rho\mathbf{I} - \mathbf{\Gamma}_{\mathcal{Y}}$; i.e., $\rho\mathbf{I} - \mathbf{\Gamma}_{\mathcal{Y}} = \mathbf{R}_{\mathcal{Y}}^T\mathbf{R}_{\mathcal{Y}}$, (48) can be reformulated as

$$\hat{\mathbf{s}}_d = \arg\min_{\mathbf{s}_d \in \{\pm 1\}^{K(P-L)}} \left\|\mathbf{R}_{\mathcal{Y}}\mathbf{s}_d - (\mathbf{R}_{\mathcal{Y}}^T)^{-1}\left(\mathbf{Z}_{\mathcal{Y}}^T\mathbf{s}_t\right)\right\|_2^2. \quad (49)$$

By applying a sphere decoding algorithm to the ILS in (49), the semiblind ML solution $\hat{\mathbf{s}}_d$ is obtained.

VII. SIMULATION RESULTS

In the following subsections, we will use simulations to study the bit error performance and complexity of the blind SDR-ML detector and the blind ML sphere decoding detector. The performance of the two proposed detectors will also be compared with that of some other suboptimal blind detectors, such as the cyclic ML method [13], [14] and the norm relaxed ML method in Section II. (We reiterate that the norm relaxed ML method is essentially equivalent to the SVD method in [13], and the subspace method in [18]; see Section III). We are particularly interested in the case where the frame length P is small to moderate; For sufficiently large P it has been shown by simulations [13], [14], [18] that computationally simple blind detectors, such as the cyclic ML method and the subspace method in [18] can yield near ML performance. However, for the model in (6) to be valid, the fading channel coefficients must remain static for P blocks. When P is large, this requirement may be violated in scenarios where the channel coherent time is small.

The algorithm settings for all simulation examples are as follows. Following the work in [25], the SDR algorithm consists of the interior-point algorithm in [40] and the Goemans-Williamson randomization. The solution tolerance parameter for the interior-point algorithm is set to $\epsilon = 0.01$, and the number of Goemans-Williamson randomizations is 20. The same settings apply to the modified SDR algorithm. We choose the computationally cheap norm-relaxed ML detector to construct the modified SDR algorithm. The norm relaxed ML detector is also used to initialize the search radius of the sphere decoding algorithms.

A. Bit Error Performance: Blind Case

We consider the real-valued full-rate OSTBC with $M_t = 3$ and $T = K = 4$ [7]. The BPSK constellation is used. The number of receiver antennas is $M_r = 4$. At each trial of the simulations, the MIMO channel coefficients are randomly generated following an i.i.d. circular Gaussian distribution (i.e., an i.i.d. Rayleigh channel). We assume that the symbol s_{11} is known at the receiver, so that the sign ambiguity problem can be resolved for all blind detectors tested here. The blind detectors tested are the SDR-ML detector, the ML detector by sphere decoding, the norm relaxed ML detector, the cyclic ML detector [13] with the norm relaxed ML detector providing the initialization, and the Swindlehurst-Leus subspace detector [17]. As the SDR algorithm and the modified SDR algorithm provide exactly the same symbol decision, we only plotted the BER performance of the former. (The difference between the two SDR algorithms lies in the complexity, which will be shown in Section VII-C.) Fig. 1(a) plots the BER performance of the blind detectors versus the SNR, when the block length is fixed at $P = 8$. We see that the BERs of the SDR-ML and sphere decoding ML detectors are very close. Moreover, the BER performance of the SDR-ML and sphere decoding ML detectors is significantly better than that of the other suboptimal detectors. In Fig. 1(b) we plot the BERs of the various detectors against the block length P, when the SNR is fixed at $2\mathrm{E}\{\|\mathbf{H}\|_F\}/\mathcal{N}_o = 14$dB. Again, the ML sphere decoding and SDR-ML detectors yield BER performance considerably better than that of the other suboptimal detectors.

The aforedescribed simulation was repeated with the number of receiver antennas reduced to $M_r = 1$. The Swindlehurst-Leus subspace detector [17] is excluded in this test because it is applicable only to MIMO channels satisfying a full column rank condition. The results are plotted in Fig. 2. The figure shows that the cyclic ML and norm relaxed ML detectors fail to work properly, while the ML sphere decoding and SDR-ML detectors manage to exhibit consistent BER performance.

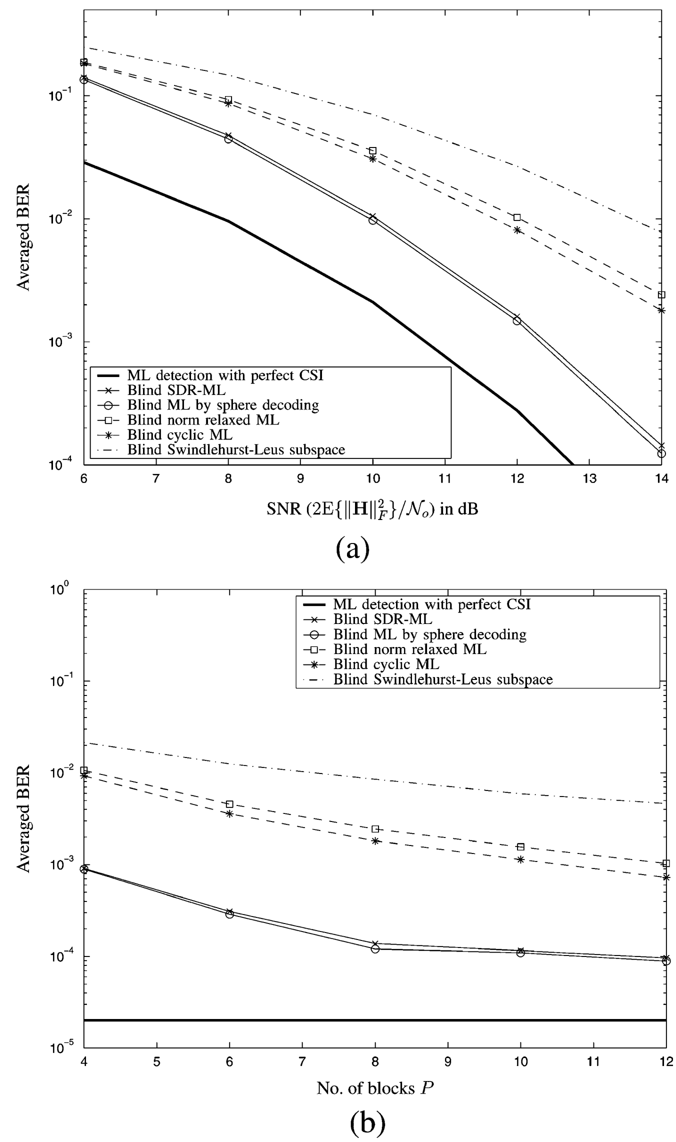

Fig. 1. BER performance in a 4 receiver antenna case. (a) BERs versus SNRs when the block length is $P = 8$. (b) BERs versus block lengths when the SNR is fixed at 14 dB.

The above sets of simulations indicate that the approximate blind ML detector provided by SDR yields negligible bit error performance degradation compared to the exact blind ML detector provided by sphere decoding.

B. Bit Error Performance: Semiblind Case

In this example, we apply the pilot-symbol-based semiblind ML detection techniques described in Section VI to the 3×4 BPSK OSTBC used in the previous example. The number of receiver antennas is $M_r = 6$, the total number of code blocks is $P = 8$, and the number of pilot symbol blocks is $L = 1$. The setting $L = 1$ represents a situation where the number of pilot symbols is small (4 only). We tested the performance of the SDR-ML detector, the ML sphere decoding detector, and the cyclic ML detector, and the conventional pilot-assisted detection method [13], [14]. The results, given in Fig. 3, show again that

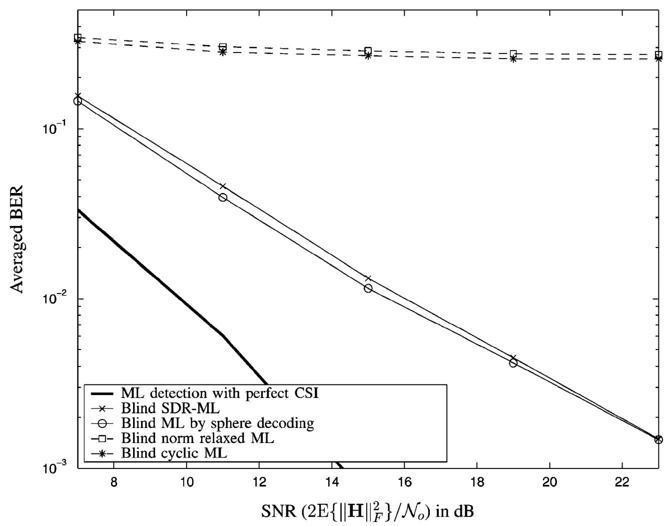

Fig. 2. BER performance versus the SNRs in a single receiver case. The block length is $P = 8$.

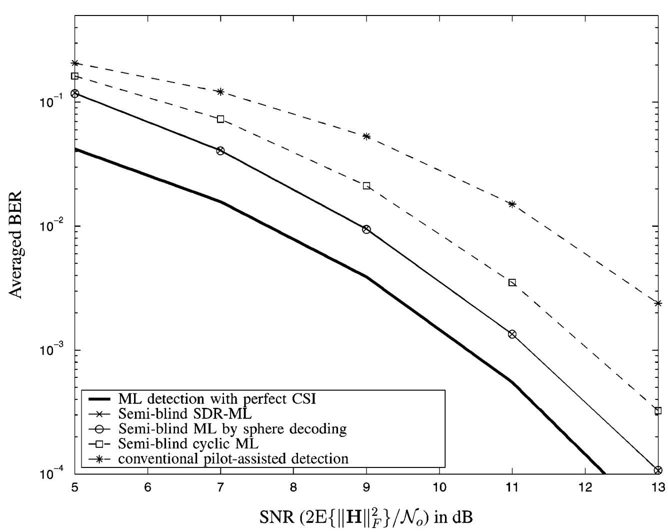

Fig. 3. BER performance in the semiblind case.

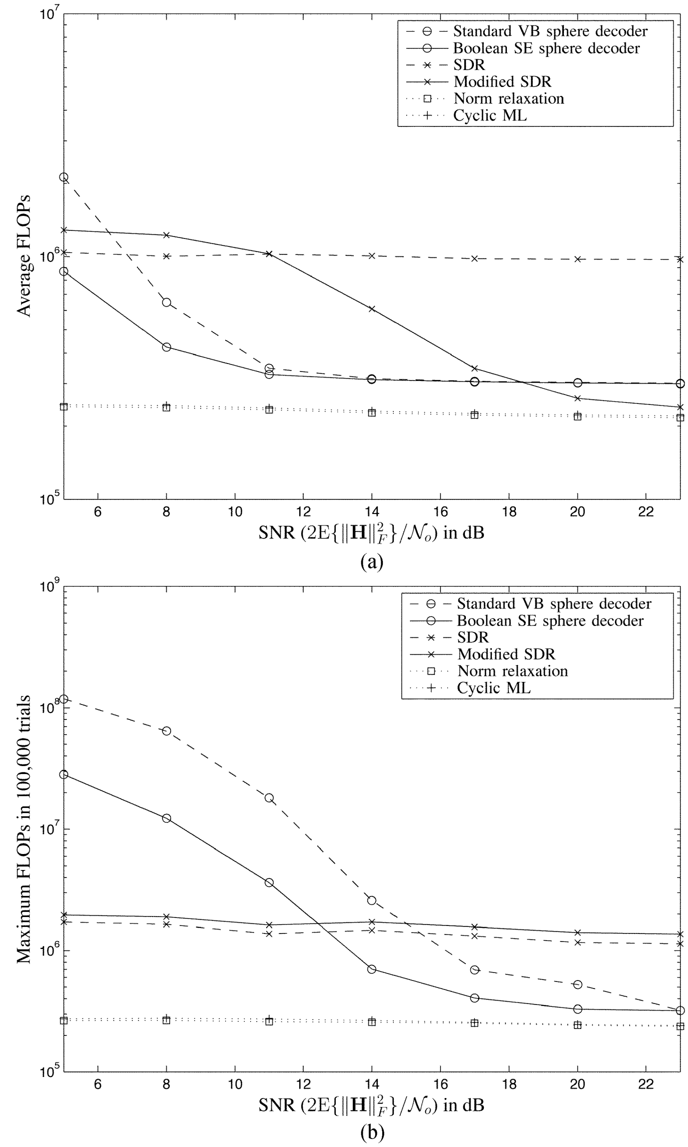

Fig. 4. Average and worst-case complexity performance of the SDR and sphere decoding algorithms.

the SDR and sphere decoding ML detectors achieve better BER performance than the other methods.

C. Complexity Comparison

Since SDR and sphere decoding offer very similar bit error performance, it is interesting to compare the complexity performance of the two approaches. We use the same simulation settings as in Section VII-A to evaluate the computational costs of the SDR algorithm, the modified SDR algorithm (developed in Section IV-C), the standard VB sphere decoder [34], and the Boolean SE sphere decoder (developed in Section V). For comparison this numerical complexity study also includes the norm relaxed ML and cyclic ML detectors, since the previous examples indicated that those two detectors provide reasonable BER performance for OSTBCs. We recall that the norm relaxed ML detector is used to initialize the sphere decoding algorithms and provide the preliminary decision for the modified SDR algorithm.

Fig. 4(a) shows the average (expected) complexity performance of the various algorithms. The complexity is measured by counting the total number of floating point operations (FLOPs) required by a detector process, and 100 000 independent simulation trials were used to evaluate the average complexity. We see that the Boolean SE sphere decoder generally provides better average complexity performance than the standard VB sphere decoder, and the SDR algorithms. The average complexity results in Fig. 4(a) also show that sphere decoding is more efficient as the SNR increases. On the other hand, at high SNRs the modified SDR algorithm provides improved average complexity performance over the (original) SDR. Like sphere decoding, the modified SDR algorithm has its average complexity decreasing with SNR. In fact, for SNRs greater than 20 dB, the average complexity of the modified SDR algorithm becomes comparable to that of the competitive Boolean SE sphere decoder.

In Fig. 4(b), we study the worst-case complexity performance of the various algorithms. The worst-case complexity is measured by picking the largest FLOPs in 100 000 independent trials. The figures shows that at low SNRs, the two sphere decoders exhibit poor worst-case complexity performance. We also note by comparing Fig. 4(b) and (a) that the difference between the worst-case and average performance of sphere decoding can be very large especially at low SNRs. As for the original SDR algorithm, the average complexity and worst-case complexity are almost the same. This indicates that the complexity of the original SDR algorithm is almost constant for every problem instance, fixing the problem size. The worst-case complexity of the modified SDR algorithm is slightly above that of the original SDR algorithm. This is because in the worst case, the complexity of the modified SDR algorithm includes that of norm relaxed ML detection and the process of checking the optimality of norm relaxed ML decision, as well as that of the original SDR algorithm.

In order to further illustrate the complexity behaviors of SDR and sphere decoding, we consider evaluating the following *computational outage probability*

$$P[\text{no. of FLOPs of a detector} > C_{\text{limit}}] \tag{50}$$

given a complexity bound C_{limit}. Fig. 5(a) and (b) shows the computational outage probabilities of the original and modified SDR algorithms. The block length is fixed at $P = 8$. The figures illustrate that when the SNR increases, there is a smaller probability that the modified SDR complexity reaches the original SDR complexity. The reason for this interesting behavior is as follows: The modified SDR algorithm does not run the more expensive SDR interior-point optimization process when it can assure the optimality of the norm relaxation decision. When the SNR increases, there is a higher probability that the norm relaxed ML detector provides the correct decision. As a result, the execution of the original SDR process becomes less frequent as the SNR increases. This also explains why the modified SDR algorithm exhibits good average complexity performance at high SNRs in Fig. 4(a).

Fig. 6(a) plots the computational outage probabilities of the Boolean SE sphere decoder when the SNRs are at 5 dB and 11 dB. The block length is fixed at $P = 8$. We observe that the computational outage probability becomes heavily tailed when the SNR decreases. In particular, for $\text{SNR} = 5$ dB there is a small probability that the complexity of the sphere decoder is higher than the worst-case complexity of the SDR algorithm. Fig. 6(b) shows the computational outage probabilities for various block lengths. The SNR is fixed at 11 dB, which corresponds to a BER of the order of 10^{-3}. The figure indicates that the sphere decoding complexity distribution also becomes heavily tailed when the problem size increases. The figure also shows that for $P = 20$, it is possible that the Boolean SE sphere decoder yields a complexity much greater than the worst-case complexity of SDR.

From the complexity results, we have the following conclusions.

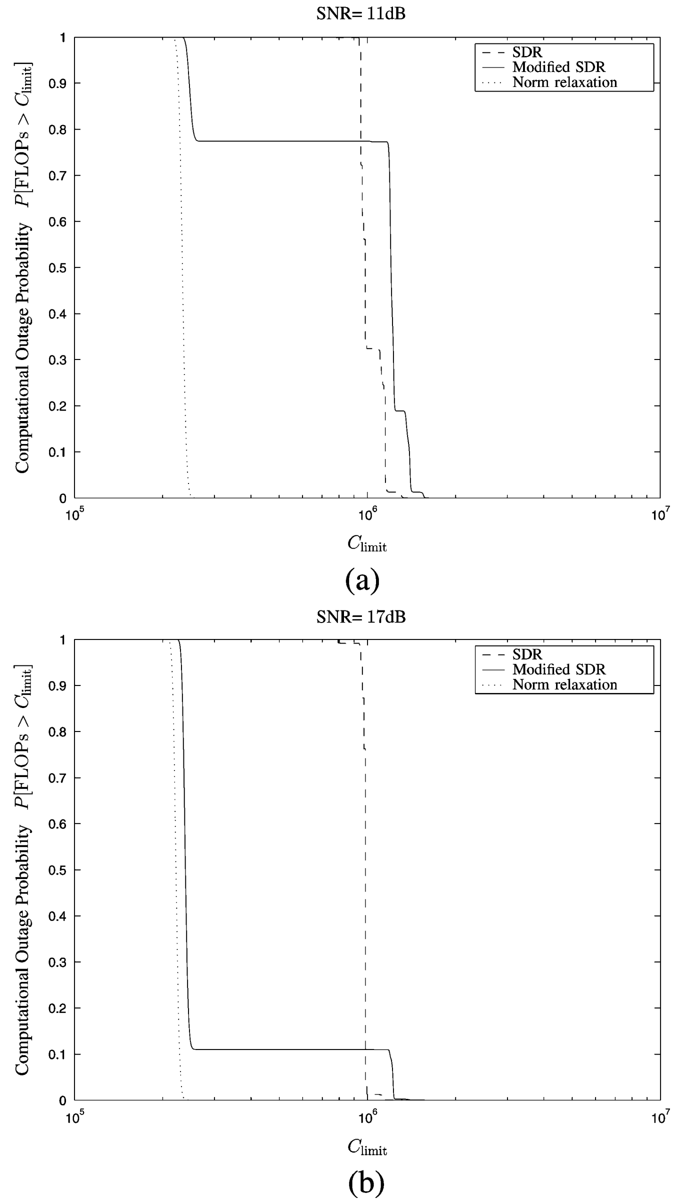

Fig. 5. Computational outage probabilities of the original and modified SDR algorithms. The block length is fixed at $P = 8$.

1) Sphere decoding methods, particularly the Boolean SE sphere decoder, generally provide good average complexity performance, especially in the high SNR regime. Unfortunately, the distribution of the sphere decoding complexity is heavily tailed when the SNR is low or when the problem size is large. In other words, the worst-case sphere decoding complexity is poor in those situations.
2) SDR provides good worst-case complexity for all SNRs, but its average complexity performance is generally higher than that of the Boolean SE sphere decoder. The modified SDR algorithm offers an extra advantage at high SNRs, namely that the average complexity performance of the modified SDR can be comparable to that of sphere decoding.

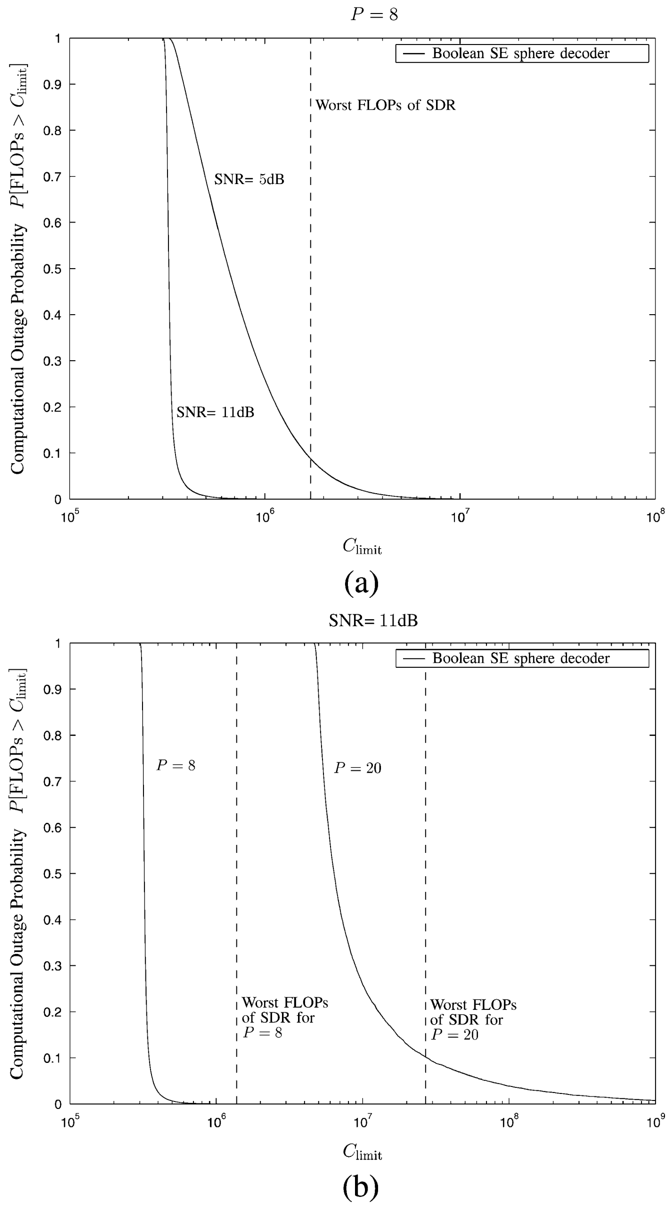

Fig. 6. Computational outage probabilities of the Boolean SE sphere decoder.

VIII. Conclusion and Discussion

In this paper, we have realized the blind ML detector for the BPSK or QPSK OSTBC scheme, using two alternatives called SDR and sphere decoding. SDR and sphere decoding are conceptually different techniques, but both are capable of effectively handling the blind ML problem with reasonable computational efficiency. For each alternative, we have developed implementation methods that can substantially reduce the computational cost. It has been demonstrated via simulation that the blind SDR-ML algorithm and the blind ML sphere decoding algorithm provide almost identical bit error performance. Simulation results have also indicated that the two blind ML implementations exhibit significantly better bit error performance than several existing blind detectors. Using numerical complexity evaluations, we have illustrated that the two algorithms are competitive in terms of computational efficiency, as well. Sphere decoding tends to provide a lower average complexity, but its worst-case complexity can be high, especially at low SNRs and for large problem sizes. SDR offers better worst-case complexity, in that its complexity depends on the problem size in a polynomial-time manner. Furthermore, we have proposed a modified SDR algorithm that can gain substantial average complexity reduction at high SNRs.

The OSTBC scheme has been well known to be attractive for its low coherent ML detection complexity compared to other MIMO and STBC schemes. It is interesting to mention that this work further reveals that OSTBCs are attractive from a blind ML detection standpoint, as well. Although blind ML detection for OSTBCs turns to be more complex requiring methods such as SDR and sphere decoding (which are tools used to handle the problem of coherent ML detection for general linear dispersion STBC schemes), blind ML detection for generic STBC schemes poses a problem that is considerably more complex to solve than that for OSTBCs.

In this development, we have placed our emphasis on the deterministic blind ML framework. That is, the MIMO channel is assumed to be a deterministic unknown. It is interesting to mention another framework, namely the stochastic blind ML in which the unknown channel is assumed to be random following certain distribution. The two frameworks are generally different, but they become closely related in the OSTBC scenario: Under the assumption of i.i.d. Rayleigh distributed channels, the stochastic blind ML OSTBC detector can be shown to be equivalent to the deterministic blind ML OSTBC detector [19] (also [38]).

This paper may be extended to several directions. First, in MIMO communications there has been much interest in soft maximum-*a posteriori* (MAP) detection. The present paper may be applied to the soft MAP scenario by using the max-log approximation idea in [32]. Second, analyzing the sphere decoding complexity would be useful in fully understanding the potential of sphere decoding in the blind ML OSTBC detection context. In the case of coherent MIMO ML detection, this aspect has been considered in [44] and [46]. The results in [44] and [46] do not appear to be directly applicable to the blind ML OSTBC detection problem here, and it would be interesting to see how the existing complexity analysis results can be applied to blind ML OSTBC detection.

Appendix I
Proof of Theorem 1

We first note a basic property that $\mathcal{L}_{\mathrm{SDR}} \geq \mathcal{L}_{\mathrm{ML}}$ and $\mathcal{L}_{\mathrm{NR}} \geq \mathcal{L}_{\mathrm{ML}}$, which is a direct consequence of relaxation. Applying these inequalities to (29), (29) is shown to be equivalent to

$$\mathcal{L}_{\mathrm{SDR}} \leq \mathcal{L}_{\mathrm{NR}}. \quad (51)$$

Hence, it suffices to prove that (51) is true. Let $\hat{\mathbf{S}}$ denote the solution of the SDR problem in (23). By Mercer's theorem, $\hat{\mathbf{S}}$ can be represented by

$$\hat{\mathbf{S}} = \sum_{i=1}^{KP} \mu_i \mathbf{q}\mathbf{q}^T \quad (52)$$

where $\{\mu_i, \mathbf{q}_i\} \in [0,\infty) \times \mathbb{R}^{KP}$ is the ith eigenvalue and (unit norm) eigenvector pair of $\hat{\mathbf{S}}$. It follows that

$$\mathcal{L}_{\mathrm{SDR}} = \mathrm{tr}\{\hat{\mathbf{S}}\mathbf{G}\} = \sum_{i=1}^{KP} \mu_i \mathbf{q}_i^T \mathbf{G} \mathbf{q}_i. \quad (53)$$

For each of the quadratic form in (53)

$$\mathbf{q}_i^T \mathbf{G} \mathbf{q}_i \leq \max_{\|\mathbf{q}\|_2^2 = 1} \mathbf{q}^T \mathbf{G} \mathbf{q}. \quad (54)$$

Moreover, the SDR solution $\hat{\mathbf{S}}$ has $\hat{S}_{ii} = 1$ for all i, which implies that

$$\sum_{i=1}^{KP} \mu_i = \mathrm{tr}\{\hat{\mathbf{S}}\} = KP. \quad (55)$$

Substituting (54) and (55) into (53), we obtain

$$\mathcal{L}_{\mathrm{SDR}} \leq KP \max_{\|\mathbf{q}\|_2^2 = 1} \mathbf{q}^T \mathbf{G} \mathbf{q} \quad (56)$$

which shows that (51) is true.

Appendix II
Proof of Theorem 2

The Lagriangian of the SDR problem in (23) is given by

$$L(\mathbf{S}, \boldsymbol{\lambda}, \mathbf{Z}) = \mathrm{tr}\{\mathbf{S}\mathbf{G}\} - \boldsymbol{\lambda}^T(\mathrm{diag}(\mathbf{S}) - \mathbf{1}_{KP}) + \mathrm{tr}\{\mathbf{S}\mathbf{Z}\} \quad (57)$$

where $\mathrm{diag}(\mathbf{S}) = [S_{11}, S_{22}, \ldots, S_{KP,KP}]^T \in \mathbb{R}^{KP}$ contains the diagonal of $\mathbf{S}, \mathbf{1}_{KP}$ is a KP-dimensional all one vector, $\boldsymbol{\lambda} \in \mathbb{R}^{KP}$ represents the Lagriangian multipliers for the equality constraints $S_{ii} = 1$ for all i, and $\mathbf{Z} \succeq \mathbf{0}$ is the Lagriangian multiplier for the constraint $\mathbf{S} \succeq \mathbf{0}$. Using the KKT condition [47], [48], it can be shown that if there exists $(\mathbf{S}, \boldsymbol{\lambda}, \mathbf{Z})$ such that

$$\nabla_{\mathbf{S}} L = \mathbf{G} + \mathbf{Z} - \mathrm{Diag}(\boldsymbol{\lambda}) = \mathbf{0} \quad (58a)$$
$$\nabla_{\boldsymbol{\lambda}} L = -\mathrm{diag}(\mathbf{S}) + \mathbf{1}_{KP} = \mathbf{0} \quad (58b)$$
$$\mathbf{Z}\mathbf{S} = \mathbf{0} \quad (58c)$$
$$\mathbf{S} \succeq \mathbf{0} \quad (58d)$$
$$\mathbf{Z} \succeq \mathbf{0} \quad (58e)$$

then $\mathbf{S}$ is an optimal SDR solution. Now suppose that there is a vector $\tilde{\mathbf{s}} \in \{\pm 1\}^{KP}$ which satisfies

$$\mathrm{Diag}(\tilde{\mathbf{s}} \odot (\mathbf{G}\tilde{\mathbf{s}})) - \mathbf{G} \succeq \mathbf{0}. \quad (59)$$

Let

$$\mathbf{S} = \tilde{\mathbf{s}}\tilde{\mathbf{s}}^T. \quad (60)$$

Such an $\mathbf{S}$ satisfies (58b) and (58d). Substituting (60) and (58a) into (58c), the following relation is shown

$$\begin{aligned}\boldsymbol{\lambda} &= -[\mathrm{Diag}(\tilde{\mathbf{s}})]^{-1}\mathbf{G}\tilde{\mathbf{s}} \\ &= -\tilde{\mathbf{s}} \odot (\mathbf{G}\tilde{\mathbf{s}}).\end{aligned} \quad (61)$$

Putting (61) into (58a), we obtain

$$\mathbf{Z} = \mathrm{Diag}(\tilde{\mathbf{s}} \odot (\mathbf{G}\tilde{\mathbf{s}})) - \mathbf{G}. \quad (62)$$

It follows from (59) that $\mathbf{Z} \succeq \mathbf{0}$, which is the KKT condition in (58). As all the KKT conditions are satisfied, (60) is an optimal SDR solution.

To verify that an optimal SDR solution $\mathbf{S} = \tilde{\mathbf{s}}\tilde{\mathbf{s}}^T, \tilde{\mathbf{s}} \in \{\pm 1\}^{KP}$ is also an optimal ML solution, we consider the following inequality

$$\begin{aligned}\mathcal{L}_{\mathrm{ML}} &= \max_{\mathbf{s} \in \{\pm 1\}^{KP}} \mathbf{s}^T \mathbf{G} \mathbf{s} \\ &\geq \tilde{\mathbf{s}}^T \mathbf{G} \tilde{\mathbf{s}} = \mathcal{L}_{\mathrm{SDR}}\end{aligned} \quad (63)$$

where $\mathcal{L}_{\mathrm{ML}}$ and $\mathcal{L}_{\mathrm{SDR}}$ are, respectively, the maximal objective function values of the ML and SDR problems in (22) and (23). Since SDR is a relaxation of the ML problem

$$\mathcal{L}_{\mathrm{ML}} \leq \mathcal{L}_{\mathrm{SDR}} = \tilde{\mathbf{s}}^T \mathbf{G} \tilde{\mathbf{s}}. \quad (64)$$

It is then clear from (63) and (64) that $\tilde{\mathbf{s}}^T \mathbf{G} \tilde{\mathbf{s}} = \mathcal{L}_{\mathrm{ML}}$, and, therefore, $\tilde{\mathbf{s}}$ is an optimal ML solution.

Acknowledgment

The authors would like to thank J. Jaldén of the Royal Institute of Technology (KTH), Stockholm, Sweden, who showed us how to improve the computational efficiency of an SDR interior-point optimization code by exploiting symmetric positive definite matrix structures in certain numerical matrix operations.

References

[1] A. F. Naguib, N. Seshadri, and A. R. Calderbank, "Increasing data rate over wireless channels," *IEEE Signal Process. Mag.*, pp. 76–92, May 2000.
[2] D. Gesbert, M. Shafi, D.-S. Shiu, P. J. Smith, and A. Naguib, "From theory to practice: An overview of MIMO space-time coded wireless systems," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 3, pp. 281–302, 2003.
[3] L. Zheng and D. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 5, pp. 1073–1096, 2003.
[4] A. J. Paulraj, D. A. Gore, R. U. Nabar, and H. Bölcskei, "An overview of MIMO communications—A key to gigabit wireless," in *Proc. IEEE*, vol. 92, 2004, pp. 198–218.
[5] S. N. Diggavi, N. Al-Dhahir, A. Stamoulis, and A. R. Calderbank, "Great expectations: The value of spatial diversity in wireless networks," *Proc. IEEE*, vol. 92, no. 2, pp. 219–270, 2004.
[6] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, 1998.
[7] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1456–1467, 1999.

[8] P. Stoica and G. Ganesan, "Space-time block codes: A maximum SNR approach," *IEEE Trans. Inf. Theory*, vol. 47, no. 4, pp. 1650–1656, 2001.

[9] W. Su and X.-G. Xia, "On space-time codes from complex orthogonal designs," *Wireless Pers. Commun.*, vol. 25, pp. 1–26, 2003.

[10] X.-B. Liang, "Orthogonal designs with maximal rates," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2468–2503, 2003.

[11] M. O. Damen, H. El Gamal, and N. C. Beaulieu, "Linear threaded algebraic space-time constellations," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2372–2388, 2003.

[12] X. Ma and G. B. Giannakis, "Full-diversity full-rate complex-field space-time coding," *IEEE Trans. Signal Process.*, vol. 51, no. 11, pp. 2917–2930, 2003.

[13] P. Stoica and G. Ganesan, "Space-time block codes: Trained, blind, and semi-blind detection," *Digital Signal Process.*, vol. 13, pp. 93–105, 2003.

[14] E. G. Larsson, P. Stoica, and J. Li, "Orthogonal space-time block codes: Maximum likelihood detection for unknown channels and unstructured interferences," *IEEE Trans. Signal Process.*, vol. 51, no. 2, pp. 362–372, 2003.

[15] S. Talwar, M. Viberg, and A. Paulraj, "Blind separation of synchronous co-channel digital signals using an antenna array—Part I: Algorithms," *IEEE Trans. Signal Process.*, vol. 44, no. 5, pp. 1184–1197, 1996.

[16] I. Ziskind and M. Wax, "Maximum likelihood localization of multiple sources by alternating projection," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 36, no. 10, pp. 1553–1560, 1988.

[17] A. L. Swindlehurst and G. Leus, "Blind and semi-blind equalization for generalized space-time block codes," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2589–2498, 2002.

[18] S. Shahbazpanahi, A. B. Gershman, and J. H. Manton, "Closed-form blind MIMO channel estimation for orthogonal space–time block codes," *IEEE Trans. Signal Process.*, vol. 53, no. 12, pp. 4506–4517, Dec. 2005.

[19] W.-K. Ma, P. C. Ching, T. N. Davidson, and X.-G. Xia, "Blind maximum-likelihood decoding for orthogonal space-time block codes: A semidefinite relaxation approach," in *Proc. IEEE 2003 Global Commun. Conf.*, Dec. 2003.

[20] E. G. Larsson and P. Stoica, *Space-Time Coding for Wireless Communications*. Cambridge, MA: Cambridge University Press, May 2003.

[21] M. Ajtai, "The shortest vector problem in L_2 is NP-hard for randomized reductions," in *Proc. 30th Ann. ACM Symp. Theory Comput.*, 1998, pp. 10–19.

[22] M. X. Goemans and D. P. Williamson, "Improved approximation algorithms for maximum cut and satisfiability problem using semi-definite programming," *J. ACM*, vol. 42, pp. 1115–1145, 1995.

[23] Y. E. Nesterov, "Semidefinite relaxation and nonconvex quadratic optimization," *Optimiz. Meth. Software*, vol. 9, pp. 140–160, 1998.

[24] S. Zhang, "Quadratic maximization and semidefinite relaxation," *Math. Program.*, vol. 87, pp. 453–465, 2000.

[25] W.-K. Ma, T. N. Davidson, K. M. Wong, Z.-Q. Luo, and P. C. Ching, "Quasimaximum-likelihood multiuser detection using semi-definite relaxation with applications to synchronous CDMA," *IEEE Trans. Signal Process.*, vol. 50, no. 4, pp. 912–922, 2002.

[26] J. Jaldén, C. Martin, and B. Ottersten, "Semidefinite programming for detection in linear systems—Optimality conditions and space-time decoding," *Proc. 2003 Int. Conf. Acoust., Speech Signal Process.*, vol. 4, pp. 9–12, 2003.

[27] M. Kisialiou and Z.-Q. Luo, "Performance analysis of quasimaximum-likelihood detector based on semi-definite programming," in *Proc. 2005 Int. Conf. Acoust., Speech Signal Process.*, Mar. 2005.

[28] P. H. Tan and L. K. Rasmussen, "The application of semidefinite programming for detection in CDMA," *IEEE J. Sel. Areas Commun.*, vol. 19, no. 8, pp. 1442–1449, 2001.

[29] M. Abdi, H. El Nahas, A. Jard, and E. Moulines, "Semidefinite positive relaxation of the maximum-likelihood criterion applied to multiuser detection in a CDMA context," *IEEE Signal Process. Lett.*, vol. 9, no. 6, pp. 165–167, 2002.

[30] X. M. Wang, W.-S. Lu, and A. Antoniou, "A near-optimal multiuser detector for CDMA systems using semidefinite programming relaxation," *IEEE Trans. Signal Process.*, vol. 51, no. 9, pp. 2446–2450, 2003.

[31] Q. Li, E.-W. Bai, and Z. Ding, "Blind source separation of signals with known alphabets using ϵ-approximation algorithms," *IEEE Trans. Signal Process.*, vol. 51, no. 1, pp. 1–10, 2003.

[32] B. Steingrimsson, Z.-Q. Luo, and K. M. Wong, "Soft quasimaximum-likelihood detection for multiple-antenna wireless channels," *IEEE Trans. Signal Process.*, vol. 51, no. 11, pp. 2710–2719, 2003.

[33] U. Fincke and M. Pohst, "Improved methods for calculating vectors of short length in a lattice, including a complexity analysis," *Math. Comput.*, vol. 44, pp. 463–471, 1985.

[34] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channels," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1639–1642, 1999.

[35] O. Damen, A. Chkeif, and J.-C. Belfiore, "Lattice code decoder for space-time codes," *IEEE Commun. Lett.*, vol. 4, no. 5, pp. 161–163, 2000.

[36] E. Agrell, T. Eriksson, A. Vardy, and K. Zeger, "Closest point searches in lattices," *IEEE Trans. Inf. Theory*, vol. 48, no. 2, pp. 2201–2214, 2002.

[37] M. O. Damen, H. El Gamal, and G. Caire, "On maximum-likelihood detection and the search for the closest lattice point," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2389–2402, 2003.

[38] E. G. Larsson, P. Stoica, and J. Li, "On maximum-likelihood detection and decoding for space-time coding systems," *IEEE Trans. Signal Process.*, vol. 50, no. 4, pp. 937–944, 2002.

[39] L. Vandenberghe and S. Boyd, "Semidefinite programming," *SIAM Rev.*, vol. 38, pp. 49–95, 1996.

[40] C. Helmberg, F. Rendl, R. J. Vanderbei, and H. Wolkowicz, "An interior-point method for semidefinite programming," *SIAM J. Optimiz.*, vol. 6, no. 2, pp. 342–361, 1996.

[41] R. M. Freund and S. Mizuno *et al.*, "Interior point methods: Current status and future directions," in *High Performance Optimization*, H. Frenk *et al.*, Eds. Boston, MA: Kluwer Academic, 2000, pp. 441–446.

[42] S. Poljak, F. Rendl, and H. Wolkowicz, "A recipe for semidefinite relaxation for (0, 1)-quadratic programming," *J. Global Optimiz.*, vol. 7, pp. 51–73, 1995.

[43] J. Jaldén, B. Ottersten, and W.-K. Ma, "Reducing the average complexity of ML detection using semidefinite relaxation," in *Proc. 2005 Int. Conf. Acoust., Speech and Signal Processing*, Mar. 2005.

[44] B. Hassibi and H. Vikalo, "Maximum-likelihood decoding and integer least-squares: The expected complexity," in *Multiantenna Channels: Capacity, Coding and Signal Processing*, J. Foschini and S. Verdu, Eds: American Mathematical Soc., 2003.

[45] P. Stoica, H. Vikalo, and B. Hassibi, "Joint maximum-likelihood channel estimation and signal detection for SIMO channels," in *Proc. 2003 Int. Conf. Acoust., Speech and Signal Processing*, vol. 4, 2003, pp. 13–16.

[46] J. Jaldén and B. Ottersten, "On the complexity of sphere decoding in digital communications," *IEEE Trans. Signal Process.*, vol. 53, no. 4, pp. 1474–1484, 2005.

[47] F. Rendl, "Semidefinite programming and combinatorial optimization," *Appl. Numer. Math.*, vol. 29, pp. 255–281, 1999.

[48] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge University Press, 2004.

Wing-Kin Ma (M'01) received the B.Eng. degree (with first class honors) in electrical and electronic engineering from the University of Portsmouth, Portsmouth, U.K., in 1995, and the M.Phil. and Ph.D. degrees, both in electronic engineering, from the Chinese University of Hong Kong (CUHK) in 1997 and 2001, respectively.

Since August 2005, he has been an Assistant Professor with the Department of Electrical Engineering and the Institute of Communications Engineering, the National Tsing Hua University, Taiwan. Previously, he has held research positions in McMaster University, Canada, CUHK, Hong Kong, and the University of Melbourne, Australia. His research interests are in signal processing for communications and statistical signal processing.

Dr. Ma's Ph.D. dissertation was commended to be "of very high quality and well deserved honorary mentioning" by the Faculty of Engineering, CUHK, in 2001.

Ba-Ngu Vo was born in Saigon, Vietnam. He received the B.Sc./B.E. degree (with first class honors) and the Ph.D. degree from the University of Western Australia (UWA), Perth, in 1994 and 1997, respectively.

He is currently a Senior Lecturer with the Department of Electrical and Electronic Engineering, University of Melbourne, Australia. His research interests include stochastic geometry, random sets, multi-target tracking, optimization, and signal processing.

Timothy N. Davidson (M'96) received the B.Eng. (Hons. I) degree in electronic engineering from the University of Western Australia (UWA), Perth, in 1991 and the D.Phil. degree in engineering science from the University of Oxford, U.K., in 1995.

He is currently an Associate Professor with the Department of Electrical and Computer Engineering, McMaster University, Hamilton, ON, Canada, where he holds the (Tier II) Canada Research Chair in Communication Systems. His research interests are in signal processing, communications, and control, with current activity focused on signal processing for digital communication systems. He has held research positions with the Communications Research Laboratory, McMaster University, the Adaptive Signal Processing Laboratory, UWA, and the Australian Telecommunications Research Institute, Curtin University of Technology, Perth, Western Australia.

Dr. Davidson was awarded the 1991 J. A. Wood Memorial Prize (for "the most outstanding [UWA] graduand" in the pure and applied sciences), and the 1991 Rhodes Scholarship for Western Australia.

Pak-Chung Ching (SM'90) received the B.Eng. (Hons.) and Ph.D. degrees in electrical engineering and electronics from the University of Liverpool, U.K., in 1977 and 1981, respectively.

From 1981 to 1982, he was a research officer with the School of Electrical Engineering, University of Bath, U.K. During 1982–1984, he was a Lecturer with the Department of Electronic Engineering of the then Hong Kong Polytechnic University. Since 1984, he has been with the Chinese University of Hong Kong, where he is presently Dean of Engineering and a full professor with the Department of Electronic Engineering. He has taught courses in digital signal processing, stochastic processes, speech processing, and communication systems. His research interests include adaptive filtering, time delay estimation, statistical signal processing, and hands-free speech communication.

Dr. Ching has actively participated in many professional activities. He was the Chairman of the IEEE Hong Kong Section during 1993–1994. He is a member of the Signal Processing Theory and Methods Technical Committee of the IEEE Signal Processing Society. He is serving as an Associate Editor for the IEEE SIGNAL PROCESSING LETTERS since January 2002, and was also an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING from 1997 to 2000. He is a member of the Accreditation Board and Fellowship Committee of the Hong Kong Institution of Engineers and a Council member of the IEE, U.K. He has been involved in organizing many international conferences, including the 1997 IEEE International Symposium on Circuits and Systems where he was the Vice-Chairman, and the 2003 IEEE International Conference on Acoustics, Speech and Signal Processing where he served as the Technical Program Co-chair. He is a Fellow of the IEE and HKIE.