

Optimized Power Allocation for Pairwise Cooperative Multiple Access

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Abstract—Multiple access schemes in which the transmitting nodes are allowed to cooperate have the potential to provide higher quality of service than conventional schemes. In the class of pair-wise cooperative multiple access schemes in which channel state information is available at the transmitters, the allocation of transmission power plays a key role in the realization of these quality of service gains. Unfortunately, the natural formulation of the power allocation problem for full-duplex cooperative schemes is not convex. It is shown herein that this non-convex formulation can be simplified and recast in a convex form. In fact, closed-form expressions for the optimal power allocation for each point on the boundary of an achievable rate region are obtained. In practice, a half-duplex cooperative scheme, in which the channel resource is partitioned in such a way that interference is avoided, may be preferred over a full-duplex scheme. The channel resource is often partitioned equally, but we develop an efficient algorithm for the joint allocation of power and the channel resource for a modified version of an existing half-duplex cooperative scheme. We demonstrate that this algorithm enables the resulting scheme to attain a significantly larger fraction of the achievable rate region for the full duplex case than the underlying scheme that employs a fixed resource allocation.

Index Terms—Achievable rate, convex optimization, cooperative communications, decode-and-forward, resource allocation.

I. INTRODUCTION

IN CONVENTIONAL multiple access schemes each node attempts to communicate its message directly to the destination node; e.g., the base station in a cellular wireless system. While such schemes can be implemented in a straightforward manner, alternative schemes in which nodes are allowed to cooperate have the potential to improve the quality of service that is offered to the transmitting nodes by enlarging the achievable rate region and by reducing the probability of outage; e.g., [1]–[4]. The basic principle of cooperative multiple access is for the nodes to mutually relay (components of) their messages to the destination node, and hence the design of such schemes

involves the development of an appropriate composition of several relay channels [5]–[7]. In particular, power and other communication resources, such as time-frequency cells/dimensions, must be allocated to the direct transmission and cooperation tasks. The realization of the potential improvement in quality of service provided by cooperation is contingent on this allocation (among other things), and the development of efficient algorithms for optimal power and resource allocation for certain classes of cooperative multiple access schemes forms the core of this paper.

We will focus on cooperative multiple access schemes in which the transmitting nodes cooperate in pairs and have access to full channel state information. The transmitting nodes will cooperate by (completely) decoding the cooperative messages transmitted by their partners, and hence the cooperation strategy can be broadly classified as being of the decode-and-forward type. We will consider an independent block fading model for the channels between the nodes, and will assume that the coherence time is long. This enables us to neglect the communication resources assigned to the feeding back of channel state information to the transmitters, and also suggests that an appropriate system design objective would be to enlarge the achievable rate region for the given channel realization.

We will begin our development with the derivation (in Sections II and III) of closed-form expressions for optimal power allocations for cooperative schemes that are allowed to operate in full-duplex mode; i.e., schemes that allow each node to simultaneously transmit and receive in the same time-frequency cell. Although the demands on the communication hardware required to facilitate full-duplex operation, such as sufficient electrical isolation between the transmission and reception modules and perfect echo cancellation, are unlikely to be satisfied in wireless systems with reasonable cost, the full-duplex case represents an idealized scenario against which more practical systems can be measured. It also provides a simplified exposition of the principles of our approach. The performance required from the communication hardware can be substantially relaxed by requiring each node to communicate in a half-duplex fashion; e.g., [1]–[4]. However, half-duplex operation requires the allocation of both power and the channel resource. In Section V, we will develop an efficient jointly optimal power and resource allocation algorithm for a (modified) block-based version of the half-duplex scheme in [2, Sec. III]. (The scheme in [2, Sec. III] employs a fixed, and equal, resource allocation.) We will demonstrate that the ability of the proposed scheme to partition the channel resource according to the rate requirements of each node enables it to achieve a larger fraction of the achievable rate region of the full duplex case than the underlying scheme.

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An impediment to the development of reliable, efficient power allocation algorithms for full-duplex cooperative multiple access has been that the direct formulation of the power allocation problem is not convex unless the transmission scheme is constrained to avoid interference. However, by examining the structure of this problem, we show that the non-convex direct formulation can be transformed into a convex one. In particular, we will show in Section II that for a given rate requirement for one of the nodes, the power allocation problem for the full-duplex case can be transformed into a convex problem that has a closed-form solution. In addition to the computational efficiencies that this closed form provides, the ability to directly control the rate of one of the nodes can be convenient in the case of heterogeneous traffic at the cooperative nodes, especially if one node has a constant rate requirement and the other is dominated by “best effort” traffic. The derivation of our closed-form expressions involved the independent discovery of some of the observations in [8] and [9] regarding the properties of the optimal solution to the sum-rate optimization problem. In particular, using different techniques from those in [8] and [9], we independently showed [10] that for each node, one of the components of the optimal power allocation is zero (although which one depends on the scenario), and that the optimization of the remaining powers can be formulated as a convex optimization problem. In [8] and [9], these observations were used to derive a power allocation algorithm with an ergodic achievable rate objective and long-term average power constraints, whereas our focus is on a setting in which the channel coherence time is long. A distinguishing feature of our approach is that the convex optimization problem for the remaining powers admits a closed-form solution.

The development of reliable, efficient, power allocation algorithms for half-duplex cooperative multiple access schemes with fixed resource allocation is simpler than that for the full-duplex case, because interference is explicitly avoided and the problem becomes convex. However, the joint allocation of power and the channel resource remains non-convex. In Section V, we consider a half-duplex scheme based on that in [2, Sec. III], and show that for a given rate requirement for one of the nodes the maximal achievable rate of the other node is a quasi-concave function of the resource allocation parameter. Therefore, we can construct an efficient algorithm for the optimal resource allocation using a standard quasi-convex search. At each stage of the search, a convex optimization problem with just two variables is solved. The complexity reduction that we obtain by exploiting the underlying quasi-convexity suggests that it may be possible to develop an online implementation of the jointly optimal power and resource allocation algorithm without resorting to approximation.

II. FULL-DUPLEX MODEL

A block diagram of the model for full-duplex pair-wise cooperative multiple access is provided in Fig. 1; see [1] and [11]. A superposition block Markov coding scheme with backward decoding was proposed for this system in [1], and we will adopt that scheme herein. To describe that coding scheme, we let $w_{i0}(n)$ denote the n th message to be sent directly from node i to the destination node (node 0), and let $w_{ij}(n)$ denote the n th

message to be sent from node i to the destination node with the cooperation of node j . At the n th (block) channel use, node i transmits the codeword

$$X_i = X_{i0} + X_{ij} + U_i \quad (1)$$

where $X_{i0}(w_{i0}(n), w_{ij}(n-1), w_{ji}(n-1))$ carries the information sent by node i directly to the destination node, $X_{ij}(w_{ij}(n), w_{ij}(n-1), w_{ji}(n-1))$ carries the information that is sent by node i to the destination node via node j ,¹ and $U_i(w_{ij}(n-1), w_{ji}(n-1))$ carries the cooperative information. (Note that all three components of X_i depend on the cooperative messages sent in the previous block.) We will let P_{i0} , P_{ij} , and P_{U_i} denote the power allocated to each component on the right-hand side of (1), and will define $P_i = P_{i0} + P_{ij} + P_{U_i}$. Assuming, as in [1], that perfect isolation and echo cancellation are achieved and that each transmitter knows the phase of the channels into which it transmits and has the means to cancel this phase, the received signal at each node can be written as

$$Y_0 = K_{10}X_1 + K_{20}X_2 + Z_0 \quad (2a)$$

$$Y_1 = K_{21}X_2 + Z_1 \quad (2b)$$

$$Y_2 = K_{12}X_1 + Z_2 \quad (2c)$$

respectively, where K_{ij} is the magnitude of the channel gain between node i and node j , and Z_i represents the additive zero-mean white circular complex Gaussian noise with variance σ_i^2 at node i . We define the (power) gain-to-noise ratio of each channel to be $\gamma_{ij} = K_{ij}^2/\sigma_j^2$. The transmitting nodes engage the channel in this way for N (block) channel uses, and the destination node employs backward decoding once all N blocks have arrived [1]. (The cooperating nodes employ forward decoding.)

The data rate of node i in the previous model is $R_i = R_{i0} + R_{ij}$, where R_{i0} is the rate of the message transmitted directly to the destination node, w_{i0} , and R_{ij} is the rate of the message transmitted with the cooperation of node j , w_{ij} . Under the assumption that all the channel parameters γ_{ij} are known at both transmitting nodes, an achievable rate region for a given channel realization is the closure of the convex hull of the rate pairs (R_1, R_2) that satisfy the following constraints [1]:²

$$R_{i0} \leq \log(1 + \gamma_{i0}P_{i0}) \quad (3a)$$

$$R_{10} + R_{20} \leq \log(1 + \gamma_{10}P_{10} + \gamma_{20}P_{20}) \quad (3b)$$

$$R_i \leq R_{i0} + \log\left(1 + \frac{\gamma_{ij}P_{ij}}{1 + \gamma_{ij}P_{i0}}\right) \quad (3c)$$

$$R_1 + R_2 \leq \log\left(1 + \gamma_{10}P_1 + \gamma_{20}P_2 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}}\right). \quad (3d)$$

Here, (3a) and (3b) bound the conventional multiple access region (with no cooperation), and (3c) and (3d) capture the impact of cooperation. A natural design objective would be to operate the system in Fig. 1 at rates that approach the boundary of the region specified in (3), subject to constraints on the power transmitted from each node. The power allocation required to do so can be found by maximizing a convex combination of R_1 and

¹The destination node also receives X_{ij} directly.

²All logarithms are to base 2, and all rates are in bits per two dimensions.

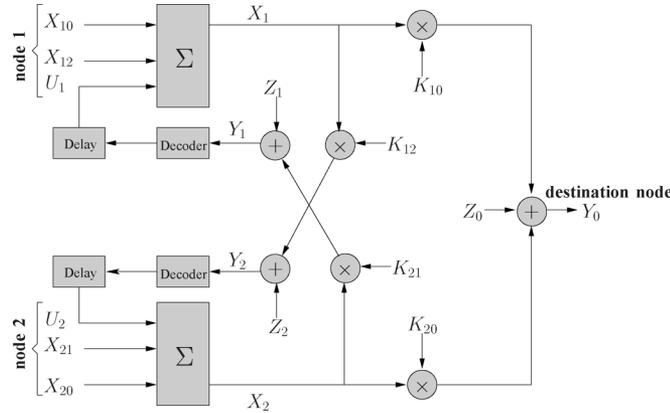


Fig. 1. Full-duplex pair-wise cooperative multiple access.

R_2 subject to (3) and a bound on the transmitted powers,³ or by maximizing R_i for a given value of R_j , subject to (3) and the bound on the transmitted powers. Unfortunately, the direct formulation of both these problems is not convex in the transmitted powers, due to the interference components in (3c). The lack of convexity renders the development of a reliable efficient algorithm for the solution of the direct formulation fraught with difficulty. However, in Section III, we will show that by adopting the latter of these two problems, the direct formulation can be transformed into a convex optimization problem that in most scenarios can be analytically solved to obtain closed-form expressions for the optimal power allocation. The overall strategy for obtaining this closed-form solution involves three main steps.

Step 1) For a given (feasible⁴) value of R_2 , denoted $R_{2,\text{tar}}$, find a closed-form expression for the powers that maximize R_1 subject to (3) and the bound on the transmitted powers; i.e., solve

$$\max_{P_{i0}, P_{ij}, P_{U_i}} R_1 \quad (4a)$$

$$\text{subject to } 0 \leq P_{i0} + P_{ij} + P_{U_i} \leq \bar{P}_i \quad (4b)$$

$$\text{and equation (3) with } R_2 = R_{2,\text{tar}}. \quad (4c)$$

Step 2) For a given (feasible) value of R_1 , denoted $R_{1,\text{tar}}$, find a closed-form expression for the powers that maximize R_2 subject to (3) and the bound on the transmitted powers. The formulation of this problem is the (algebraically) symmetric image of that in (4), in the sense that the powers and the rates of nodes 1 and 2 simply exchange roles.

Step 3) The achievable rate region of the system proposed in [1] is the convex hull of the rate regions obtained in Steps 1) and 2). When a desired rate pair on the boundary of this convex hull is achieved by the solution to Steps 1) or 2), an optimal power allocation is

³That problem that can be viewed as a special case of the problem in [8] and [9], which involves power allocation over a distribution of channel realizations with an ergodic achievable rate objective and long-term average power constraints.

⁴The set of feasible values for R_2 is $[0, R_{2,\text{max}}]$, where a closed-form expression for $R_{2,\text{max}}$ can be obtained by solving the problem in Step 2) for $R_{1,\text{tar}} = 0$. Analogously, a closed-form expression for $R_{1,\text{max}}$ can be obtained by solving (4) for $R_{2,\text{tar}} = 0$.

obtained directly. When this is not the case, a standard time-sharing strategy is applied.

In a conventional multiple access scheme, all points on the boundary of the capacity region can be obtained by time sharing (if necessary) between rate pairs that can be achieved by successive decoding of the messages from each node [13], and this significantly simplifies the system. A related result holds for the cooperative multiple access scheme, and significantly simplifies the power allocation problem. In particular, it is shown in Appendix I that in each of Steps 1) and 2) it is sufficient to consider two simplified problems in which the direct messages from each node are decoded sequentially—one in which the direct message from node 1 is decoded first (and is cancelled before the remaining messages are decoded), and one in which the direct message of node 2 is decoded first. Furthermore, solving Step 1) with the direct message of node 1 decoded first results in the same set of constraints on the rates [i.e., the same simplification of (3)] as solving Step 2) with the direct message of node 1 decoded first. Also, solving Step 1) with the direct message of node 2 decoded first results in the same set of constraints on the rates as solving Step 2) with the direct message of node 2 decoded first. Therefore, in Step 1) it is sufficient to consider only the case in which the direct message of node 1 is decoded first, and in Step 2) it is sufficient to consider only the case in which the direct message of node 2 is decoded first. Moreover, these two problems are (algebraically) symmetric images of each other. Therefore, we will explicitly state our closed-form solution only for the problem in Step 1) in which the direct message of node 1 is decoded first.

A key observation in the derivation of our closed-form expressions for the optimal power allocation is that the monotonicity of the logarithm implies that for positive constants a and c , and nonnegative constants b and d , the function $\log((a + bx)/(c + dx))$ is monotonic in $x \in [0, P]$. Hence, the optimal solution to

$$\max_x \log\left(\frac{a + bx}{c + dx}\right) \quad (5a)$$

$$\text{subject to } 0 \leq x \leq P \quad (5b)$$

is

$$x^* = \begin{cases} P, & \text{if } b - \frac{ad}{c} > 0 \\ 0, & \text{if } b - \frac{ad}{c} < 0. \end{cases} \quad (6)$$

(When $b - ad/c = 0$ all $x \in [0, P]$ are optimal.) Since problems of the form in (5) appear in two of the underlying components of (4), and since (6) has two important cases, we will need to consider four cases in order to solve Step 1). (These four cases also arise in [8] and [9], although in a different way.) In each case, we exploit the fact that since we are attempting to maximize R_1 , the upper bound in (4b) on the transmitted power for node 1 will be active at optimality. In order to simplify our exposition, we have collected the closed-form expressions for the optimal power allocations in Table I, and to simplify the derivation of these expressions, we will consider each of the four cases in a separate subsection of Section III. Before moving to those derivations, we point out that in each of the four cases in Table I, for each node (at least) one of the components of the optimal

TABLE I
POWER ALLOCATIONS FOR POINTS ON THE BOUNDARY OF THE ACHIEVABLE RATE REGION FOR THE FULL-DUPLEX COOPERATIVE SYSTEM, IN CASES 1–3 AND IN STEP 1) OF CASE 4. GIVEN A FEASIBLE RATE FOR NODE 2, $R_{2,\text{tar}}$, WE DEFINE $\Phi_{2j} = (2^{R_{2,\text{tar}}} - 1)/\gamma_{2j}$, AND $\tilde{\Phi}_{20,4} = \max\{\Phi_{20}, (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}\}$

Case	Scenario	A, B^{\S}	B^{\P}	P_{10}^*	P_{12}^*	P_{U1}^*	P_{20}^*	P_{21}^*	P_{U2}^*
1	$\gamma_{10} \leq \gamma_{12}, \gamma_{20} \leq \gamma_{21}$	(17)	> 0	0	$\bar{P}_1 - P_{U1}^*$	(18a)	0	Φ_{21}	$\bar{P}_2 - \Phi_{21}$
			≤ 0	0	\bar{P}_1	0	0	$\check{P}'_2 - P_{U2}^*$	$[0, \check{P}'_2 - \Phi_{21}]$
2	$\gamma_{10} > \gamma_{12}, \gamma_{20} \leq \gamma_{21}$	(22)	> 0	$\bar{P}_1 - P_{U1}^*$	0	(18a)	0	Φ_{21}	$\bar{P}_2 - \Phi_{21}$
			≤ 0	\bar{P}_1	0	0	0	$\check{P}'_2 - P_{U2}^*$	$[0, \check{P}'_2 - \Phi_{21}]$
3	$\gamma_{10} \leq \gamma_{12}, \gamma_{20} > \gamma_{21}$	(17)	> 0	0	$\bar{P}_1 - P_{U1}^*$	(18a)	Φ_{20}	0	$\bar{P}_2 - \Phi_{20}$
			≤ 0	0	\bar{P}_1	0	$\check{P}'_2 - P_{U2}^*$	0	$[0, \check{P}'_2 - \Phi_{20}]$
4 [†]	$\gamma_{10} > \gamma_{12}, \gamma_{20} > \gamma_{21}$	(17)	> 0	0	$\bar{P}_1 - P_{U1}^*$	(18a)	$\tilde{\Phi}_{20,4}$	0	$\bar{P}_2 - \tilde{\Phi}_{20,4}$

[§] A and B are parameters in the expression in (18a).

[¶] If $B \leq 0$ there is a set of power allocations for node 2, rather than a unique allocation. The term \check{P}'_2 is an arbitrary power in $[\check{P}_2, \bar{P}_2]$, where \bar{P}_2 is defined in (19), and $\check{P}_2 \in [\check{P}_2, \bar{P}_2]$, where \check{P}_2 is defined in (23). Those allocations for which $\check{P}'_2 = \bar{P}_2$ (or $\check{P}'_2 = \check{P}_2$) use the minimum power required to achieve the boundary. (Note that $\bar{P}_2 \geq \Phi_{21}$ and $\check{P}_2 \geq \Phi_{21}$.)

[†] In Case 4, the achievable rate region is the convex hull of this region, the analogous region generated in Step 2, and the conventional multiple access region; see Section III-D.

power allocation is zero. That observation is a key step in the derivation of these expressions, because once it is made the remaining design problem is convex. (These observations were made independently of [8] and [9], in which they arise from an analysis of the problem of optimizing the sum rate.) The particular powers that are zero imply that in Case 1 both nodes use cooperative transmission only; in Case 2 node 1 uses direct transmission only and node 2 uses only cooperative transmission; and in Case 3 node 1 uses cooperative transmission only, and node 2 uses only direct transmission.

III. DERIVATION OF EXPRESSIONS IN TABLE I

A. Case 1: $\gamma_{10} \leq \gamma_{12}$ and $\gamma_{20} \leq \gamma_{21}$

In this case, the cooperation channel for both nodes has a higher gain-to-noise ratio than the direct channel. Following the discussion in Section II, we will assume that node 1's direct message will be decoded first, and hence, the constraint on $R_{2,\text{tar}}$ in (3c) can be written as

$$R_{2,\text{tar}} \leq \log(1 + \gamma_{20}P_{20}) + \log\left(1 + \frac{\gamma_{21}P_{21}}{1 + \gamma_{21}P_{20}}\right). \quad (7)$$

Furthermore, since $P_i \leq \bar{P}_i$ the constraint on R_1 in (3d) can be written as

$$R_1 \leq \log\left(1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}}\right) - R_{2,\text{tar}}. \quad (8)$$

Our first step in the solution of (4) is to determine the powers of node 2 such that (7) is satisfied and the bound on the right-hand side of (8) is maximized. To do so, we need to maximize $P_{U2} = \bar{P}_2 - (P_{20} + P_{21})$, which is the portion of the power node 2 uses to send the cooperative codeword. This can be done by minimizing the power required to satisfy (7). The power used to satisfy (7) is the sum of the powers allocated to the direct and

indirect codewords, namely $S_2 = P_{20} + P_{21}$. To determine P_{20} and P_{21} such that S_2 is minimized, we rewrite (7) as

$$R_{2,\text{tar}} \leq \log(1 + \gamma_{20}P_{20}) + \log\left(\frac{1 + \gamma_{21}S_2}{1 + \gamma_{21}P_{20}}\right) = \log\left(\frac{1 + \gamma_{20}P_{20}}{1 + \gamma_{21}P_{20}}\right) + \log(1 + \gamma_{21}S_2). \quad (9)$$

From (9), it can be seen that in order to minimize S_2 such that (9) holds, we need to make the second term in (9) as small as possible. This can be achieved by making the first term large. (Recall that $S_2 = P_{20} + P_{21}$.) That is, we seek to

$$\max_{P_{20}} \log\left(\frac{1 + \gamma_{20}P_{20}}{1 + \gamma_{21}P_{20}}\right) \quad \text{subject to} \quad 0 \leq P_{20} \leq S_2.$$

This problem has the form of (5), and since $\gamma_{21} \geq \gamma_{20}$, the solution is $P_{20}^* = 0$, and hence, $P_{21} = \bar{P}_2 - P_{U2}$. For that power allocation, (7) can be written as

$$R_{2,\text{tar}} \leq \log(1 + \gamma_{21}P_{21}) \quad (10)$$

and hence the minimum power required for $R_{2,\text{tar}}$ to be achievable is $S_2^* = P_{21}^* = (2^{R_{2,\text{tar}}} - 1)/\gamma_{21}$. As in Table I, we define $\Phi_{21} = (2^{R_{2,\text{tar}}} - 1)/\gamma_{21}$. The remaining power available for the cooperative codeword is $P_{U2}^* = \bar{P}_2 - \Phi_{21}$.

We now consider the optimization of the remaining powers $\{P_{10}, P_{12}, P_{U1}\}$ so that R_1 is maximized (subject to $R_{2,\text{tar}}$ being achievable). The rate of node 1 has two constraints

$$R_1 \leq \log(1 + \gamma_{10}P_{10}) + \log\left(1 + \frac{\gamma_{12}P_{12}}{1 + \gamma_{12}P_{10}}\right) \quad (11)$$

$$R_1 \leq \log\left(1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}^*}\right) - R_{2,\text{tar}}. \quad (12)$$

To simplify (11), we let $S_1 = P_{10} + P_{12} = \bar{P}_1 - P_{U1}$. This enables us to write (11) as

$$R_1 \leq \log(1 + \gamma_{10}P_{10}) + \log\left(\frac{1 + \gamma_{12}S_1}{1 + \gamma_{12}P_{10}}\right) \quad (13)$$

$$= \log\left(\frac{1 + \gamma_{10}P_{10}}{1 + \gamma_{12}P_{10}}\right) + \log(1 + \gamma_{12}S_1). \quad (14)$$

For a given value of P_{U1} , S_1 is fixed and (14) can be maximized by solving

$$\max_{P_{10}} \log\left(\frac{1 + \gamma_{10}P_{10}}{1 + \gamma_{12}P_{10}}\right) \quad \text{subject to} \quad 0 \leq P_{10} \leq S_1.$$

This problem takes the form in (5), and since $\gamma_{12} \geq \gamma_{10}$, the solution is $P_{10}^* = 0$ and $P_{12} = \bar{P}_1 - P_{U1}$. The remaining two constraints on R_1 are (12) and $R_1 \leq \log(1 + \gamma_{12}(\bar{P}_1 - P_{U1}))$, and the remaining design variable is P_{U1} . Therefore, we have reduced the problem in (4) to

$$\max_{P_{U1}} \min\{\beta_1(P_{U1}), \beta_2(P_{U1})\} \quad (15a)$$

$$\text{subject to} \quad 0 \leq P_{U1} \leq \bar{P}_1 \quad (15b)$$

where

$$\beta_1(P_{U1}) = \log(1 + \gamma_{12}(\bar{P}_1 - P_{U1})) \quad (16a)$$

$$\beta_2(P_{U1}) = \log\left(1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}^*}\right) - R_{2,\text{tar}}. \quad (16b)$$

In order to solve (15) analytically, we observe that the argument of the logarithm in $\beta_1(P_{U1})$ is linearly decreasing in P_{U1} while the argument of the logarithm in $\beta_2(P_{U1})$ is concave increasing. Therefore, the solution of (15) is the value of P_{U1} for which the two upper bounds on R_1 intersect (i.e., $\beta_1(P_{U1}) = \beta_2(P_{U1})$), so long that value of P_{U1} satisfies $0 \leq P_{U1} \leq \bar{P}_1$. (A similar observation was made in [12] in the context of relay channels.) Equating $\beta_1(P_{U1})$ and $\beta_2(P_{U1})$ results in a quadratic equation in P_{U1} , and to express the solution of that quadratic equation we define

$$A = \frac{\gamma_{12}}{2\sqrt{\gamma_{10}\gamma_{20}}} 2^{R_{2,\text{tar}}} \quad (17a)$$

$$B = \frac{[2^{R_{2,\text{tar}}}(1 + \gamma_{12}\bar{P}_1) - (1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2)]}{(2\sqrt{\gamma_{10}\gamma_{20}})}. \quad (17b)$$

If $B > 0$, then the optimal power allocation for node 1 is⁵

$$P_{U1}^* = \frac{2AB + P_{U2}^* - \sqrt{(2AB + P_{U2}^*)^2 - 4A^2B^2}}{2A^2} \quad (18a)$$

$$P_{10}^* = 0 \quad (18b)$$

$$P_{12}^* = \bar{P}_1 - P_{U1}^* \quad (18c)$$

where $P_{U2}^* = \bar{P}_2 - \Phi_{21}$.

In some scenarios, the gain-to-noise ratios of the channels may be such that $B \leq 0$ for small values of $R_{2,\text{tar}}$. In that case, it can be shown that $\beta_1(P_{U1}) \leq \beta_2(P_{U1})$ for all admissible values of P_{U1} , and hence, the problem in (15) simplifies

⁵It can be analytically shown that $0 \leq P_{U1}^* \leq \bar{P}_1$. Furthermore, the condition that $B > 0$ guarantees that the argument of the square root in (18a) is positive.

to the maximization of $\beta_1(P_{U1})$, for which the optimal power allocation is $P_{U1}^* = 0$, and hence, $P_{12}^* = \bar{P}_1$. The fact that $\beta_1(P_{U1}) \leq \beta_2(P_{U1})$, for all admissible P_{U1} means that for the given $R_{2,\text{tar}}$, node 2 does not have to use all its allowed power in order for node 1 to achieve its maximum achievable rate (while node 2 achieves its target rate). In fact, the minimum (total) power that node 2 must use is the power that would make B in (17b) zero; i.e.,

$$\tilde{P}_2 = \frac{(2^{R_{2,\text{tar}}}(1 + \gamma_{12}\bar{P}_1) - (1 + \gamma_{10}\bar{P}_1))}{\gamma_{20}}. \quad (19)$$

Since $\gamma_{10} \leq \gamma_{12}$ and $\gamma_{20} \leq \gamma_{21}$, we have that $\tilde{P}_2 \geq \Phi_{21}$. If $\tilde{P}_2 > \Phi_{21}$, the additional power $\tilde{P}_2 - \Phi_{21}$ can be partitioned arbitrarily between X_{21} or U_2 ; see Table I.⁶

An alternative perspective on scenarios in which $B \leq 0$ is provided by the observation that if $[0, R_{2,B}]$ denotes the interval, if any, of values of $R_{2,\text{tar}}$ that result in $B \leq 0$, then for all $R_{2,\text{tar}} \in [0, R_{2,B}]$ the optimal value of R_1 is equal to its maximum possible value. That is, for these cases the provision of a nonzero rate of up to $R_{2,B}$ to node 2 does not reduce the achievable rate for node 1. Such scenarios arise naturally in conventional multiple access systems. This property is illustrated in the achievable rate regions in Fig. 3 for the cases in which $E(K_{12}) = 0.63$ and $E(K_{12}) = 0.71$. The boundaries of these regions are constant for small values of $R_{2,\text{tar}}$.

As discussed in Step 1), the target rate $R_{2,\text{tar}}$ is achievable if (and only if) it lies in $[0, R_{2,\text{max}}]$, where $R_{2,\text{max}}$ can be found from the solution to Step 2) with target rate $R_{1,\text{tar}} = 0$. Using a similar derivation to that above, we can show that under the conditions of Case 1, the solution to Step 2) with $R_{1,\text{tar}} = 0$ has $P_{10}^* = 0$, $P_{12}^* = 0$, and $P_{U1}^* = \bar{P}_1$, and that the constraints on R_2 reduce to

$$R_2 \leq \log(1 + \gamma_{21}P_{21}) \quad (20a)$$

$$R_2 \leq \log\left(1 + \gamma_{10}P_1 + \gamma_{20}P_2 + 2\sqrt{\gamma_{10}\gamma_{20}\bar{P}_1(\bar{P}_2 - P_{21})}\right). \quad (20b)$$

Therefore, the problem of finding $R_{2,\text{max}}$ has been reduced to finding the value of $P_{21} \in [0, \bar{P}_2]$ that maximizes the minimum of the two constraints in (20). A problem of this type arose in (15), and hence, by applying techniques similar to those that followed (15) one can obtain a closed-form solution for the optimal value of P_{21} , and hence, a closed-form expression for $R_{2,\text{max}}$. Actually, this expression for $R_{2,\text{max}}$ applies in any situation in which $\gamma_{21} \geq \gamma_{20}$. That is, it also applies to Case 2. For Cases 3 and 4, in which $\gamma_{21} < \gamma_{20}$, one can use similar arguments to show that $R_{2,\text{max}}$ is the same as that for the conventional multiple access region, namely $\log(1 + \gamma_{20}\bar{P}_2)$.

B. Case 2: $\gamma_{10} > \gamma_{12}$ and $\gamma_{20} \leq \gamma_{21}$

In this case, the direct channel for node 1 has a higher gain-to-noise ratio than its cooperation channel, but for node 2 the op-

⁶Since $P_{U1} = 0$, there is no coherent combining of U_1 and U_2 at the destination node, but U_2 can still play an active role in the backward decoder at the destination node; cf., [1, Appendix]. That said, setting $P_{21} = \bar{P}_2$ and $P_{U2} = 0$ has the potential to simplify the encoder at node 2 and the decoder at the destination node.

posite is true. Using a similar argument to Case 1, the minimum value of $P_{20} + P_{21}$ required for $R_{2,\text{tar}}$ to be achievable occurs when $P_{20} = 0$. Thus, an optimal power distribution for the second node is $P_{21}^* = \Phi_{21} = (2^{R_{2,\text{tar}}} - 1)/\gamma_{21}$, $P_{20}^* = 0$, $P_{U2}^* = \bar{P}_2 - P_{21}^*$. Therefore, the constraint in (3c) for node 1 reduces to that in (11). However, in this case, it can be shown that the choices $P_{12}^* = 0$ and $P_{10}^* = \bar{P}_1 - P_{U1}^*$ maximize the constraint in (11). The two constraints on R_1 will be (3d) and $R_1 \leq \log(1 + \gamma_{10}(\bar{P}_1 - P_{U1}))$. That is, for Case 2, we have reduced the problem in (4) to

$$\max_{P_{U1}} \min \{\beta_3(P_{U1}), \beta_2(P_{U1})\} \quad (21a)$$

$$\text{subject to } 0 \leq P_{U1} \leq \bar{P}_1 \quad (21b)$$

where $\beta_3(P_{U1}) = \log(1 + \gamma_{10}(\bar{P}_1 - P_{U1}))$ and $\beta_2(P_{U1})$ was defined in (16b). By analogy to Case 1, the solution to this problem is the intersection point between the two terms inside the minimum function, so long as that value lies in $[0, \bar{P}_1]$. Let us define

$$A = \sqrt{\frac{\gamma_{10}}{4\gamma_{20}}} 2^{R_{2,\text{tar}}} \quad (22a)$$

$$B = \frac{(2^{R_{2,\text{tar}}}(1 + \gamma_{10}\bar{P}_1) - (1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2))}{(2\sqrt{\gamma_{10}\gamma_{20}})} \quad (22b)$$

If $B > 0$, the optimal value of P_{U1} has the same form as (18a). If $B \leq 0$, it can be shown that $\beta_3(P_{U1}) \leq \beta_2(P_{U1})$ for all admissible values of P_{U1} , and hence that $P_{U1}^* = 0$. As in Case 1, if $B \leq 0$ then node 2 can reduce its total transmission power, in this case to

$$\check{P}_2 = (2^{R_{2,\text{tar}}} - 1) \frac{(1 + \gamma_{10}\bar{P}_1)}{\gamma_{20}} \quad (23)$$

and there is a range of optimal values for the pair (P_{21}, P_{U2}) ; see Table I.

C. Case 3: $\gamma_{10} \leq \gamma_{12}$ and $\gamma_{20} > \gamma_{21}$

In this case, the cooperation channel of node 1 has a higher gain-to-noise ratio than the direct channel, while this property is reversed for node 2. This case is (algebraically) symmetric to Case 2, which means that it is optimal to set $P_{10}^* = 0$, $P_{21}^* = 0$, $P_{20}^* = \Phi_{20} = (2^{R_{2,\text{tar}}} - 1)/\gamma_{20}$, and $P_{U2}^* = \bar{P}_2 - P_{20}^*$. The problem in (4) can then be written in the same form as (15), except that $R_{2,\text{tar}} = \log(1 + \gamma_{20}P_{20}^*)$. Therefore, if we define A and B as in (17), then if $B > 0$ the optimal power allocation for node 1 will have the same form as (18). If $B \leq 0$, then $P_{U1}^* = 0$, and node 2 can reduce its total power to \check{P}_2 .

D. Case 4: $\gamma_{10} > \gamma_{12}$ and $\gamma_{20} > \gamma_{21}$

In this case, for each node the cooperation channel is “weaker” than the direct channel. Therefore, we expect rather modest gains, if any, from cooperation. These expectations materialize in our solution. To solve the problem in Step 1), namely (4), under the assumption that node 1’s direct message

will be decoded first, we observe that the constraint set (3) can be written as

$$R_{2,\text{tar}} \leq \log(1 + \gamma_{20}P_{20}) + \log\left(1 + \frac{\gamma_{21}P_{21}}{1 + \gamma_{21}P_{20}}\right) \quad (24a)$$

$$R_1 \leq \log\left(1 + \frac{\gamma_{10}P_{10}}{1 + \gamma_{20}P_{20}}\right) + \log\left(1 + \frac{\gamma_{12}P_{12}}{1 + \gamma_{12}P_{10}}\right) \quad (24b)$$

$$R_1 \leq \log\left(1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}}\right) - R_{2,\text{tar}} \quad (24c)$$

where we have used the power constraint $P_i \leq \bar{P}_i$. To simplify (24b), we let $S_1 = P_{10} + P_{12} = \bar{P}_1 - P_{U1}$. This enables us to write (24b) as

$$R_1 \leq \log\left(\frac{(1 + \gamma_{20}P_{20}) + \gamma_{10}P_{10}}{1 + \gamma_{20}P_{20}}\right) + \log\left(\frac{1 + \gamma_{12}S_1}{1 + \gamma_{12}P_{10}}\right) \\ = \log\left(\frac{(1 + \gamma_{20}P_{20}) + \gamma_{10}P_{10}}{1 + \gamma_{12}P_{10}}\right) + \log\left(\frac{1 + \gamma_{12}S_1}{1 + \gamma_{20}P_{20}}\right). \quad (25)$$

For a given value of P_{U1} , S_1 is fixed and (25) can be maximized by solving

$$\max_{P_{10}} \log\left(\frac{(1 + \gamma_{20}P_{20}) + \gamma_{10}P_{10}}{1 + \gamma_{12}P_{10}}\right) \\ \text{subject to } 0 \leq P_{10} \leq S_1.$$

This problem takes the form of (5), and hence, the solution is

$$P_{10}^* = \begin{cases} 0, & \text{if } P_{20} > \frac{(\gamma_{10}/\gamma_{12} - 1)}{\gamma_{20}} \\ S_1, & \text{if } P_{20} < \frac{(\gamma_{10}/\gamma_{12} - 1)}{\gamma_{20}}. \end{cases} \quad (26)$$

Consider the first case in (26), namely $P_{20} > (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}$. In that case, the constraint set in (24) can be written as

$$R_{2,\text{tar}} \leq \log(1 + \gamma_{20}P_{20}) + \log\left(1 + \frac{\gamma_{21}P_{21}}{1 + \gamma_{21}P_{20}}\right) \quad (27a)$$

$$R_1 \leq \log(1 + \gamma_{12}P_{12}) \quad (27b)$$

$$R_1 \leq \log\left(1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}}\right) - R_{2,\text{tar}}. \quad (27c)$$

Our next step in the solution of (4) is to determine the powers of node 2 such that (27a) is satisfied and the bound on the right-hand side of (27c) is maximized. To do so we need to maximize $P_{U2} = \bar{P}_2 - (P_{20} + P_{21})$. This can be done by minimizing the power required to satisfy (27a). The power used to satisfy (27a) is $S_2 = P_{20} + P_{21}$, and to determine P_{20} and P_{21} such that S_2 is minimized, we rewrite (27a) as

$$R_{2,\text{tar}} \leq \log(1 + \gamma_{20}P_{20}) + \log\left(\frac{1 + \gamma_{21}S_2}{1 + \gamma_{21}P_{20}}\right) \\ = \log\left(\frac{1 + \gamma_{20}P_{20}}{1 + \gamma_{21}P_{20}}\right) + \log(1 + \gamma_{21}S_2). \quad (28)$$

From (28), it can be seen that in order to minimize S_2 such that (28) holds, we need to make the second term in (28) as small as

possible. This can be achieved by making the first term large; i.e.,

$$\max_{P_{20}} \log \left(\frac{1 + \gamma_{20} P_{20}}{1 + \gamma_{21} P_{20}} \right) \quad \text{subject to} \quad 0 \leq P_{20} \leq S_2.$$

This problem has the form of (5), and since $\gamma_{20} \geq \gamma_{21}$, the solution is $P_{21}^* = 0$, and hence, $P_{20} = S_2 = \bar{P}_2 - P_{U2}$. For those power allocations, (27a) can be written as

$$R_{2,\text{tar}} \leq \log(1 + \gamma_{20} P_{20}). \quad (29)$$

The minimum value of P_{20} required for (29) to be satisfied is $\Phi_{20} = (2^{R_{2,\text{tar}}} - 1)/\gamma_{20}$, and hence, an optimal value for P_{20} under the conditions of the first case of (26) is $P_{20}^* = \max\{\Phi_{20}, (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}\}$. The remaining power available for node 2's cooperative codeword is $P_{U2}^* = \bar{P}_2 - P_{20}^*$.

We now consider the optimization of the remaining powers (P_{10}, P_{12}, P_{U1}) so that R_1 is maximized. Since we are focusing on the first case in (26), $P_{12}^* = \bar{P}_1 - P_{U1}$, and hence, the problem of maximizing R_1 can be written as

$$\max_{P_{U1}} \min\{\beta_1(P_{U1}), \beta_2(P_{U1})\} \quad (30a)$$

$$\text{subject to} \quad 0 \leq P_{U1} \leq \bar{P}_1 \quad (30b)$$

where $\beta_1(P_{U1})$ and $\beta_2(P_{U1})$ were defined in (16). Therefore, as we have seen in Case 1, the solution of (30) is the value of P_{U1} for which the two upper bounds on R_1 intersect, so long as this value lies in $[0, \bar{P}_1]$. If we define A and B as in (17), then if $B > 0$ the optimal power allocations for node 1 in the first case of (26) are given by the expressions in (18); see Table I. If $B \leq 0$, then $\beta_1(P_{U1}) \leq \beta_2(P_{U1})$ for all admissible values of P_{U1} , and in order to maximize $\min\{\beta_1(P_{U1}), \beta_2(P_{U1})\}$, we would choose $P_{U1}^* = 0$ and hence $P_{12}^* = \bar{P}_1$. With that choice, the achievable rate R_1 for a given target rate $R_{2,\text{tar}}$ is $R_1 = \log(1 + \gamma_{12} \bar{P}_1)$. However, a feature of the scenario in Case 4 is that for values of $R_{2,\text{tar}}$ that result in a negative value of B , the conventional multiple access scheme provides a larger value for R_1 than that derived previously; see Appendix II for the derivation. Therefore, in the first case of (26), if $R_{2,\text{tar}}$ is such that $B \leq 0$, then the optimal solution is to perform conventional multiple access.

In the second case of (26), namely $P_{20} < (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}$, the constraint set in (24) can be written as

$$R_{2,\text{tar}} \leq \log(1 + \gamma_{20} P_{20}) + \log \left(1 + \frac{\gamma_{21} P_{21}}{1 + \gamma_{21} P_{20}} \right) \quad (31a)$$

$$R_1 \leq \log \left(1 + \frac{\gamma_{10} P_{10}}{1 + \gamma_{20} P_{20}} \right) \quad (31b)$$

$$R_1 \leq \log \left(1 + \gamma_{10} \bar{P}_1 + \gamma_{20} \bar{P}_2 + 2\sqrt{\gamma_{10} \gamma_{20} P_{U1} P_{U2}} \right) - R_{2,\text{tar}}. \quad (31c)$$

It can be shown that letting $P_{20} = S_2$ and $P_{21} = 0$ increases (31c), but at the same time it increases the interference term in (31b). This suggests that the optimal values of P_{20} and P_{21} might both be nonzero, and hence, the power allocation problem

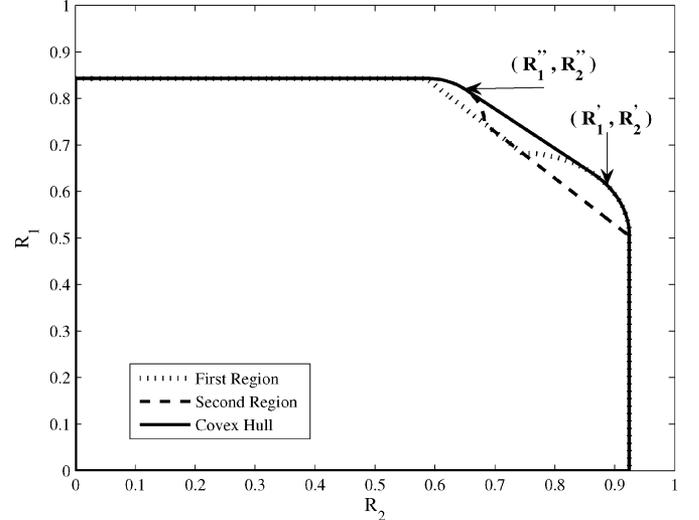


Fig. 2. Convex hull of the two achievable rate regions in Case 4. $P_1 = P_2 = 2.0$, $\sigma_0^2 = \sigma_1^2 = \sigma_2^2 = 1$, $E(K_{12}) = E(K_{21}) = 0.55$, $E(K_{10}) = 0.63$, $E(K_{20}) = 0.67$.

appears to become significantly more complex to solve. However, in Appendix III, we show that it is sufficient to study the first case of (26), namely $P_{20} > (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}$, and the symmetric case that arises when solving Step 2). In particular, we show in Appendix III, that the convex hull of the regions obtained by solving Step 1) for $P_{20} > (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}$ and Step 2) for $P_{10} > (\gamma_{20}/\gamma_{21} - 1)/\gamma_{10}$, and the conventional multiple access region contains all other regions that would result from solving Step 1) for $P_{20} < (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}$ and solving Step 2) for $P_{10} < (\gamma_{20}/\gamma_{21} - 1)/\gamma_{10}$.

In Cases 1–3, our proposed closed-form solution to (4) generates the achievable rate region directly. However, as stated in the previous paragraph, in the present case (Case 4) the most convenient description of the achievable rate region is via the convex hull of the rates generated by solving (4) in a special subcase, those generated by the solution of the (algebraically) symmetric image of that problem, and the conventional multiple access region. Fig. 2 shows the construction of the optimal rate region. The dotted curve is the union of the region generated by the power allocation in Table I and the conventional multiple access region. The dashed curve is the union of the region generated by the corresponding solution for Step 2) and the conventional multiple access region. The solid curve is the convex hull of the two component regions and hence is the optimized achievable rate region. The inner pentagon in Fig. 2 is the conventional (non-cooperative) multiple access region, and hence, the cooperative gain in Case 4 is clear. (Recall that in Case 4 the direct channels are stronger than the cooperation channels, and hence, the cooperative gain is expected to be modest.) Points on the interval (R_1', R_2') to (R_1'', R_2'') in Fig. 2 are not achieved by the solution of the problem for Step 1) nor that for Step 2), but can be achieved using standard time sharing techniques in which the system operates at the point (R_1', R_2') for a fraction ρ of the block length, and at the point (R_1'', R_2'') for the remainder of the block. Although we do not have a closed-form expression for

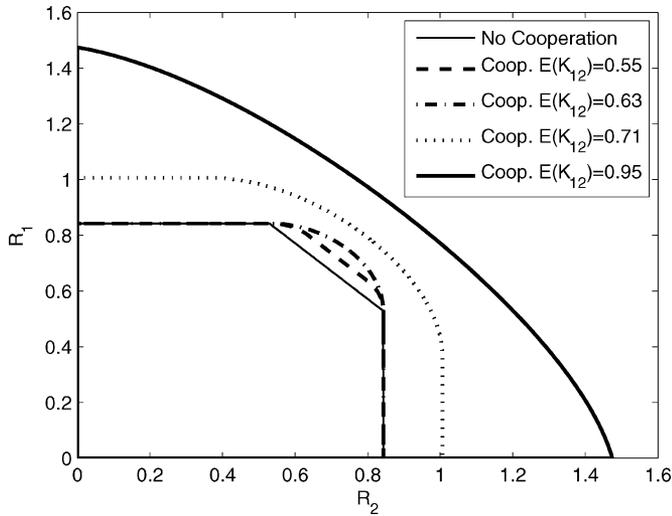


Fig. 3. Achievable rate region in a case of statistically symmetric direct channels. $P_1 = P_2 = 2.0$, $\sigma_0^2 = \sigma_1^2 = \sigma_2^2 = 1$, $E(K_{10}) = E(K_{20}) = 0.63$.

the points (R'_1, R'_2) and (R''_1, R''_2) at this time, they can be determined from the solution of an auxiliary convex optimization problem. (For reasons of brevity, we have omitted those details.)

IV. SIMULATION RESULTS

In order to verify our derivations, we used our closed-form expressions to compute the average achievable rate regions in scenarios corresponding to those chosen for [1, Fig. 2]. The resulting regions are plotted in Fig. 3 and, as expected, they match the corresponding regions in [1, Fig. 2]. In the scenarios considered, the channels were independent block fading channels with long coherence times. The channel gains were Rayleigh distributed, the Gaussian noise variances were normalized to 1, and the transmission powers of the cooperating nodes were set to be equal $\bar{P}_1 = \bar{P}_2 = 2$. (Recall that each node has full channel state information.) As in [1, Fig. 2], Fig. 3 is constructed for the case of a channel with symmetric statistics, in the sense that the direct channels between each node and the destination node is Rayleigh fading with the same mean value $E(K_{10}) = E(K_{20}) = 0.63$. Different curves are plotted for different values of the mean value of the inter-node channel $E(K_{12})$. (For each realization $K_{12} = K_{21}$.) The average achievable rate region was obtained by taking the direct sum of the achievable rate regions for each channel realization and then dividing by the number of realizations. Like [1, Fig. 2], Fig. 3 demonstrates advantages of cooperative multiple-access, especially when the gain of the cooperative channels is (often) significantly larger than the gain of the direct channels.

In addition to the average achievable rate region, it is interesting to observe the optimal power allocations. Fig. 4 shows the allocation of the different power components for one channel realization in which $K_{10} = K_{20} = 0.4$ and $K_{12} = K_{21} = 0.7$. (These gains satisfy the conditions of Case 1 in our closed-form solution.) Fig. 4 plots the optimal power components that maximize the rate R_1 for each value of the rate R_2 . We note from Fig. 4 that there is one power component for each node that is

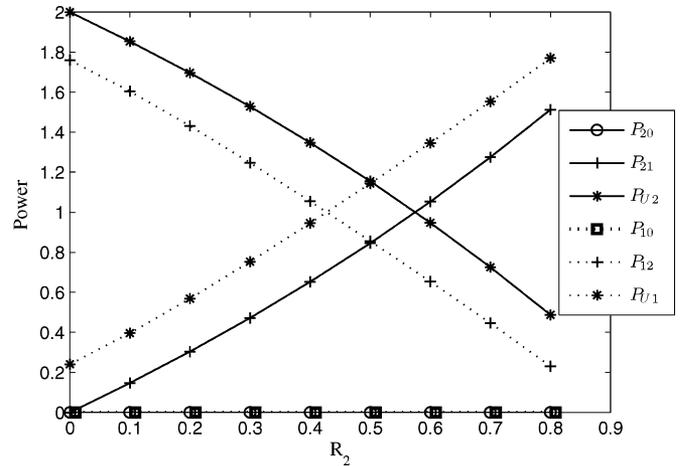


Fig. 4. Power allocation for a channel realization that satisfies the conditions of Case 1.

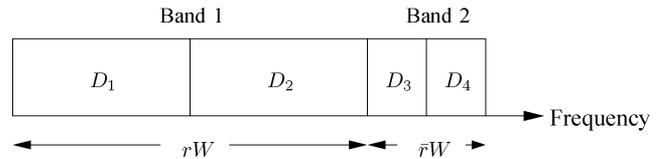


Fig. 5. Partitioning of the total bandwidth W into two bands and four subchannels, where $\bar{r} = 1 - r$.

zero for all values of R_2 ; i.e., in this case $P_{10} = P_{20} = 0$. We also note that the curves for P_{12} and P_{21} intersect at the same value for R_2 as the curves for P_{U1} and P_{U2} . This intersection point represents the equal rate point at which $R_1 = R_2$. Fig. 4 also illustrates that as R_2 increases, node 2 allocates more power to P_{21} to increase the data rate sent to node 1. As R_2 increases, node 1 has to reduce its data rate, and this is reflected in the decreasing amount of power that is allocated to P_{12} .

V. JOINTLY OPTIMAL POWER AND RESOURCE ALLOCATION FOR A HALF-DUPLEX SCHEME

The full-duplex cooperative multiple access scheme studied in Section IV places demands on the communication hardware that can be difficult to satisfy with a reasonable cost. Therefore, in this section, we consider (a modified version of) an existing half-duplex cooperative scheme that partitions the channel resource in order to avoid interference at the receivers, and hence, can be implemented using conventional communication hardware. In practice, the channel resource is often partitioned equally, but we develop an efficient algorithm for the joint allocation of power and the channel resource for this scheme, and we demonstrate that this algorithm enables the resulting scheme to attain a significantly larger fraction of the achievable rate region for the full duplex case than the underlying scheme.

The half-duplex cooperation scheme that we will consider is a modified version of a block-based version of the scheme proposed in [2, Sec. III]. The modification is that the system bandwidth is partitioned into two bands of fractional bandwidth r and $\bar{r} = 1 - r$, $0 \leq r \leq 1$, respectively, rather than having r fixed to

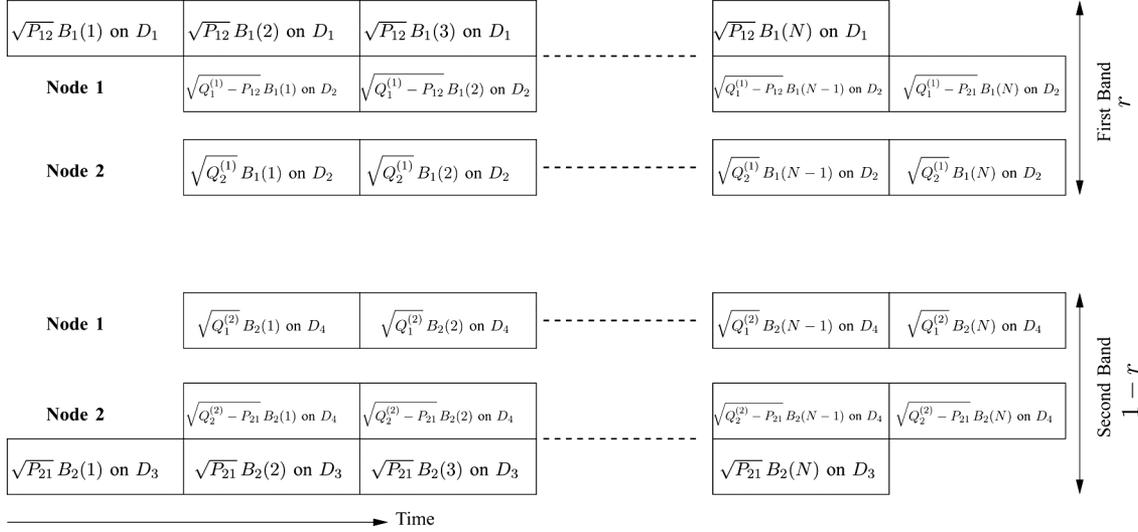


Fig. 6. Modified half-duplex cooperation scheme. In the first band, node 2 acts as a regenerative relay for node 1, and in the second band, these roles are reversed.

$r = 1/2$.⁷ In the first band, node 2 acts as a relay for node 1 and does not attempt to transmit its own data, while in the second band, node 1 acts solely as a relay for node 2. The chosen relaying strategy is of the regenerative decode-and-forward type, and exploits coherent combining at the destination node. (The chosen scheme does not require backward decoding.) Each band is partitioned into two orthogonal subchannels.⁸ Those in the first band will be denoted D_1 and D_2 and those in the second band will be denoted D_3 and D_4 . We will adopt a repetition-based transmission strategy within each band, and hence, the subchannels D_1 and D_2 will each contain half of the bandwidth of the first band, and D_3 and D_4 will each contain half of the bandwidth of the second band. A typical bandwidth allocation for this system is given in Fig. 5.

The transmission strategy envisioned for this system is depicted in Fig. 6, where we have used $Q_i^{(k)}$ to denote the power allocated by node i to the k th band, and hence, the total power transmitted by node i is $P_i = Q_i^{(1)} + Q_i^{(2)}$. Let us consider the operation of the first band, in which node 2 acts as solely a relay for node 1. In the n th block channel use, $2 \leq n \leq N - 1$, node 1 transmits a new codeword (or new segment of a larger codeword), $B_1(n)$, on subchannel D_1 with power P_{12} , and repeats its previous codeword, $B_1(n-1)$, on subchannel D_2 with power $Q_1^{(1)} - P_{12}$. In that same block, node 2 receives $B_1(n)$ on subchannel D_1 , and regeneratively retransmits $B_1(n-1)$ on subchannel D_2 with power $Q_2^{(1)}$ and with the phase corrected so that it coherently combines at the destination node with the repeated transmission of $B_1(n-1)$ by node 1. The first block channel use contains only the first codeword (segment) from node 1, and the last block involves only repetition of the previous codeword, but the impact of these end effects can be neglected when N is

⁷It is possible to construct equivalent systems in which the communication resource that is partitioned is a time interval or the components of a (large) set of orthogonal spreading codes, rather than the system bandwidth, but the principles that underly our approach are the same in those cases. For ease of exposition we will focus on the case of bandwidth allocation.

⁸Again, for ease of exposition we will focus on systems in which the subchannels are synthesized in frequency.

large. In the second band, the roles are reversed, with powers as shown in Fig. 6.

The half-duplex cooperative multiple access scheme described has been designed for scenarios in which the cooperative channels are stronger than the direct channels; i.e., $\gamma_{12} \geq \gamma_{10}$ and $\gamma_{21} \geq \gamma_{20}$, which corresponds to Case 1 in Table I. In that case, the achievable rate region for the proposed scheme is the convex hull of all rate pairs (R_1, R_2) that satisfy the following constraints:

$$R_1 \leq \frac{r}{2} \log \left(1 + \frac{\gamma_{12} P_{12}}{\frac{r}{2}} \right) \quad (32a)$$

$$R_1 \leq \frac{r}{2} \log \left(1 + \frac{\gamma_{10} P_{12} + \left(\sqrt{\gamma_{10} (Q_1^{(1)} - P_{12})} + \sqrt{\gamma_{20} Q_2^{(1)}} \right)^2}{\frac{r}{2}} \right) \quad (32b)$$

$$R_2 \leq \frac{(1-r)}{2} \log \left(1 + \frac{\gamma_{21} P_{21}}{\frac{(1-r)}{2}} \right) \quad (32c)$$

$$R_2 \leq \frac{(1-r)}{2} \log \left(1 + \frac{\gamma_{20} P_{21} + \left(\sqrt{\gamma_{20} (Q_2^{(2)} - P_{21})} + \sqrt{\gamma_{10} Q_1^{(2)}} \right)^2}{\frac{(1-r)}{2}} \right). \quad (32d)$$

The constraints in (32a) and (32c) ensure that the messages to the relaying nodes can be reliably decoded, while the constraints in (32b) and (32d) arise from the combination of the repeated direct transmissions and the regeneratively relayed signals. In particular, in (32b), the term $\gamma_{10} P_{12}$ is the SNR of the direct transmission of $B_1(n)$ to the destination node on subchannel D_1 in block n , and the term $\left(\sqrt{\gamma_{10} (Q_1^{(1)} - P_{12})} + \sqrt{\gamma_{20} Q_2^{(1)}} \right)^2$ is the SNR of the coherent combination of the repetition of $B_1(n)$ by node 1 on subchannel D_2 in block $n+1$ and the regenerative retransmission of $B_1(n)$ on subchannel D_2 in block $n+1$ by node 2. (The relay receives $B_1(n)$ on subchannel D_1 in block

n.) The roles of nodes 1 and 2 are reversed in the second band, leading to (32d).⁹

For a given value of the fractional bandwidth allocation, r , we can determine the achievable rate region by maximizing R_1 for each value of R_2 subject to (32) and the constraint that $P_i \leq \bar{P}_i$. Using techniques similar to those in Section II, that problem can be simplified to (see Appendix IV for a derivation)

$$\max_{Q_1^{(1)}, Q_2^{(1)}} \frac{r}{2} \log \left(1 + \frac{\gamma_{12} f_1(Q_1^{(1)}, Q_2^{(1)})}{\frac{r}{2}} \right) \quad (33a)$$

$$\text{subject to } \gamma_{20}^2 (Q_2^{(1)})^2 - B_0 Q_2^{(1)} - 2\gamma_{10}\gamma_{20} Q_1^{(1)} Q_2^{(1)} + C_0 Q_1^{(1)} + \gamma_{10}^2 (Q_1^{(1)})^2 + D_0 \leq 0 \quad (33b)$$

$$f_1(Q_1^{(1)}, Q_2^{(1)}) - Q_1^{(1)} \leq 0 \quad (33c)$$

$$0 \leq Q_1^{(1)} \leq \bar{P}_1, \quad 0 \leq Q_2^{(1)} \leq \bar{P}_2 \quad (33d)$$

where $f_1(Q_1^{(1)}, Q_2^{(1)})$ and the constants B_0 , C_0 , and D_0 are defined in Appendix IV. Appendix IV also contains the modifications that need to be made to (33) in the case where r is a time-sharing parameter rather than a bandwidth sharing parameter.

The formulation in (33) gives the optimal power allocation for a given value of r . However, our goal is to find the value of r that enables the largest R_1 ; i.e., the largest optimal value in (33). It is clear that the optimal value of r depends on the target value of R_2 ; i.e., the different points on the boundary of the achievable rate region are not achieved with the same r . Therefore, in order to determine the achievable rate region, we will have to optimize over both the powers and the resource sharing parameter, r . Although the problem in (33) is not convex in r and the powers, the following result, which is proven in Appendix V, will enable us to develop an efficient algorithm for the optimal value of r .

Theorem 1: For a given rate R_2 , the maximum achievable rate R_1 in (33) is a quasi-concave function of the resource sharing parameter r .

Since the maximum achievable value of R_1 is quasi-concave in r , we can determine the optimal value of r using a standard search method for quasi-convex problems [14]. At each step in the search, a problem based on (33) with the current value for r is solved. Since the problem in (33) can be efficiently solved, and since the quasi-convex search can be efficiently implemented, the optimal design of r and the powers P_{12} , P_{21} , $Q_1^{(1)}$, $Q_1^{(2)}$, $Q_2^{(1)}$, and $Q_2^{(2)}$ can be efficiently obtained.

To illustrate the performance of the modified half-duplex cooperation multiple access scheme, we have plotted the average achievable rate regions for several multiple access schemes in Figs. 7 and 8. We consider the case of no cooperation (i.e., conventional multiple access), an optimally power loaded block-based version of the half-duplex strategy in [2, Sec. III],¹⁰ the proposed half-duplex strategy and the full-duplex strategy from

⁹It is relatively straightforward to obtain an expression for the achievable rate region of the proposed scheme in the other cases in Table I. In particular, if $\gamma_{ij} < \gamma_{i0}$, one would switch off the relaying activity of node j (i.e., set $Q_j^{(i)} = 0$), and the rate constraint on communication from node i to node j would be removed. That said, in such cases the proposed scheme operates like a repetition-based orthogonal multiple access scheme, and for certain target rates for one of the users, one may be able to achieve a larger rate for the other user by time sharing between the proposed scheme and a conventional multiple access scheme.

¹⁰Recall that the proposed scheme reduces to a block-based version of the scheme in [2, Sec. III] when r is set to $1/2$. Therefore, the optimal power loading for that scheme can be found by solving (33) with $r = 1/2$.

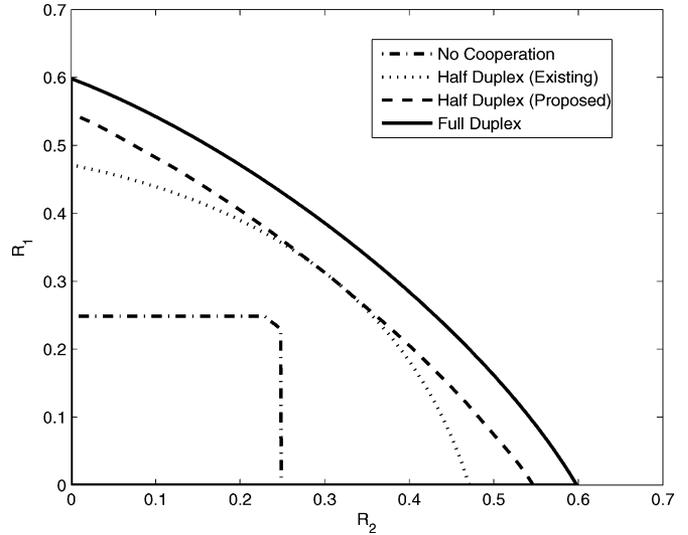


Fig. 7. Achievable rate region for the no cooperation case, the existing cooperative strategy in [2], the proposed cooperative strategy and the full duplex model in case of statistically symmetric direct channels. $P_1 = P_2 = 1.0$, $\sigma_0^2 = \sigma_1^2 = \sigma_2^2 = 1$, $E(K_{10}) = E(K_{20}) = 0.3$, $E(K_{12}) = E(K_{21}) = 0.5$.

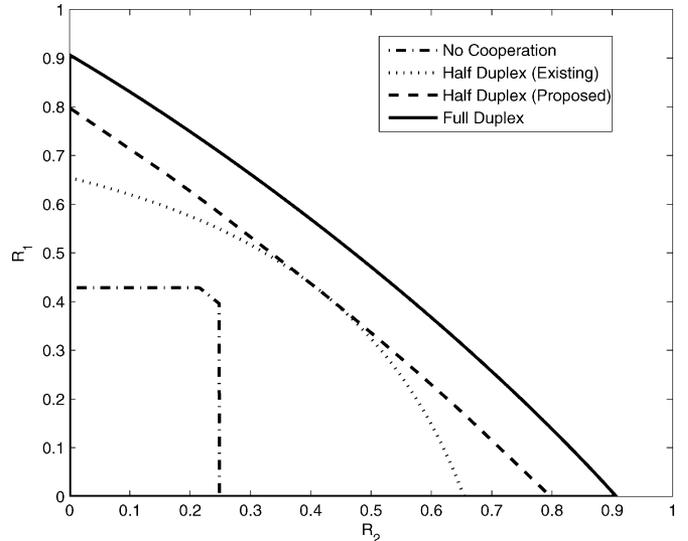


Fig. 8. Achievable rate region for the no cooperation case, the existing cooperative strategy in [2], the proposed cooperative strategy and the full duplex model in case of statistically asymmetric direct channels. $P_1 = P_2 = 1.0$, $\sigma_0^2 = \sigma_1^2 = \sigma_2^2 = 1$, $E(K_{10}) = 0.4$, $E(K_{20}) = 0.3$, $E(K_{12}) = E(K_{21}) = 0.7$.

Section II. In each scenario, the noise variance and the transmitted powers were normalized to one, and the channel gains were Rayleigh distributed. In Fig. 7, the direct channel gains are statistically symmetric with $E(K_{10}) = E(K_{20}) = 0.3$, and $K_{12} = K_{21}$ with $E(K_{12}) = 0.5$. In Fig. 8, the direct channel gains are statistically asymmetric with $E(K_{10}) = 0.4$, $E(K_{20}) = 0.3$, and $K_{12} = K_{21}$ with $E(K_{12}) = 0.7$. In both Figs. 7 and 8, the proposed half-duplex strategy provides a significant expansion of the achievable rate region over that of the half-duplex strategy in [2] when the rates of the nodes are significantly different. This is a consequence of the efficient algorithm for finding the optimal resource sharing parameter. Since $r = 1/2$ is the optimal value of r when $R_1 = R_2$, boundaries of the achievable rate regions of the proposed strategy and that in [2] overlap at this point.

VI. CONCLUSION

In this paper, we have analyzed the power allocation problem in full-duplex cooperative multiple access schemes and the joint power and resource allocation problem in a class of half-duplex cooperative multiple access schemes. In the full duplex case, our analysis revealed an underlying convex optimization problem that has a closed-form solution, while in the half-duplex case we exposed the quasi-convexity of the resource allocation problem and exploited that property to develop an efficient algorithm. In doing so, we demonstrated that a significantly larger fraction of the achievable rate region of the full duplex case can be obtained with the proposed scheme. Moreover, the reduction that we have obtained in the complexity of finding the optimal power and resource allocation suggests that it may be possible to avoid approximations in the development of online algorithms.

APPENDIX I

SUFFICIENCY OF THE CORNER POINTS ON THE CONVENTIONAL MULTIPLE ACCESS REGION

Let us begin with a point on the line connecting the two corner points of the conventional multiple access region; i.e., the (R_{10}, R_{20}) region. Any point on that line can be achieved by time sharing between the two corners, [13]. That is

$$R_{10} \leq \rho \log \left(1 + \frac{\gamma_{10} P_{10}}{1 + \gamma_{20} P_{20}} \right) + \bar{\rho} \log (1 + \gamma_{10} P_{10}) \quad (34)$$

$$R_{20} \leq \rho \log (1 + \gamma_{20} P_{20}) + \bar{\rho} \log \left(1 + \frac{\gamma_{20} P_{20}}{1 + \gamma_{10} P_{10}} \right) \quad (35)$$

where $\rho \in [0, 1]$ is a time sharing constant and $\bar{\rho} = 1 - \rho$. Since $R_1 = R_{10} + R_{12}$ and $R_2 = R_{20} + R_{21}$, we can write the constraints on R_1 and R_2 as

$$R_1 \leq \rho \log \left(1 + \frac{\gamma_{10} P_{10}}{1 + \gamma_{20} P_{20}} \right) + \bar{\rho} \log (1 + \gamma_{10} P_{10}) + \log \left(1 + \frac{\gamma_{12} P_{12}}{1 + \gamma_{12} P_{10}} \right) \quad (36)$$

$$R_2 \leq \rho \log (1 + \gamma_{20} P_{20}) + \bar{\rho} \log \left(1 + \frac{\gamma_{20} P_{20}}{1 + \gamma_{10} P_{10}} \right) + \log \left(1 + \frac{\gamma_{21} P_{21}}{1 + \gamma_{21} P_{20}} \right) \quad (37)$$

$$R_1 + R_2 \leq \log \left(1 + \gamma_{10} P_1 + \gamma_{20} P_2 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}} \right) \quad (38)$$

which can be written as

$$R_1 \leq \rho R_1^* + \bar{\rho} R_1^{**} \quad (39a)$$

$$R_2 \leq \rho R_2^* + \bar{\rho} R_2^{**} \quad (39b)$$

$$R_1 + R_2 \leq \log \left(A_0 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}} \right) \quad (39c)$$

where

$$R_1^* = \log \left(1 + \frac{\gamma_{10} P_{10}}{1 + \gamma_{20} P_{20}} \right) + \log \left(1 + \frac{\gamma_{12} P_{12}}{1 + \gamma_{12} P_{10}} \right)$$

$$R_1^{**} = \log (1 + \gamma_{10} P_{10}) + \log \left(1 + \frac{\gamma_{12} P_{12}}{1 + \gamma_{12} P_{10}} \right)$$

$$R_2^* = \log (1 + \gamma_{20} P_{20}) + \log \left(1 + \frac{\gamma_{21} P_{21}}{1 + \gamma_{21} P_{20}} \right)$$

$$R_2^{**} = \log \left(1 + \frac{\gamma_{20} P_{20}}{1 + \gamma_{10} P_{10}} \right) + \log \left(1 + \frac{\gamma_{21} P_{21}}{1 + \gamma_{21} P_{20}} \right)$$

$$A_0 = 1 + \gamma_{10} P_1 + \gamma_{20} P_2.$$

Now, consider maximizing the region defined by the constraints in (39), subject to the power constraints in (4b). Any point on the boundary of the maximized region can be obtained by solving the following problem (maximize the weighted sum of rates):

$$\max_{\mathbf{p}} \mu_1 R_1 + \mu_2 R_2 \quad \text{subject to (39a)–(39c), (4b)}$$

where \mathbf{p} is a vector containing all the power components. For simplicity, we will focus on the case in which $\mu_2 \geq \mu_1$. The proof for the case in which $\mu_2 < \mu_1$ is analogous. If $\mu_2 \geq \mu_1$, then the previous problem can be written as

$$\max_{\mathbf{p}} \mu_1 (R_1 + R_2) + (\mu_2 - \mu_1) R_2 \quad (40a)$$

$$\text{subject to (39a)–(39c), (4b).} \quad (40b)$$

The sum rate constraint in (39c) can be written as

$$R_1 + R_2 \leq \min \left\{ \log \left(A_0 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}} \right), \right. \\ \left. \rho R_1^* + \bar{\rho} R_1^{**} + \rho R_2^* + \bar{\rho} R_2^{**} \right\} \quad (41)$$

$$= \min \{ f(\mathbf{p}), g(\mathbf{p}) \} \quad (42)$$

where

$$f(\mathbf{p}) = \log \left(A_0 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}} \right)$$

$$g(\mathbf{p}) = \log (1 + \gamma_{10} P_{10} + \gamma_{20} P_{20}) + \log \left(1 + \frac{\gamma_{12} P_{12}}{1 + \gamma_{12} P_{10}} \right) + \log \left(1 + \frac{\gamma_{21} P_{21}}{1 + \gamma_{21} P_{20}} \right).$$

Now, (40) can be written as

$$\max_{\mathbf{p}} \mu_1 \min \{ f(\mathbf{p}), g(\mathbf{p}) \} + (\mu_2 - \mu_1) R_2 \quad (43a)$$

$$\text{subject to (39a)–(39c), (4b).} \quad (43b)$$

Substituting for R_2 from (39b), the objective in (43a) can be rewritten as

$$\max_{\mathbf{p}} \mu_1 \min \{ f(\mathbf{p}), g(\mathbf{p}) \} + (\mu_2 - \mu_1) (\rho R_2^* + \bar{\rho} R_2^{**}). \quad (44)$$

Furthermore, (44) can be bounded as

$$\begin{aligned} & \max_{\mathbf{p}} \rho [\mu_1 \min \{f(\mathbf{p}), g(\mathbf{p})\} + (\mu_2 - \mu_1)R_2^*] \\ & \quad + \bar{\rho} [\mu_1 \min \{f(\mathbf{p}), g(\mathbf{p})\} + (\mu_2 - \mu_1)R_2^{**}] \\ & \leq \rho \max_{\mathbf{p}} [\mu_1 \min \{f(\mathbf{p}), g(\mathbf{p})\} + (\mu_2 - \mu_1)R_2^*] \\ & \quad + \bar{\rho} \max_{\mathbf{p}} [\mu_1 \min \{f(\mathbf{p}), g(\mathbf{p})\} + (\mu_2 - \mu_1)R_2^{**}]. \end{aligned} \quad (45)$$

The first term in the right-hand side of (45) represents the maximization of the rate region assuming that the direct message of node 1 is decoded first, while the second term represents the maximization of the rate region assuming that the direct message of node 2 is decoded first. Therefore, it is clear from (45) that time sharing between the two regions corresponding to the two corner points of conventional multiple access region [i.e., the (R_{10}, R_{20}) region] contains all the regions corresponding to the points on the line connecting the two corners. Hence, it is sufficient to study only the two regions corresponding to the two corner points on the (R_{10}, R_{20}) region.

APPENDIX II

THE FIRST CASE OF (26) WITH $B < 0$

In the first case of (26), if B is negative, then $(1 + \gamma_{12}\bar{P}_1) < (1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2)/2^{R_{2,\text{tar}}}$. Taking the logarithm of both sides, we have that

$$\begin{aligned} \log(1 + \gamma_{12}\bar{P}_1) & < \log\left(\frac{1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2}{2^{R_{2,\text{tar}}}}\right) \\ & = \log(1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2) - R_{2,\text{tar}}. \end{aligned} \quad (46)$$

Given the conditions on the channel gains for Case 4 to arise, we also have $\log(1 + \gamma_{12}\bar{P}_1) < \log(1 + \gamma_{10}\bar{P}_1)$. The conventional multiple access scheme provides an achievable R_1 that equals

$$\min\{\log(1 + \gamma_{10}\bar{P}_1), \log(1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2) - R_{2,\text{tar}}\}. \quad (47)$$

Since both terms in the minimization in (47) are larger than $\log(1 + \gamma_{12}\bar{P}_1)$, in this scenario the conventional multiple access scheme provides a higher achievable rate than that offered by a scheme that presumes cooperation.

APPENDIX III

PROOF OF SUFFICIENCY OF THE FIRST CASE OF (26) AND ITS SYMMETRIC IMAGE

For simplicity, we let

$$f(P_{U1}, P_{U2}) = \log\left(1 + \gamma_{10}\bar{P}_1 + \gamma_{20}\bar{P}_2 + 2\sqrt{\gamma_{10}\gamma_{20}P_{U1}P_{U2}}\right).$$

When $P_{20} < (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}$, the constraint set (24) can be written as

$$R_2 \leq \log(1 + \gamma_{20}P_{20}) + \log\left(1 + \frac{\gamma_{21}P_{21}}{1 + \gamma_{21}P_{20}}\right) \quad (48a)$$

$$R_1 \leq \log\left(1 + \frac{\gamma_{10}P_{10}}{1 + \gamma_{20}P_{20}}\right) \quad (48b)$$

$$R_1 + R_2 \leq f(P_{U1}, P_{U2}). \quad (48c)$$

The first term in the right-hand side of (48a), and the term on the right-hand side of (48b) represent the conventional multiple access region. Therefore, this constraint set can be rewritten as

$$R_2 \leq \log(1 + \gamma_{20}P_{20}) + \log\left(1 + \frac{\gamma_{21}P_{21}}{1 + \gamma_{21}P_{20}}\right) \quad (49a)$$

$$R_1 \leq \log(1 + \gamma_{10}P_{10}) \quad (49b)$$

$$R_1 + R_2 \leq \min\left\{f(P_{U1}, P_{U2}), \log\left(\frac{1 + \gamma_{10}P_{10} + \gamma_{20}P_{20}}{1 + \gamma_{21}P_{20}}\right) + \log(1 + \gamma_{21}S_2)\right\}. \quad (49c)$$

Solving Step 1) with $P_{20} > (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}$, using (27) and (29), gives the following constraint set:

$$R_2 \leq \log(1 + \gamma_{20}P_{20}), \quad R_1 \leq \log(1 + \gamma_{12}P_{12}) \quad (50a)$$

$$R_1 + R_2 \leq f(P_{U1}, P_{U2}). \quad (50b)$$

Similarly, solving Step 2) with $P_{10} > (\gamma_{20}/\gamma_{21} - 1)/\gamma_{10}$ gives the following constraint set:

$$R_2 \leq \log(1 + \gamma_{21}P_{21}), \quad R_1 \leq \log(1 + \gamma_{10}P_{10}) \quad (51a)$$

$$R_1 + R_2 \leq f(P_{U1}, P_{U2}). \quad (51b)$$

Now consider the generic problem of maximizing a weighted sum of the rates R_1 and R_2 , under the second case of (26), namely

$$\max_{\mathbf{p}} \mu_1 R_1 + \mu_2 R_2 \quad \text{subject to (49), (4b), } P_{20} < \frac{(\gamma_{10} - 1)}{\gamma_{12}} \quad (52)$$

where \mathbf{p} is the vector containing all the power components to be allocated. The region resulting from (52) is contained in the region that is achievable if the constraint $P_{20} < (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}$ is removed, namely

$$\max_{\mathbf{p}} \mu_1 R_1 + \mu_2 R_2 \quad \text{subject to (49), (4b).} \quad (53)$$

The problem in (53) can be written as

$$\max_{\mathbf{p}} a(R_1 + R_2) + bR_1 \quad \text{subject to (49), (4b)} \quad (54)$$

where $a = \mu_2$ and $b = \mu_1 - \mu_2$ are constants. The region resulting from the problem in (54) is contained in the following region:

$$\max_{\tilde{\mathbf{p}}} a \max_{P_{21}} (R_1 + R_2) + b \max_{P_{21}} R_1 \quad (55a)$$

$$\text{subject to (49), (4b)} \quad (55b)$$

where $\tilde{\mathbf{p}}$ contains all the power components to be allocated, except P_{21} . The objective in (56) can be expanded as

$$\begin{aligned} & \max_{\tilde{\mathbf{p}}} a \max_{P_{21}} \\ & \min\left\{f(P_{U1}, P_{U2}), \log\left(\frac{1 + \gamma_{10}P_{10} + \gamma_{20}P_{20}}{1 + \gamma_{21}P_{20}}\right) + \log(1 + \gamma_{21}S_2)\right\} \\ & \quad + b \log(1 + \gamma_{10}P_{10}). \end{aligned} \quad (56)$$

For a constant sum $S_2 = P_{20} + P_{21}$, maximizing over P_{21} takes the form of (5), and hence

$$P_{21}^* = \begin{cases} 0, & \text{if } P_{10} < \frac{(\frac{\gamma_{20}}{\gamma_{21}} - 1)}{\gamma_{10}} \\ S_2, & \text{if } P_{10} > \frac{(\frac{\gamma_{20}}{\gamma_{21}} - 1)}{\gamma_{10}}. \end{cases} \quad (57)$$

For the first case of (57), namely $P_{10} < (\gamma_{20}/\gamma_{21} - 1)/\gamma_{10}$, the expression in (56) can be written as

$$\max_{\bar{P}} a \min \{f(P_{U1}, P_{U2}), \log(1 + \gamma_{10}P_{10} + \gamma_{20}P_{20})\} + b \log(1 + \gamma_{10}P_{10}). \quad (58)$$

Since $f(P_{U1}, P_{U2}) \geq \log(1 + \gamma_{10}P_{10} + \gamma_{20}P_{20})$, (58) can be written as

$$\max_{\bar{P}} a \log(1 + \gamma_{10}P_{10} + \gamma_{20}P_{20}) + b \log(1 + \gamma_{10}P_{10}). \quad (59)$$

The region resulting from (59) is the same as the conventional multiple access region. In the second case of (57), namely $P_{10} > (\gamma_{20}/\gamma_{21} - 1)/\gamma_{10}$, the problem in (56) can be written as

$$\max_{\bar{P}} a \min \{f(P_{U1}, P_{U2}), \log(1 + \gamma_{10}P_{10}) + \log(1 + \gamma_{21}P_{21}^*)\} + b \log(1 + \gamma_{10}P_{10}). \quad (60)$$

The region resulting from solving (60) is the same as the region resulting from the constraint set in (51).

Therefore, given the containment arguments that gave rise to (53) and (55b), the regions resulting from the case in which $P_{20} < (\gamma_{10}/\gamma_{12} - 1)/\gamma_{20}$ is contained in the convex hull of the conventional multiple access region and the region resulting from the constraint set in (51). Using similar arguments, it can be shown that the region resulting from solving Step 2) with $P_{10} < (\gamma_{20}/\gamma_{21} - 1)/\gamma_{10}$ is contained in the convex hull of the conventional multiple access region and the region resulting from the constraint set in (50).

APPENDIX IV DERIVATION OF (33)

For given values of r and R_2 we must choose values for P_{21} , $Q_1^{(2)}$, and $Q_2^{(2)}$ such that (32c) and (32d) are satisfied. For (32c) to be satisfied with minimum power we should choose $P_{21} = (1-r)(2^{2R_2/(1-r)} - 1)/(2\gamma_{21})$ and for (32d) to be satisfied $Q_1^{(2)}$ and $Q_2^{(2)}$ should satisfy the following inequality:

$$\gamma_{20}^2(Q_2^{(2)})^2 - 2A_0\gamma_{20}Q_2^{(2)} - 2\gamma_{10}\gamma_{20}Q_1^{(2)}Q_2^{(2)} + (4\gamma_{10}\gamma_{20}P_{21} - 2A_0\gamma_{10})Q_1^{(2)} + \gamma_{10}^2(Q_1^{(2)})^2 + \gamma_0 \leq 0 \quad (61)$$

where $A_0 = (1-r)(2^{2R_2/(1-r)} - 1)/2$. Using the relations between $(Q_1^{(1)}, Q_2^{(1)})$ and $(Q_1^{(2)}, Q_2^{(2)})$, (61) becomes

$$\gamma_{20}^2(Q_2^{(1)})^2 - B_0Q_2^{(1)} - 2\gamma_{10}\gamma_{20}Q_1^{(1)}Q_2^{(1)} + C_0Q_1^{(1)} + \gamma_{10}^2(Q_1^{(1)})^2 + D_0 \leq 0 \quad (62)$$

where

$$\begin{aligned} B_0 &= 2\gamma_{10}\gamma_{20}P_1 + 2A_0\gamma_{20} - 2\gamma_{20}^2P_2 \\ C_0 &= 2\gamma_{10}\gamma_{20}P_2 + 2A_0\gamma_{10} - 4\gamma_{10}\gamma_{20}P_{21} - 2\gamma_{10}^2P_1 \\ D_0 &= A_0^2 + \gamma_{20}^2P_2^2 + \gamma_{10}^2P_1^2 - 2\gamma_{10}\gamma_{20}P_1P_2 - 2A_0\gamma_{20}P_2 \\ &\quad - 2A_0\gamma_{10}P_1 + 4\gamma_{10}\gamma_{20}P_{21}P_1. \end{aligned} \quad (63)$$

Now, (32a) and (32b) can be written as

$$R_1 \leq \frac{r}{2} \log \left(1 + \frac{\gamma_{12}P_{12}}{\frac{r}{2}} \right) \quad (64a)$$

$$R_1 \leq \frac{r}{2} \cdot \log \left(1 + \frac{\gamma_{10}Q_1^{(1)} + \gamma_{20}Q_2^{(1)} + 2\sqrt{\gamma_{10}\gamma_{20}}\sqrt{Q_2^{(1)}(Q_1^{(1)} - P_{12})}}{\frac{r}{2}} \right). \quad (64b)$$

The problem of maximizing R_1 can now be written as

$$\max_{P_{12}, Q_1^{(1)}, Q_2^{(1)}} \min \{R_{11}, R_{12}\} \quad (65a)$$

$$\text{subject to } \gamma_{20}^2(Q_2^{(1)})^2 - B_0Q_2^{(1)} - 2\gamma_{10}\gamma_{20}Q_1^{(1)}Q_2^{(1)} + C_0Q_1^{(1)} + \gamma_{10}^2(Q_1^{(1)})^2 + D_0 \leq 0 \quad (65b)$$

$$0 \leq Q_1^{(1)} \leq \bar{P}_1 \quad (65c)$$

$$0 \leq Q_2^{(1)} \leq \bar{P}_2 \quad (65d)$$

where R_{11} and R_{12} are the terms on the right-hand side of (64a) and (64b), respectively. This problem can be shown to be concave in P_{12} , $Q_1^{(1)}$, and $Q_2^{(1)}$ as follows. First, R_{11} is concave in P_{12} . The function inside the logarithm in R_{12} has a negative semi-definite Hessian, namely (66), shown at the bottom of the page. Since the logarithm is concave non-decreasing function, then R_{12} is a concave function and since the minimum of two concave functions is a concave function then the objective function is concave. The first constraint can be shown to be convex by evaluating the Hessian. The Hessian is

$$\begin{pmatrix} 2\gamma_{10}^2 & -2\gamma_{10}\gamma_{20} \\ -2\gamma_{10}\gamma_{20} & 2\gamma_{20}^2 \end{pmatrix} \succeq 0 \quad (67)$$

which is positive semidefinite and hence the constraint is convex. The last three constraints are linear constraints, and

$$\frac{\sqrt{\gamma_{10}\gamma_{20}}}{2(Q_2^{(1)})^{3/2}(Q_1^{(1)} - P_{12})^{3/2}} \times \begin{pmatrix} -(Q_2^{(1)})^2 & Q_2^{(1)}(Q_1^{(1)} - P_{12}) & (Q_2^{(1)})^2 \\ Q_2^{(1)}(Q_1^{(1)} - P_{12}) & -(Q_1^{(1)} - P_{12})^2 & -Q_2^{(1)}(Q_1^{(1)} - P_{12}) \\ (Q_2^{(1)})^2 & -Q_2^{(1)}(Q_1^{(1)} - P_{12}) & -(Q_2^{(1)})^2 \end{pmatrix} \preceq 0. \quad (66)$$

hence the problem in (65) is concave and a global maximum can be efficiently obtained. However, as we show in the next paragraph, the variable P_{12} can be determined analytically.

We can solve (64a) and (64b) to obtain the value of P_{12} that maximizes the minimum of both equations in terms of $Q_1^{(1)}$ and $Q_2^{(1)}$. Since the term inside the logarithm in (64a) is linearly increasing in P_{12} and the term inside the logarithm in (64b) is concave decreasing in P_{12} , the intersection of these two curves is the target point. This will result in $P_{12}^* = f_1(Q_1^{(1)}, Q_2^{(1)})$, where

$$f_1(Q_{11}, Q_{21}) = \frac{\frac{\sigma_0^2 K_{12}^2}{\sigma_2^2} (K_{10}^2 Q_{11} + K_{20}^2 Q_{21}) - 4K_{10}^2 K_{20}^2 Q_{21}}{\left(\frac{\sigma_0^2 K_{12}^2}{\sigma_2^2}\right)^2} + \frac{2K_{10} K_{20} \sqrt{K_{10}^2 - \frac{\sigma_0^2 K_{12}^2}{\sigma_2^2}} \sqrt{K_{20}^2 Q_{21}^2 - \frac{\sigma_0^2 K_{12}^2}{\sigma_2^2} Q_{11} Q_{21}}}{\left(\frac{\sigma_0^2 K_{12}^2}{\sigma_2^2}\right)^2}. \quad (68)$$

By substituting the expression for $f_1(Q_1^{(1)}, Q_2^{(1)})$ into (65), we obtain (33). By evaluating the Hessian, it can be verified that $f_1(Q_1^{(1)}, Q_2^{(1)})$ is a concave function of $Q_1^{(1)}$ and $Q_2^{(1)}$. In the case that $\gamma_{12} Q_1^{(1)} \leq \gamma_{10} Q_1^{(1)} + \gamma_{20} Q_2^{(1)}$ the derivation simplifies in that $R_{11} \leq R_{12}$ for all admissible $Q_1^{(1)}$ and $Q_2^{(1)}$. In that case, the optimal value of P_{12} is $Q_1^{(1)}$ and $f_1(Q_1^{(1)}, Q_2^{(1)}) = Q_1^{(1)}$.

In Section V and the previous proof, the parameter r was a bandwidth sharing parameter. However, it is easy to envision an equivalent time sharing system. In this paragraph, we point out some minor adjustments to the formulation that are required in the time sharing case. In the case of time sharing, the average power transmitted by node i is $P_i = rQ_i^{(1)} + (1-r)Q_i^{(2)}$. Therefore, the constraints in (33d) should be changed to $\min(P_{1,\text{peak}}, \bar{P}_1/r) \geq Q_1^{(1)} \geq \max(0, (\bar{P}_1 - (1-r)P_{1,\text{peak}})/r)$, where $P_{1,\text{peak}}$ is the value of the peak power for the transmitter of node 1. Constraining $Q_1^{(1)}$ to be less than \bar{P}_1/r guarantees that $Q_1^{(2)} = (\bar{P}_1 - rQ_1^{(1)})/(1-r)$ will not take negative values, while constraining $Q_1^{(1)}$ to be greater than $(\bar{P}_1 - (1-r)P_{1,\text{peak}})/r$ guarantees that $Q_1^{(2)}$ will not exceed the value of peak power for the transmitter of node 1. Using similar arguments, the constraint in (33d) should be changed to $\min(P_{2,\text{peak}}, \bar{P}_2/r) \geq Q_2^{(1)} \geq \max(0, (\bar{P}_2 - (1-r)P_{2,\text{peak}})/r)$.

APPENDIX V

PROOF OF QUASI-CONCAVITY OF R_1 IN r

Using the substitution $Q_1^{(2)} = \bar{P}_1 - Q_1^{(1)}$ and $Q_2^{(2)} = \bar{P}_2 - Q_2^{(1)}$, (32d) can be written as a function of P_{21} , $Q_1^{(1)}$ and $Q_2^{(1)}$. For given values of $Q_1^{(1)}$ and $Q_2^{(1)}$ we can solve (32a) and (32b) to find the value of P_{12} that maximizes the minimum of the two equations in terms of $Q_1^{(1)}$ and $Q_2^{(1)}$. [A similar analysis led to (68).] Similarly, we can solve (32c) and (32d) for P_{21} . Hence, the constraints on R_1 and R_2 reduce to

$$R_1 \leq \frac{r}{2} \log \left(1 + \frac{f_1(Q_1^{(1)}, Q_2^{(1)})}{\frac{r}{2}} \right) \quad (69a)$$

$$R_2 \leq \frac{(1-r)}{2} \log \left(1 + \frac{f_2(Q_1^{(1)}, Q_2^{(1)})}{\frac{(1-r)}{2}} \right) \quad (69b)$$

where $f_1(Q_1^{(1)}, Q_2^{(1)})$ was defined in (68) and $f_2(Q_1^{(1)}, Q_2^{(1)})$ has a similar form. It can be shown that $f_1(Q_1^{(1)}, Q_2^{(1)})$ and $f_2(Q_1^{(1)}, Q_2^{(1)})$ are concave functions of $Q_1^{(1)}$ and $Q_2^{(1)}$ by evaluating the Hessian.

We will now show that the maximum achievable R_1 for a given R_2 is quasi-concave in r by proving that the set of values of r such that the maximum achievable R_1 is greater than certain level $R_{1,\text{tar}}$ is a convex set. For a given value of R_2 , if a value of r , say r_0 , is to be such that there exist transmission powers that provide a rate R_1 that is greater than $R_{1,\text{tar}}$, these powers must satisfy

$$f_1(Q_1^{(1)}, Q_2^{(1)}) \geq g_1(r) \quad (70a)$$

$$f_2(Q_1^{(1)}, Q_2^{(1)}) \geq g_2(r) \quad (70b)$$

for $r = r_0$, where $g_1(r) = (r/2)(2^{2R_{1,\text{tar}}/r} - 1)$ and $g_2(r) = ((1-r)/2)(2^{2R_2/(1-r)} - 1)$. The functions $g_1(r)$ and $g_2(r)$ can be shown to be convex functions in r for $r \in (0, 1)$. Now, assume that there exist transmission powers $Q_1^{(1)} = X^{(1)}$ and $Q_2^{(1)} = Y^{(1)}$ such that (70) holds for $r = r_1$, and powers $Q_1^{(1)} = X^{(2)}$ and $Q_2^{(1)} = Y^{(2)}$ such that (70) holds for $r = r_2$. To complete the proof we need to show that for any $r_3 = \lambda r_1 + (1-\lambda)r_2$ with $\lambda \in [0, 1]$ there exist transmission powers $Q_1^{(1)} = X^{(3)}$ and $Q_2^{(1)} = Y^{(3)}$ such that (70) holds for r_3 .

Consider the powers $X^{(3)} = \lambda X^{(1)} + (1-\lambda)X^{(2)}$ and $Y^{(3)} = \lambda Y^{(1)} + (1-\lambda)Y^{(2)}$. For these powers

$$f_1(X^{(3)}, Y^{(3)}) \geq \lambda f_1(X^{(1)}, Y^{(1)}) + (1-\lambda)f_1(X^{(2)}, Y^{(2)}) \quad (71a)$$

$$\geq \lambda g_1(r_1) + (1-\lambda)g_1(r_2) \quad (71b)$$

$$\geq g_1(r_3) \quad (71c)$$

where (71a) follows from concavity of $f_1(Q_1^{(1)}, Q_2^{(1)})$, (71b) follows from (70), and (71c) follows from the convexity of $g_1(r)$. A similar analysis shows that $f_2(X^{(3)}, Y^{(3)}) \geq g_2(r_3)$. That is, given $r_3 \in [r_1, r_2]$, we have constructed powers $X^{(3)}$ and $Y^{(3)}$ such that (70) holds for $r = r_3$. Therefore, the set of values of r for which the maximum achievable R_1 for a given R_2 is greater than $R_{1,\text{tar}}$ is convex, and hence the maximum achievable R_1 for a given R_2 is a quasi-concave function in r .

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