Transceiver Optimization for Multiple Access Through ISI Channels*

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A central problem in multiple access (MA) communication systems over intersymbol interference (ISI) channels is the optimal allocation of the transmitters' finite resources. One solution is for each user to allocate transmission power at each frequency according to a multi-user 'water-filling' distribution [1]. While this solution can attain the boundary of the capacity region, the optimal transmitters can be awkward to compute exactly, and may require joint (or at least successive) detection at the receiver [1]. To simplify the transmitter design and the receiver implementation, we devise efficiently-solvable optimal transmitter resource allocation formulations for MA schemes with a *linear* receiver. We do so by minimizing the Mean Square Error (MSE) of the receiver output, under the constraint of finite transmission power. The direct formulation of that problem is nonconvex, making it difficult to solve in practice, but we derive an alternative convex formulation of the problem using Linear Matrix Inequalities (LMIs). When the channel matrices are diagonal (or jointly diagonalizable), as in OFDMA-type systems, the minimum MSE (MMSE) transmitters can be realized by appropriately allocating subcarriers and power to each user, according to the relative gains of the subcarriers. This result simplifies the transceiver design problem to a Second Order Cone Program (SOCP) which can be solved by highly efficient interior point methods. In this short paper, we will simply state our results for a two-user scenario. More general results are available elsewhere [2].

Consider a quasi-synchronous block-based multiple access scheme with two users whose data vectors are $\mathbf{s}_1$ and $\mathbf{s}_2$. The received signal takes the form

$$\mathbf{x} = \mathbf{H}_1\mathbf{F}_1\mathbf{s}_1 + \mathbf{H}_2\mathbf{F}_2\mathbf{s}_2 + \mathbf{n}, \tag{1}$$

where the channel matrices $\mathbf{H}_j \in \mathbb{C}^{p\times n}$ are assumed to be known, the transmitter precoding matrices $\mathbf{F}_j$ are square, and $\mathbf{n}$ is a zero mean additive Gaussian noise vector with known covariance matrix $\mathbf{R}$. At the receiver, we will use a (block-based) linear equalizer $\hat{\mathbf{s}}_j = \mathbf{G}_j\mathbf{x}$. Although the symbol vectors $\mathbf{s}_j$ are nominally of length n, after the matrices $\mathbf{F}_j$ are designed, we usually have $r(\mathbf{F}_j) \leq n$, where $r(\cdot)$ denotes the rank of a matrix. Therefore, the actual symbol rate of user j will be reduced to $r(\mathbf{F}_j)$ symbols per block, resulting in a coding rate of $r(\mathbf{F}_j)/n$. In other words, we do not set the coding rates before the design process. Instead, they are (implicitly) optimized along with the explicit MMSE optimization of the transceivers.

For the system in (1), let $\mathbf{e}_j = \hat{\mathbf{s}}_j - \mathbf{s}_j$ denote the error vector for user j. Then the multiuser MMSE design problem is: *Find transmitters $\mathbf{F}_j$ and equalizers $\mathbf{G}_j$ such that the total* $\mathrm{MSE} = \mathrm{tr}(E(\mathbf{e}_1\mathbf{e}_1^H)) + \mathrm{tr}(E(\mathbf{e}_2\mathbf{e}_2^H))$ *is minimized, subject to* $\mathrm{tr}(\mathbf{F}_j\mathbf{F}_j^H) \leq p_j$, *where* $p_j > 0$ *are user-specified bounds on the average transmitted power for each user.* Unfortunately, this problem is not convex and hence can be difficult to solve. However, by reformulating the problem in terms of some new matrix variables it can be algebraically transformed into a (convex) Semidefinite Programme (SDP) [2]. Although such a formulation can be efficiently solved, when the channel matrices are diagonal (as in OFDMA-type

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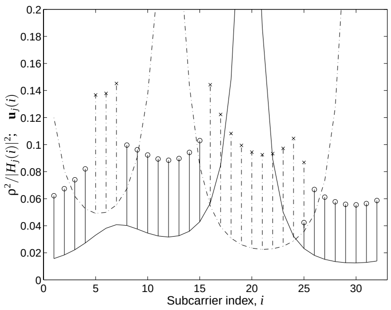

Figure 1: A typical multiuser power allocation. The curves are linear interpolations of $\rho^2/|H_j(i)|^2$ for $j = 1$ (solid) and $j = 2$ (dash-dot). The stems are of length $\mathbf{u}_j(i)$ (where this is non-zero) for $j = 1$ ('$\circ$') and $j = 2$ ('$\times$').

systems) and $\mathbf{R}$ is also diagonal, the optimal transmitters are also diagonal [2] and can be computed even more efficiently. (Diagonal transmitters simply represent power loading/subcarrier allocation and hence they are easy to implement.) To state this formally, note that the diagonal elements of $\mathbf{H}_j$ in an OFDMA-type system are $H_j(i)$, where $H_j(i)$ is the frequency response of user j's channel at frequency $\omega_i = 2\pi(i-1)/n$. If $\mathbf{R} = \text{diag}\{\rho_i^2\}$, and we let $\mathbf{u}_j(i)$ denote the power allocated by the jth user to the ith subcarrier, and introduce an auxiliary vector $\mathbf{w}$, the design problem becomes

$$\begin{aligned} \text{minimize}_{\mathbf{w},\mathbf{u}_1,\mathbf{u}_2} \quad & \sum_{i=1}^{n} \rho_i^2 \mathbf{w}(i) \\ \text{subject to} \quad & \sum_{i=1}^{n} \mathbf{u}_1(i) \le p_1, \quad \sum_{i=1}^{n} \mathbf{u}_2(i) \le p_2, \quad \mathbf{u}_1(i) \ge 0, \quad \mathbf{u}_2(i) \ge 0, \\ & \mathbf{w}(i)\left(|H_1(i)|^2 \mathbf{u}_1(i) + |H_2(i)|^2 \mathbf{u}_2(i) + \rho_i^2\right) \ge 1. \end{aligned} \tag{2}$$

There exist highly efficient (general purpose) implementations (e.g., [3]) of interior point solution methods for the above (rotated) second order cone program with total computational complexity of $O(n^{3.5}\log(1/\varepsilon))$, where $\varepsilon > 0$ is the solution accuracy. Note that (2) depends only on the *magnitude* of the subchannel gains, $H_j(i)$, and hence does not require phase estimation.

By examining the optimality conditions for (2) it can be shown [2] that in order to minimize the MSE one should allocate a subcarrier i to user 1 and a subcarrier j to user 2 only if $\frac{|H_1(i)|^2}{|H_2(i)|^2} \ge \frac{|H_1(j)|^2}{|H_2(j)|^2}$. In other words, the subcarriers are allocated to the users according to the relative ratios of the subchannel gains. For all the subcarriers which are shared by both users, the subchannel gain ratio $|H_2(i)|^2/|H_1(i)|^2$ must be the same, and hence in a fading environment the probability of more than one subcarrier being shared is vanishingly small. Of course, there may also be subcarriers which are not used by either user. These subcarriers have small subchannel gain to (subcarrier) noise ratios for both users (i.e., both $|H_1(i)|^2/\rho_i^2$ and $|H_2(i)|^2/\rho_i^2$ are small).

The results of the multi-user MMSE power loading algorithm for a typical scenario are shown in Figure 1 in a form reminiscent of water-filling. (The signal-to-noise ratio is low to enhance clarity.) Note that in this scenario, 19 subcarriers have been allocated to user 1 alone, 12 to user 2 alone, and that subcarrier 25 is shared. Also note that subcarriers 5, 6 and 7 are allocated to user 2 even though $|H_1(i)| > |H_2(i)|$ for $i = 5, 6, 7$.

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