February 7, 2012

ASSIGNMENT 2

DUE FEBRUARY 14, 2012

1. (10 points) Prove that if

$$\begin{vmatrix} \mathbf{e}(r,\theta,\varphi,t) = \mathrm{Re}\left\{\mathbf{E}(r,\theta,\varphi)e^{j\omega t}\right\} \\ \mathbf{h}(r,\theta,\varphi,t) = \mathrm{Re}\left\{\mathbf{H}(r,\theta,\varphi)e^{j\omega t}\right\} \end{matrix},$$

then the time-varying Poynting vector $\mathbf{p}$ can be represented as

$$\mathbf{p}(r,\theta,\varphi,t) = \frac{1}{2}\mathrm{Re}\left\{\mathbf{E}(r,\theta,\varphi)\times\mathbf{H}^*(r,\theta,\varphi)\right\} + \frac{1}{2}\mathrm{Re}\left\{\mathbf{E}(r,\theta,\varphi)\times\mathbf{H}(r,\theta,\varphi)\cdot e^{j2\omega t}\right\}.$$

Comment on the meaning of the two terms in the expression for $\mathbf{p}$ above.

2. (5 points) Prove that Eq. (2.3), Gauss law of electrostatics, follows from Eq. (2.2), Ampere's law, and the continuity of current law, Eq. (2.4).

3. (35 points) Prove Eq. (3.34): i.e., prove that the radiated power of a very small loop of current is

$$\Pi_{rad} = \frac{1}{12\pi}\eta\beta^4(IA)^2.$$

4. (50 points) Assume that an antenna radiates isotropically over a <u>half-space</u> above a perfectly conducting flat ground plane. If $E = 50$ mV/m (rms) at a distance of 1 km (<u>far zone</u>), find:

a) the magnitude of the **H**-field vector;
b) the vector of the average power flux density $\mathbf{p}_{av}$ (give the direction and the magnitude);
c) the total radiated power Π;
d) the radiation resistance R_r, if the antenna input current is $I = 3.5$ A (rms).