Lecture 7: Antenna Noise Temperature and System Signal-to-Noise Ratio

(Noise temperature. Antenna noise temperature. System noise temperature. Minimum detectable temperature. System signal-to-noise ratio.)

1. Noise temperature of bright bodies

The performance of a telecommunication system depends very much on the signal-to-noise ratio (SNR) at the receiver's input. The electronic circuitry of the receiver (amplifiers, mixers, etc.) has its own contribution to the noise generation. However, the antenna itself is sometimes a significant source of noise. The antenna noise can be divided into two types according to its physical source: noise due to the loss resistance of the antenna itself; and noise, which the antenna picks up from the surrounding environment.

Any object whose temperature is above the absolute zero radiates EM energy. Thus, an antenna is surrounded by noise sources, which create noise power at the antenna terminals. Here, we are not concerned with technological sources of noise, which are a subject of the electromagnetic interference science. We are also not concerned with intentional sources of EM interference (EM jamming). We are concerned with natural sources of EM noise such as sky noise and ground noise.

The concept of antenna temperature is not only associated with the EM noise. The relation between the object's temperature and the power it can generate at the antenna terminals is used in passive remote sensing (radiometry). A radiometer can create temperature images of objects. Typically, the remote object's temperature is measured by comparison with the aggregate noise due to background sources and the receiver itself.

Every object (e.g., a resistor R) with a physical temperature above zero (0° K $= -273^\circ$ C) possesses heat energy. The ***noise power per unit bandwidth*** p_h is proportional to the object's temperature and is given by Nyquist's relation:

$$p_h = kT_P,\ \text{W/Hz} \tag{7.1}$$

where

T_P is the physical temperature of the object in K (Kelvin degrees); and

k is Boltzmann's constant (1.38×10^{-23} J/K).

In the case of a resistor, this is noise power, which can be measured at the resistor's terminals with a matched load. Thus, a resistor can serve as a noise generator.

Often, we assume that heat energy is evenly distributed in the frequency band Δf. Then, the associated ***heat power*** P_h in Δf is

$$P_h = kT_P \Delta f \text{, W.} \tag{7.2}$$

The noise EM power of the object depends on the ability of the object's surface to let the heat leak out. This radiated heat power is associated with the so-called ***equivalent temperature*** or ***brightness temperature*** T_B of the body via the power-temperature relation in (7.2):

$$P_B = kT_B \Delta f \text{, W.} \tag{7.3}$$

The brightness temperature T_B is proportional to the physical temperature of the body T_P:

$$T_B = (1 - |\Gamma_s|^2) \cdot T_P = \varepsilon T_P \text{, K} \tag{7.4}$$

where

Γ_s is the reflection coefficient of the surface of the body; and

ε is what is called the ***emissivity*** of the body.

Note that the brightness power P_B relates to P_h the same way at T_B relates to T_P, i.e., $P_B = \varepsilon P_h$.

2. Antenna noise temperature

The power radiated by the body P_B, when intercepted by an antenna, generates power P_A at its terminals. The equivalent temperature associated with the received power P_A at the antenna terminals is called the ***antenna temperature*** T_A of the object, where again $P_A = kT_A \Delta f$.

The received power can be calculated if the antenna effective aperture A_e (m^2) is known and if the power density W_B (W/m^2) created by the bright body at the antenna's location is known:

$$P_A = A_e W_B \text{, W.} \tag{7.5}$$

If the body radiates isotropically in all directions, then

$$W_B = \frac{P_B}{4\pi R^2}\text{, W/m}^2 \qquad (7.6)$$

where

$P_B = kT_B\Delta f$ (W) is the brightness power radiated by the body,

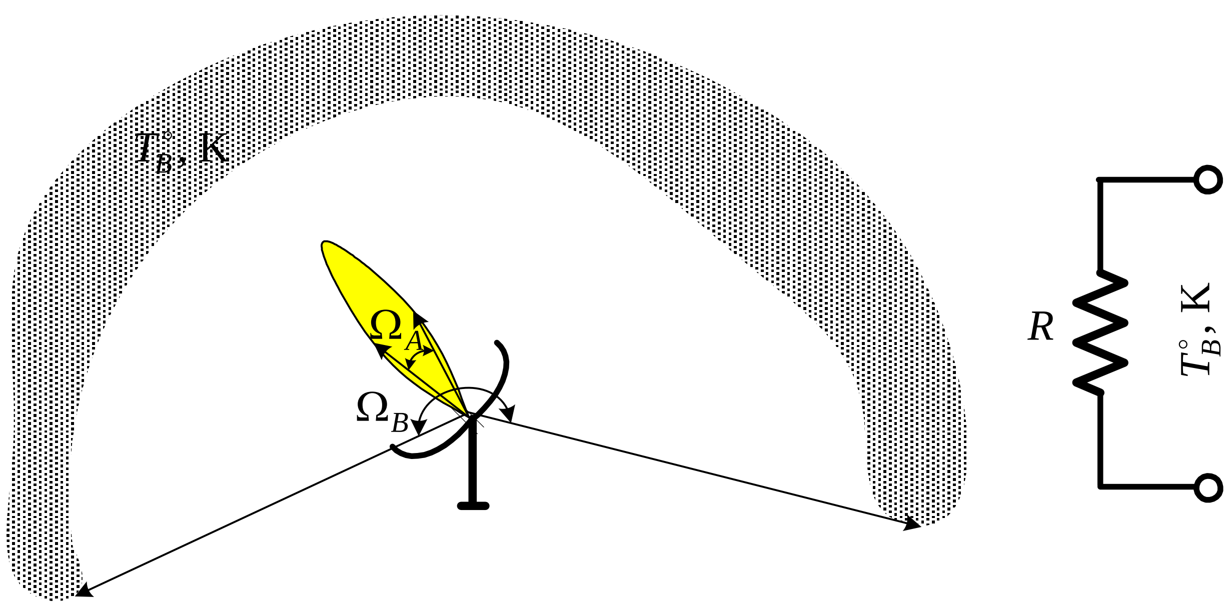
R (m) is the distance between the object and the antenna.

2.1. Antenna noise from large uniformly bright bodies (background noise)

Let us first assume that the entire antenna pattern (beam) "sees" an object of temperature T_B (K). We assume that the antenna itself is lossless, i.e., it has no loss resistance, and, therefore, it does not generate noise itself. Then, certain noise power can be measured at its terminals as

$$P_A = kT_B\Delta f\text{, W.} \qquad (7.7)$$

This is the same noise power as that of a resistor of temperature T_B (K).



The antenna temperature is related to the measured noise power as

$$P_A = kT_A\Delta f\,. \qquad (7.8)$$

In the case above (where the solid angle subtended by the noise source Ω_B is much larger than the antenna solid angle Ω_A), the antenna temperature T_A is exactly equal to the object's temperature T_B (if the antenna is loss-free):

$$T_A = T_B,\ \text{ if } \Omega_A \ll \Omega_B. \qquad (7.9)$$

2.2. Antenna noise from large bright bodies

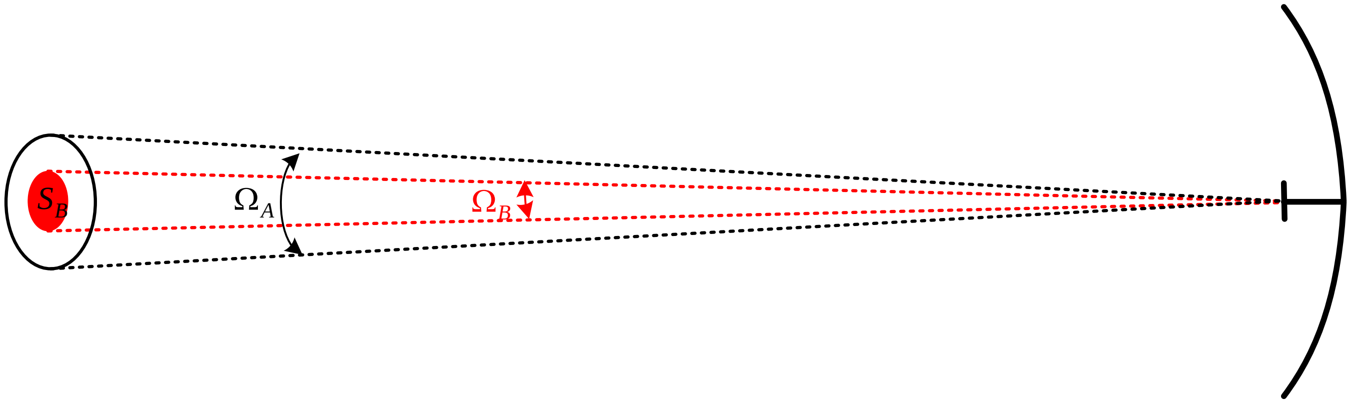
The situation described above is of practical importance to the overall antenna noise performance. When an antenna is pointed right at the night sky, its noise temperature is very low: $T_A \approx 3^\circ$ to 5° K at frequencies between 1 and 10 GHz. This is the noise temperature of the night sky. The higher the elevation angle, the less the sky temperature. Sky noise is very much dependent on the frequency. It depends on the time of the day, too. It is due to cosmic rays (emanating from the sun, the moon and other bright sky objects), to atmospheric noise and also to man-made noise, in addition to the deep-space background temperature of about 3° K at microwave frequencies.

The noise temperature of ground is about $\approx 300^\circ$ K, and it varies during the day. The noise temperature at approximately zero elevation angle (horizon) is about 100° to 150° K.

When a single large bright body is in the antenna beam, (7.9) holds. In practice, however, the antenna temperature may include contributions from several large sources, and the source under observation may be superimposed on a background of certain temperature. In order the antenna and its receiver to be able to discern an RF/microwave source (bright body) while "sweeping" the background, this source has to put out more power than the noise power of its background, i.e., it has to be "brighter" than its background. The antenna temperature is measured with the beam on and off the target. The difference is the ***antenna incremental temperature*** ΔT_A. If the bright body is large enough to "fill in" the antenna beam (and if the antenna has negligible side and back lobes), the difference between the background noise antenna temperature and the temperature when the object is in the antenna solid angle is equal to the object's temperature, $\Delta T_A = T_B$.

2.3. Antenna noise from small bright bodies

A different case arises in radiometry and radio-astronomy. The bright object subtends such a small solid angle that it is well inside the antenna solid angle when the antenna is pointed at it: $\Omega_B \ll \Omega_A$.



To separate the power due to the bright body from the background noise, the difference in the antenna temperature ΔT_A is measured with the beam on and off the object. This time, ΔT_A is ***not equal*** to the bright body temperature T_B, as it was in the case of a large object. However, both temperatures are proportional. The relation is derived below.

The noise power intercepted by the antenna depends on the antenna effective aperture A_e and on the power density W_B due to the noise source at the antenna location:

$$P_A = A_e \cdot W_B \text{, W.} \quad (7.10)$$

Assuming that the bright body radiates isotropically and expressing the effective area by the antenna solid angle, we obtain

$$P_A = \frac{\lambda^2}{\Omega_A} \cdot \frac{P_B}{4\pi R^2} \text{, W.} \quad (7.11)$$

The distance R between the noise source and the antenna is related to the effective area of the body and the solid angle Ω_B it subtends as

$$R^2 = \frac{S_B}{\Omega_B} \text{, m}^2 \quad (7.12)$$

$$\Rightarrow P_A\Omega_A = \frac{\lambda^2}{4\pi S_B} P_B\Omega_B. \quad (7.13)$$

Next, we notice that

$$\frac{\lambda^2}{4\pi S_B} = \frac{1}{G_B} = 1. \quad (7.14)$$

Here, G_B is the gain of the bright body, which is unity because we assumed in (7.11) that the body radiates isotropically. Thus,

$$P_A\Omega_A = P_B\Omega_B, \text{ if } \Omega_B \ll \Omega_A. \quad (7.15)$$

Equation (7.15) leads to the relation between the brightness temperature of the observed object T_B and the differential antenna temperature ΔT_A measured at the antenna terminals:

$$\Delta T_A = \frac{\Omega_B}{\Omega_A} T_B, \text{ K}. \quad (7.16)$$

For a large bright body, where $\Omega_B = \Omega_A$, we obtain from (7.16) the familiar result $\Delta T_A = T_B$, see (7.9).

2.4. Source flux density from noise sources

The power at the antenna terminals P_A, which corresponds to the antenna incremental temperature ΔT_A, is defined by (7.8). In radio-astronomy and remote sensing, it is often convenient to use the ***flux density*** S of the noise source:

$$S = \frac{p_h}{A_e} = \frac{k\Delta T_A}{A_e}, \text{ Wm}^{-2}\text{Hz}^{-1}. \quad (7.17)$$

Notice that S is not the Poynting vector (power flow per unit area) but rather the Poynting-vector spectral density (power flow per unit area per *hertz*). In radio-astronomy, the usual unit for source flux density is *jansky*, Jy=10^{-26} Wm^{-2}Hz^{-1}. (Karl G. Jansky was the first one to use radio waves for astronomical observations.)

From (7.17), we conclude that the measured incremental antenna temperature ΔT_A relates to the source flux density as

$$\Delta T_A = \frac{1}{k} A_e \cdot S. \quad (7.18)$$

This would be the case indeed if the antenna and the bright-body source were polarization matched. Since the bright-body source is a natural noise source, we cannot expect perfect match. In fact, an astronomical object is typically *unpolarized*, i.e., its polarization is random. Thus, about half of the bright-body flux density cannot be picked up by the receiving antenna whose polarization is fixed. For this reason, the relation in (7.18) is modified as

$$\Delta T_A = \frac{1}{2} \cdot \frac{A_e \cdot S}{k}. \tag{7.19}$$

The same correction factor should be inserted in (7.16), where the measured ΔT_A will actually correspond only to one-half of the noise temperature of the bright body:

$$\Delta T_A = \frac{1}{2} \frac{\Omega_B}{\Omega_A} T_B. \tag{7.20}$$

2.5. Antenna noise from a nonuniform noise background

In the case of a small bright body (see above), we have tacitly assumed that the gain (directivity) of the antenna is constant within the solid angle Ω_B subtended by the bright body. This is in accordance with the definition of an antenna solid angle Ω_A, which was used to obtain the ratio between ΔT_A and T_B. The solid-angle representation of the directivity of an antenna is actually quite accurate for high-directivity antennas, e.g., reflector antennas.

However, the antenna gain may be strongly dependent on the observation angle (θ, φ). In this case, the noise signals arriving from different sectors of space have different contributions to the total antenna temperature — those arriving from the direction of the maximum directivity contribute the most while those arriving from the direction of zero directivity will not contribute at all. The differential contribution from a sector of space of solid angle $d\Omega$ should, therefore, be weighted by the antenna power pattern $F(\theta, \varphi)$ in the respective direction:

$$dT_A = F(\theta, \varphi) \cdot T_B(\theta, \varphi) \cdot \frac{d\Omega}{\Omega_A}. \tag{7.21}$$

In general, the total antenna noise power is made up by the weighted contributions of all noise sources whose brightness temperature may vary with the angle of observation (θ, φ):

$$T_A = \frac{1}{\Omega_A} \oiint_{4\pi} F(\theta, \varphi) \cdot T_B(\theta, \varphi) d\Omega. \tag{7.22}$$

The expression in (7.22) is general and the previously discussed special cases are easily derived from it. For example, assume that the brightness temperature surrounding the antenna is the same at all observation angles, $T_B(\theta,\varphi) = const = T_{B0}$. Then,

$$T_A = \frac{T_{B0}}{\Omega_A} \cdot \underbrace{\oiint_{4\pi} F(\theta,\varphi) d\Omega}_{\Omega_A} = T_{B0}. \quad (7.23)$$

The above situation was already described by (7.9).

Assume now that $T_B(\theta,\varphi) = const = T_{B0}$ but only inside a solid angle Ω_B, which is much smaller than the antenna solid angle Ω_A. Outside Ω_B, $T_B(\theta,\varphi) = 0$. Since $\Omega_B \ll \Omega_A$, when the antenna is pointed at the noise source, its normalized power pattern within Ω_B is $F(\theta,\varphi) \simeq 1$. Then,

$$T_A = \frac{1}{\Omega_A} \oiint_{4\pi} F(\theta,\varphi) \cdot T_B(\theta,\varphi) d\Omega = \frac{1}{\Omega_A} \iint_{\Omega_B} 1 \cdot T_{B0} \cdot d\Omega = T_{B0} \frac{\Omega_B}{\Omega_A}. \quad (7.24)$$

This case was addressed in (7.16).

The antenna pattern strongly influences the antenna temperature. High-gain antennas (such as reflector systems), when pointed at elevation angles close to the zenith, have negligibly low noise level. However, if an antenna has significant side and back lobes, which are pointed toward the ground or the horizon, its noise power is much higher. The worst case for an antenna is when its main beam points towards the ground or the horizon, as is often the case with radar antennas or airborne antennas directed toward the earth.

Example (modified from Kraus, p. 406): A circular reflector antenna of 500 m^2 effective aperture operating at $\lambda = 20$ cm is directed at the zenith. What is the total antenna temperature assuming the sky temperature close to zenith is equal to 10° K, while at the horizon it is 150° K? Take the ground temperature equal to 300° K and assume that one-half of the minor-lobe beam area is in the back direction (toward the ground) and one-half is toward the horizon. The main beam efficiency (BE = Ω_M / Ω_A) is 0.7.

Such a large reflector antenna is highly directive and, therefore, its main beam "sees" only the sky around the zenith. The main beam efficiency is 70%. Thus, substituting in (7.22), the noise contribution of the main beam is

$$T_A^{MB} = \frac{1}{\Omega_A}(10 \times 0.7 \times \Omega_A) = 7 \text{ K}. \quad (7.25)$$

The contribution from the half back-lobe (which is half of 30% of the antenna solid angle) directed toward ground is

$$T_A^{GBL} = \frac{1}{\Omega_A}(300 \times 0.15 \times \Omega_A) = 45 \text{ K}. \quad (7.26)$$

The contribution from the half back-lobe directed toward the horizon is

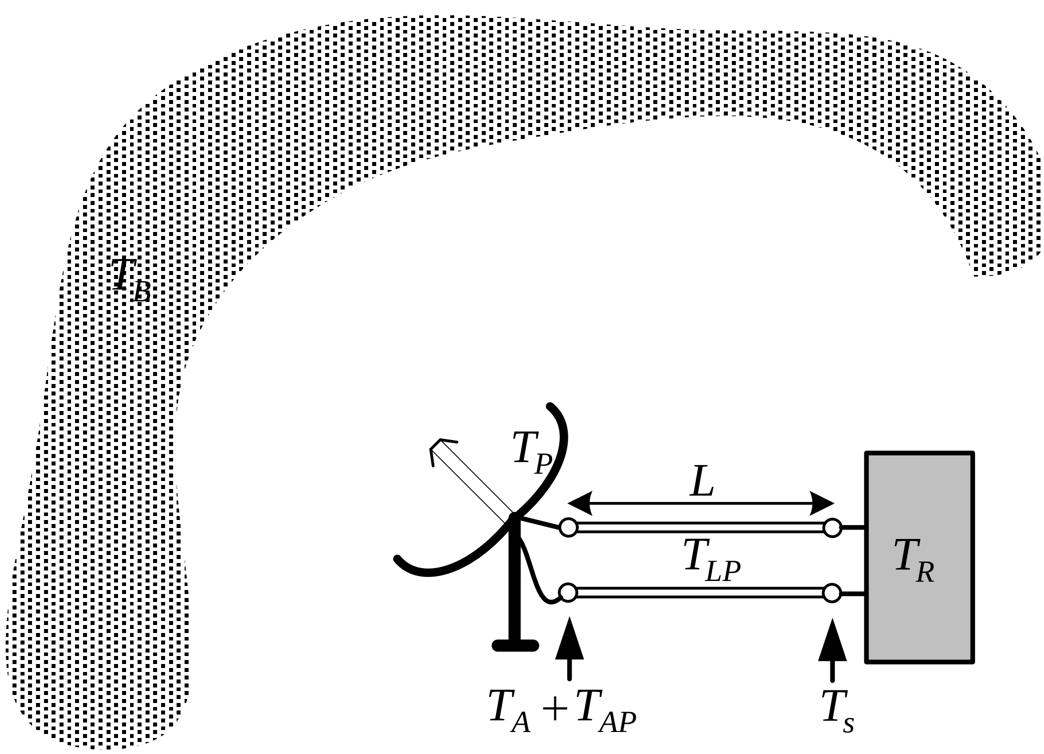
$$T_A^{HBL} = \frac{1}{\Omega_A}(150 \times 0.15 \times \Omega_A) = 22.5 \text{ K}. \quad (7.27)$$

The total noise temperature is

$$T_A = T_A^{MB} + T_A^{GBL} + T_A^{HBL} = 74.5 \text{ K}. \quad (7.28)$$

3. System noise temperature

An antenna is a part of a receiving system, which, in general, consists of a receiver, a transmission line and an antenna. All these system components have their contribution to the system noise. The system temperature (or the system noise level) is a critical factor in determining the sensitivity and the SNR of a receiving system.



If the antenna has losses, the noise temperature at its terminals consists of two terms: the antenna temperature due to the environment surrounding the antenna T_A, and the temperature due to the physical temperature of the antenna T_{AP} (the antenna acts as a resistive noise generator). The last term is related to the physical temperature of the antenna T_P as

$$T_{AP} = \left(\frac{1}{e_A} - 1\right) T_P = \frac{R_l}{R_r} T_P = \frac{P_h}{P_r} T_P, \text{ K} \tag{7.29}$$

where e_A is the radiation efficiency of the antenna $(0 \le e_A \le 1)$, R_l is the antenna loss resistance and R_r is its radiation resistance [see Lecture 4].

The transmission line itself is a source of noise if it has conduction losses. Its noise contribution at the antenna terminals is

$$T_L = \left(\frac{1}{e_L} - 1\right) T_{LP}, \text{ K.} \tag{7.30}$$

Here, $e_L = e^{-2\alpha L}$ is the ***line thermal efficiency*** $(0 \le e_L \le 1)$, T_{LP} is the physical temperature of the transmission line, α (Np/m) is the attenuation constant of the transmission line, and L is the length of the transmission line. The relation (7.30) is shown to be true for any transmission line or attenuator of transmission efficiency $e_L = P_{\text{ou}} / P_{\text{in}}$ where P_{ou} and P_{in} are the output and input power levels, respectively [see, Blake&Long, *Antennas*, 3rd ed., p. 412].

The system temperature referred to the antenna terminals includes the contributions of the antenna, the transmission line and the receiver as

$$T_{sys}^{A} = \underbrace{T_A}_{\text{external}} + \underbrace{T_P\left(\frac{1}{e_A} - 1\right)}_{T_{AP}} + \underbrace{T_{LP}\left(\frac{1}{e_L} - 1\right)}_{T_L} + \underbrace{\frac{1}{e_L} T_R}_{\text{receiver}}, \tag{7.31}$$

where T_R is the receiver noise temperature. The receiver noise temperature is given by

$$T_R = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots, \text{ K} \tag{7.32}$$

where

T_1 is the noise temperature of the first amplifying stage;

G_1 is the gain of the first amplifying stage;

T_2 is the noise temperature of the second amplifying stage;
G_2 is the gain of the second amplifying stage.

The noise temperature due to the antenna and the transmission line referred to the receiver's terminals is

$$T_{A\&L} = (T_A + T_{AP} + T_L) \cdot \underbrace{e^{-2\alpha L}}_{e_L}, \text{ K} \tag{7.33}$$

$$\Rightarrow T_{A\&L} = (T_A + T_{AP}) \cdot e^{-2\alpha L} + T_{LP}(1 - e^{-2\alpha L}), \text{ K.} \tag{7.34}$$

When we add also the receiver noise temperature T_R, we get

$$T_{sys}^R = (T_A + T_{AP}) \cdot e^{-2\alpha L} + T_{LP}(1 - e^{-2\alpha L}) + T_R, \text{ K} \tag{7.35}$$

for the system noise temperature at the receiver terminals. The relation between the system noise temperature at the receiver and at the antenna is simple:

$$T_{sys}^R = e_L T_{sys}^A . \tag{7.36}$$

Example (from Kraus, p. 410): A receiver has an antenna with a total noise temperature 50 K, a physical temperature of 300 K, and an efficiency of 99%. Its transmission line has a physical temperature of 300 K and an efficiency of 90%. The first three stages of the receiver all have 80 K noise temperature and 13 dB gain (13 dB is about 20 times the power). Find the system temperature.

The receiver noise temperature is

$$T_R = 80 + \frac{80}{20} + \frac{80}{20^2} = 84.2 \text{ K.} \tag{7.37}$$

Then, the system temperature at the antenna is

$$\begin{aligned} T_{sys}^A &= T_A + T_{AP}\left(\frac{1}{e_A} - 1\right) + T_{LP}\left(\frac{1}{e_L} - 1\right) + \frac{1}{e_L} T_R, \\ T_{sys}^A &= 50 + 300\left(\frac{1}{0.99} - 1\right) + 300\left(\frac{1}{0.9} - 1\right) + \frac{1}{0.9} 84.2 \approx 180 \text{ K.} \end{aligned} \tag{7.38}$$

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4. Minimum detectable temperature (sensitivity) of the system

The ***minimum detectable temperature, or sensitivity***, of a receiving system ΔT_{min} is defined as being equal to the RMS noise temperature of the system ΔT_{rms} referred to the antenna terminals. This is calculated as

$$\Delta T_{\text{min}} \equiv \Delta T_{\text{rms}} = \frac{k' T_{sys}^A}{\sqrt{\Delta f \cdot \tau}}, \tag{7.39}$$

where

k' is a system constant (commensurate with unity), dimensionless;
Δf is the pre-detection bandwidth of the receiver, Hz;
τ is the post-detection time constant, s.

The RMS noise temperature ΔT_{rms} is determined experimentally by pointing the antenna at a uniform brightness object and recording the signal for a sufficiently long period of time. Assume the output of the receiver is digital. Then, the RMS deviation D_{rms} of the numbers produced at the receiver (signal power) is representative of the RMS noise power:

$$D_{\text{rms}} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(a_n - a_{av})^2} \approx \Delta T_{\text{rms}}^R \text{ where } a_{av} = \frac{1}{N}\sum_{n=1}^{N} a_n . \tag{7.40}$$

ΔT_{rms} can be obtained from ΔT_{rms}^R by

$$\Delta T_{\text{rms}} = \frac{\Delta T_{\text{rms}}^R}{e_L} = \Delta T_{\text{min}} \tag{7.41}$$

where e_L is the efficiency of the transmission line [see (7.36) and (7.33)].

In order a source to be detected, it has to create an incremental antenna temperature ΔT_A which exceeds ΔT_{min}, $\Delta T_A > \Delta T_{\text{min}}$. The minimum detectable power P_{min} from a noise source is obtained by assuming that $\Delta T_A = \Delta T_{\text{min}}$. Then,

$$P_{\text{min}} = 0.5 A_e p_{\text{min}} = k \Delta T_{\text{min}} \Delta f \tag{7.42}$$

where A_e is the effective antenna area, p_{min} is the power-flow density (magnitude of Poynting vector) due to the noise source at the location of the antenna, and the factor of 0.5 accounts for the randomness of the noise-signal polarization. It follows that the minimum noise power-flow density, which can be detected is

$$p_{\min} = \frac{2k\Delta T_{\min}\Delta f}{A_e}. \tag{7.43}$$

The signal-to-noise ratio is given by

$$SNR = \frac{\Delta T_A}{\Delta T_{\min}}. \tag{7.44}$$

This SNR is used in radio-astronomy and remote sensing.

5. System signal-to-noise ratio (SNR) in communication links

The system noise power is related to the system noise temperature as

$$P_N = kT_S \Delta f_r, \text{ W}. \tag{7.45}$$

Here, $T_S = T_{sys}^A$ and Δf_r is the bandwidth of the receiver. From Friis' transmission equation, we can calculate the received power as

$$P_r = (1-|\Gamma_t|^2)(1-|\Gamma_r|^2)e_t e_r \text{PLF}\left(\frac{\lambda}{4\pi R}\right)^2 D_t(\theta_t,\varphi_t)D_r(\theta_r,\varphi_r)\cdot P_t \tag{7.46}$$

if the bandwidths of the transmitter and receiver are the same, $\Delta f_r = \Delta f_t$. Here, P_t is the transmitted power. If, however, the bandwidths are different, i.e., $\Delta f_r < \Delta f_t$, but centered at the same frequency, we have to include in (7.46) a factor of $\Delta f_r / \Delta f_t$. Normally, $\Delta f_r = \Delta f_t = \Delta f$.

Finally, the SNR becomes

$$SNR = \frac{P_r}{P_N} = \frac{(1-|\Gamma_t|^2)(1-|\Gamma_r|^2)e_t e_r \text{PLF}\left(\frac{\lambda}{4\pi R}\right)^2 D_t D_r \cdot P_t}{kT_{sys}^A \Delta f}. \tag{7.47}$$

The above equation is fundamental for the design of telecommunication systems. If T_{sys}^A in (7.47) is replaced by ΔT_{rms} from (7.39), the two SNR equations (7.44) and (7.47) represent essentially the same power ratios.