Lecture 4

Polarization of EM Waves
**definition**: the locus traced by the extremity of a *harmonic* time-varying field vector at a given observation point

- plane of polarization is orthogonal to direction of propagation
- there are three types of polarization: linear, circular, and elliptic

![Diagram of polarization types](image)

linear  
circular  
elliptic
Field Polarization: Time Domain Analysis

Field of any polarization can be represented in terms of 2 orthogonal linearly polarized components:

\[ E_x(t) = m_x \cos(\omega t) \text{ and } E_y(t) = m_y \cos(\omega t + \delta) \]

\[ \mathbf{E}(t) = \hat{x}m_x \cos(\omega t) + \hat{y}m_y \cos(\omega t + \delta) \]

Linearly (horizontally) polarized field

\[ e_H(\omega t_0) \]

\[ e_H(\omega t_0 + \pi) \]
linearly (vertically) polarized field

\[ e_V(\omega t_0) \]

\[ e_V(\omega t_0 + \pi) \]
Field Polarization: Time Domain Analysis (3)

**case 1: linear polarization of arbitrary direction**

- $\delta = n\pi$
- no constraints on magnitudes

$$E(t) = \hat{x}m_x \cos(\omega t) + \hat{y}m_y \cos(\omega t \pm n\pi), \ n = 0, 1, 2, \ldots$$

**example: direction at 45 deg.** ($\delta = 0, \ m_H = m_V$)

$$\phi = \arctan\left(\frac{m_V}{m_H}\right)$$
case 2: circular polarization

- $\delta_L = \pm \pi / 2$
- magnitudes must fulfill $m_H = m_V = m$

$$E(t) = \hat{x}m \cos(\omega t) + \hat{y}m \cos(\omega t \pm n\pi / 2), \; n = 1, 3, \ldots$$

example: clockwise circularly polarized vector, $\delta_L = \pi/2$
case 3: elliptic polarization

Elliptic polarization is any polarization different from linear or circular.

Example: counter-clockwise elliptically polarized vector

\[ m_H = 2m_V, \quad \delta_L = -\pi / 2 \]

\[ \mathbf{E}(t) = \hat{x} 2m \cos(\omega t) + \hat{y} m \cos(\omega t - \pi / 2) \]

\[ E_{H(\omega t=0)} = 1 \]
\[ E_{V(\omega t=0)} = 0 \]
\[ E_{H(\omega t=90^\circ)} = 0 \]
\[ E_{V(\omega t=90^\circ)} = 0.5 \]
any field can be represented by 2 **circularly** polarized components — one clockwise and one counter-clockwise

**example:** linearly polarized wave decomposed into two CP components
Field Polarization – Phasor Analysis

• work with decomposition into 2 orthogonal linear components

\[ \mathbf{E}(t) = \hat{x}E_x(t) + \hat{y}E_y(t) \quad \Rightarrow \quad \mathbf{\tilde{E}} = \hat{x}\tilde{E}_x + \hat{y}\tilde{E}_y \]

\[ E_x(t) = m_x \cos(\omega t - kz) \quad \Rightarrow \quad \tilde{E}_x = m_x \]
\[ E_y(t) = m_y \cos(\omega t - kz + \delta) \quad \Rightarrow \quad \tilde{E}_y = m_y e^{j\delta} \quad \Rightarrow \quad \mathbf{\tilde{E}} = \hat{x}m_x + \hat{y}m_y e^{j\delta} \]

• drop the propagation factor, \( \exp(-jkz) \), because it is common to both components and we can always choose \( z = 0 \)

• IMPORTANT: the direction of propagation (+z or −z) determines whether the wave is left-hand (LH or CCW) or right-hand (RH or CW) polarized in the case of circular or elliptical polarizations
Field Polarization – Phasor Analysis – Linear Polarization

\[ \delta = n\pi, \quad n = 0, 1, 2, \ldots \]

\[ \mathbf{E}(t) = \hat{x}m_x \cos(\omega t) + \hat{y}m_y \cos(\omega t \pm n\pi) \]

\[ \mathbf{\tilde{E}} = \hat{x}m_x \pm \hat{y}m_y \]

\[ \phi = \pm \arctan \left( \frac{m_y}{m_x} \right) \]

(a) \quad \delta = 2n\pi \quad \Rightarrow \phi > 0

(b) \quad \delta = (2n + 1)\pi \quad \Rightarrow \phi < 0
Field Polarization – Phasor Analysis – Circular Polarization

\[
m_x = m_y = m \quad \text{and} \quad \delta = \pm (\pi / 2 + n\pi), \quad n = 0, 1, 2, \ldots
\]

\[
E(t) = \hat{x}m \cos(\omega t) + \hat{y}m \cos[\omega t \pm (\pi / 2 + n\pi)] \quad \Rightarrow \quad \tilde{E} = m(\hat{x} \pm j\hat{y})
\]

\[
\begin{align*}
E &= m(\hat{x} + j\hat{y}) \\
\delta &= \pi / 2
\end{align*}
\]

RH (CW) for \(\hat{u} = -\hat{z}\)
LH (CCW) for \(\hat{u} = \hat{z}\)

\[
\begin{align*}
E &= m(\hat{x} - j\hat{y}) \\
\delta_L &= -\pi / 2
\end{align*}
\]

LH (CCW) for \(\hat{u} = -\hat{z}\)
RH (CW) for \(\hat{u} = \hat{z}\)
Field Polarization – Phasor Analysis – Circular Polarization (2)

- sense of rotation is determined when looking along the direction of propagation (be aware: in optics, the agreement is opposite)

\[ E = m(\hat{x} + j\hat{y}) \]
\[ \delta = \pi / 2 \]

- if wave propagates along \(+z\) (CCW) LH-CP wave is
\[ \tilde{E}(z) = m(\hat{x} + j\hat{y})e^{-jkz} \]

- if wave propagates along \(-z\) (CW) RH-CP wave is
\[ \tilde{E}(z) = m(\hat{x} + j\hat{y})e^{+jkz} \]
Circular Polarization: Sense of Rotation

- snapshot of E-field along direction of propagation for RH-CP wave

• phase difference $\delta$ can be any

• magnitude ratio $m_x/m_y$ can be any

• the term elliptic polarization is used to indicate polarizations other than linear or circular

• elliptic polarization has sense of rotation – the sign of $\delta$ determines the sense of rotation

\[
E(t) = \hat{x}m_x \cos(\omega t) + \hat{y}m_y \cos(\omega t + \delta)
\]

\[
\tilde{E} = \hat{x}m_x + \hat{y}m_y e^{j\delta}
\]
Field Polarization in Terms of Circularly Polarized Components

Any field can be written in terms of two CP terms – one RHCP, the other LHCP (wave propagating along \( +z \))

\[
\vec{E} = \vec{E}_R (\hat{x} - j\hat{y}) + \vec{E}_L (\hat{x} + j\hat{y})
\]

Bearing in mind that this field can also be represented in terms of two LP terms

\[
\vec{E} = \hat{x}\vec{E}_x + \hat{y}\vec{E}_y
\]

find the equations allowing for the calculation of \( E_R \) and \( E_L \) if \( E_x \) and \( E_y \) are known.
Summary: Field Polarization

- harmonic plane waves are characterized by their polarization
- polarization is defined by the locus traced by the tip of the E-field vector as time flows
- polarization can be linear, circular, or elliptic (do not identify with the other two!)
- circular and elliptic polarizations can be right-hand (clockwise) or left-hand (counter-clockwise)
- in microwave and antenna engng: the sense of rotation is determined by looking along the direction of propagation (thumb points along direction of propagation)
- every plane-wave field can be decomposed into: (1) two mutually orthogonal linearly polarized terms, or (2) two circularly polarized terms of opposite sense of rotation