UNIVERSITÀ DELLA CALABRIA
DIMES
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Introduction to Microwave Imaging Part I: Forward Models

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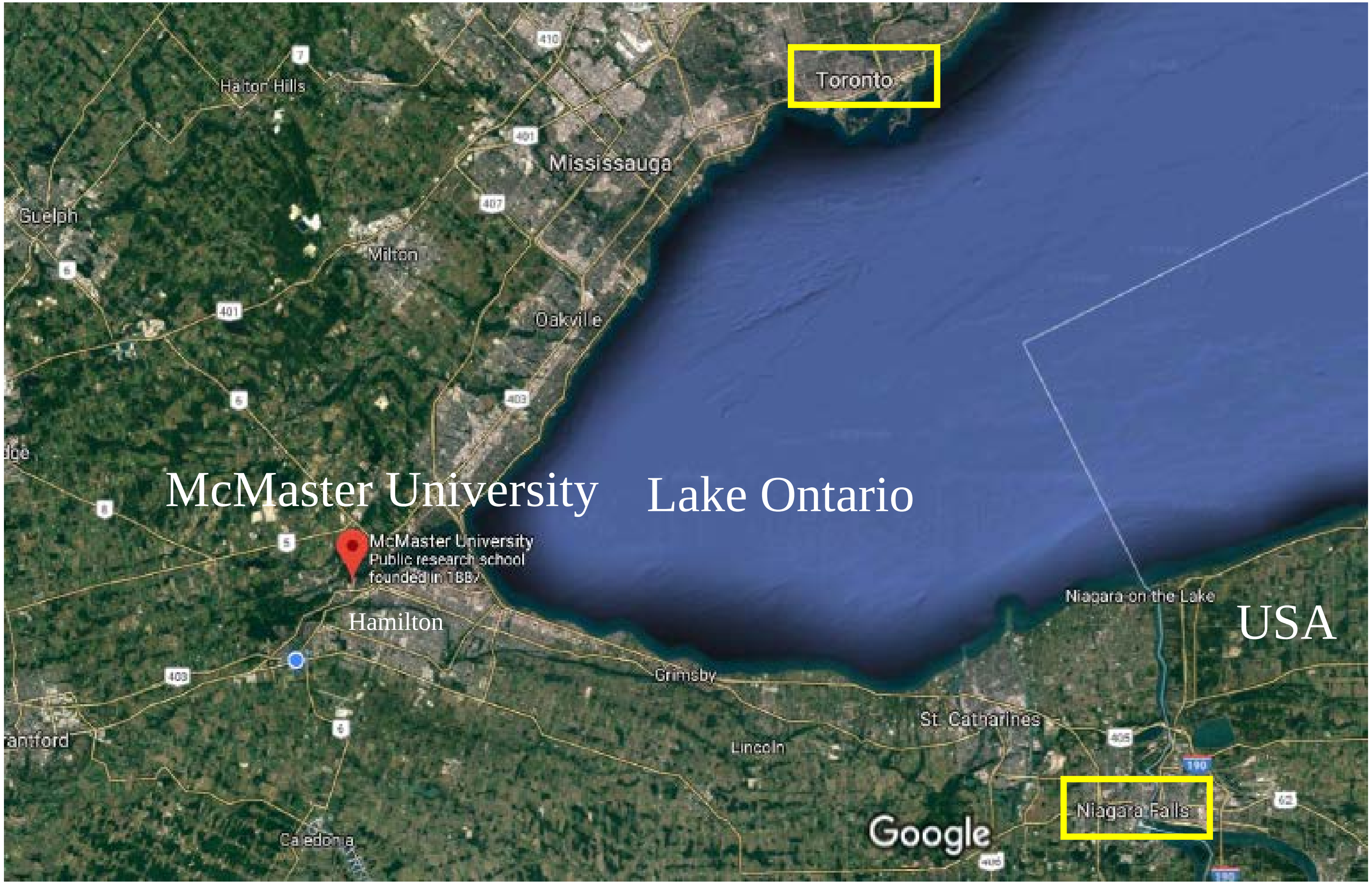
I COME FROM …



- World University Ranking 77 (Times Higher Education)
- Canada’s most research intensive university ($/faculty member)

COURSE OVERVIEW

Day 1: Introduction & Forward Models of Microwave Imaging

- Field-based Integral Solutions of the Scattering Problem in Time and Frequency
- Born and Rytov Approximations of the Forward Model of Scattering
- Scattering Parameters and Integral Solutions in Terms of *S*-parameters
- 2D Model of Tomography in Microwave Scattering

Day 2: Linear Inversion Methods

- Deconvolution Methods
  - Microwave Holography (MH)
  - Scattered Power Mapping (SPM) Image
- Reconstruction of Pulsed-radar Data
  - Synthetic Focusing: Delay and Sum (DAS

Day 3: Performance Metrics & Hardware

- Spatial Resolution
- Dynamic Range
- Data Signal-to-noise Ratio

Select Topics

- Overview of Nonlinear Inversion Methods
  - Direct Iterative Methods
  - Model-based Optimization Methods
- Tissue Imaging – Challenges and Advancements

INTRODUCTION INTO THE SUBJECT

WHY DO WE CARE ABOUT MICROWAVE IMAGING

- penetration into optically obscured objects (fog/clouds, foliage, soil, brick, concrete, clothing, walls, luggage, living tissue…)
  - the lower the frequency the better the penetration
  - frequency bands from 500 MHz well into the mm-wave bands (≤300 GHz)
- long-range radar – weather radar, airport and marine radars, automotive radars
- compact relatively cheap electronics esp. in the low-GHz range
- diverse suite of image reconstruction methods

SHORT-RANGE RADAR: NUMEROUS APPLICATIONS

whole body scanners

nondestructive testing

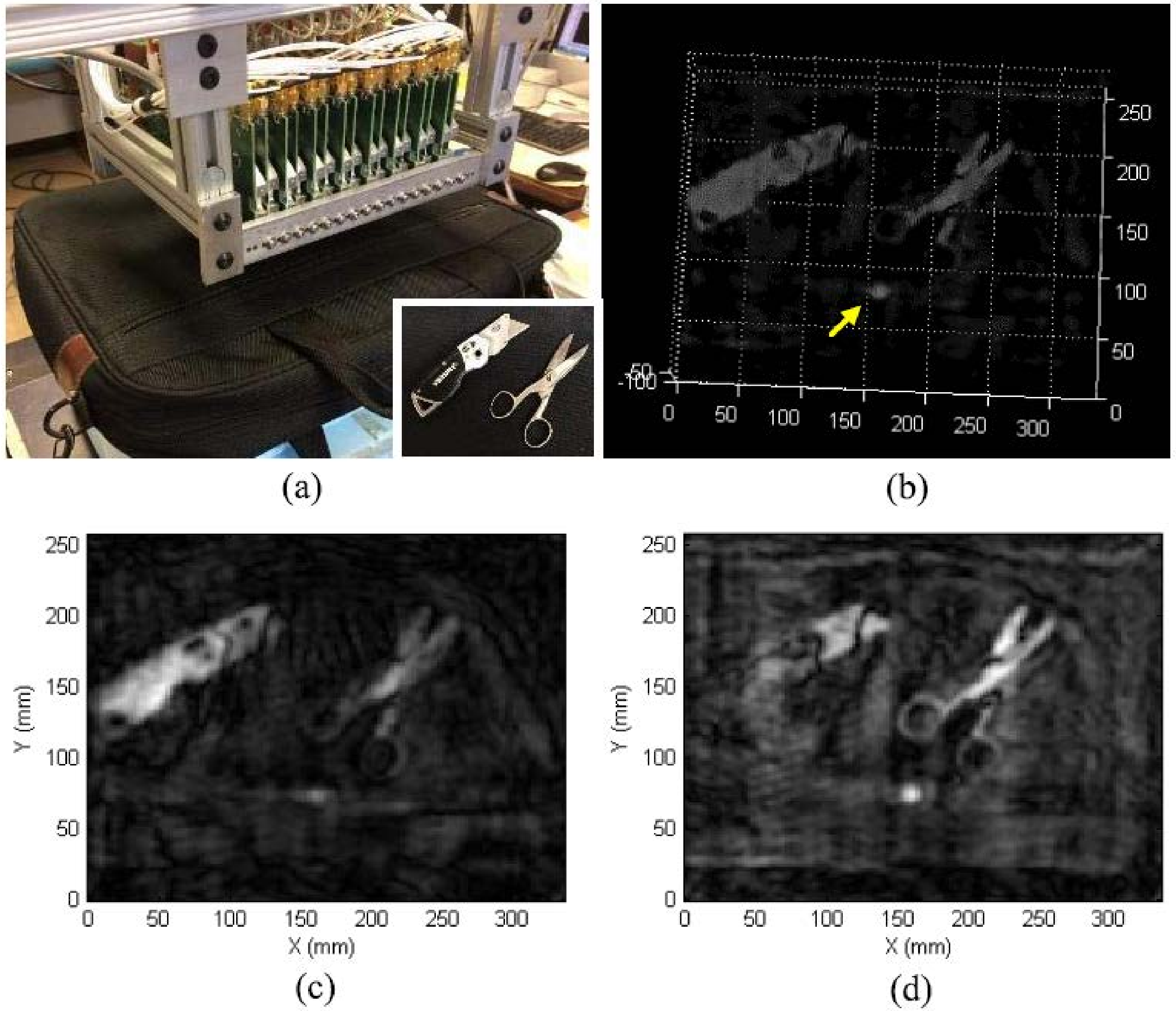
through-wall imaging

medical imaging

underground radar

RECENT APPLICATIONS: LUGGAGE INSPECTION, NDT

[Ghasr *et al.*, “Wideband microwave camera for real-time 3-D imaging,” *IEEE Trans. AP*, 2017]



20 GHz to 30 GHz frequency range

Prof. Zoughi’s team at Missouri University of Science & Technology

video

[https://youtu.be/RE-PPXmtTeA]

Fig. 15. Example of video camera utility for imaging a box cutter and a pair of scissors inside a laptop bag. (a) Picture of laptop bag in front of the camera aperture with inset showing the objects inside the bag. (b) 3-D view. (c) 2-D image slice focused on the box cutter. (d) 2-D image slice focused on the pair of scissors.

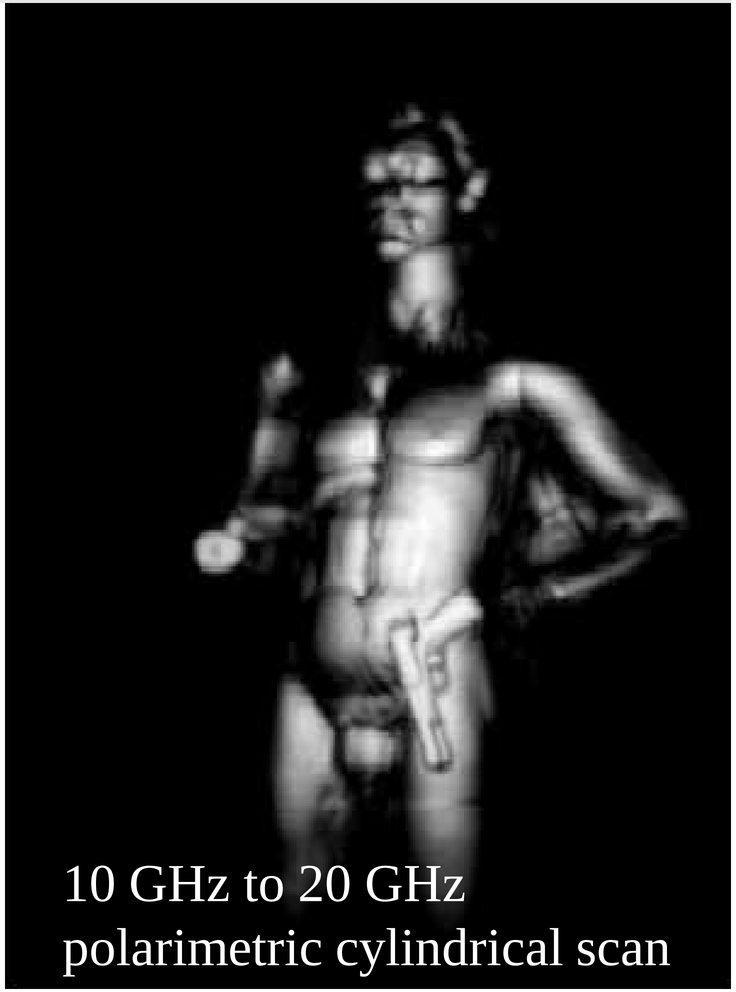
APPLICATIONS: WHOLE BODY SCANNERS

[Sheen *et al.*, "Near-field three-dimensional radar imaging techniques and applications," *Applied Optics* 2010]

Pacific Northwest National Laboratory, Washington, USA

40 GHz to 60 GHz (U band) cylindrical scan

40 GHz to 60 GHz (U band) cylindrical scan



10 GHz to 20 GHz polarimetric cylindrical scan

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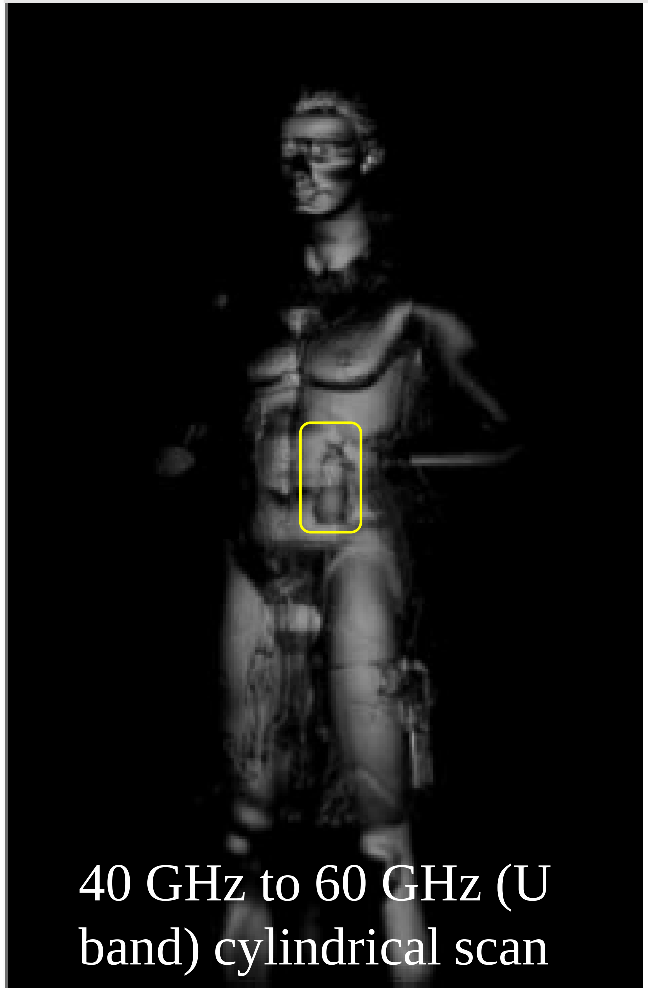

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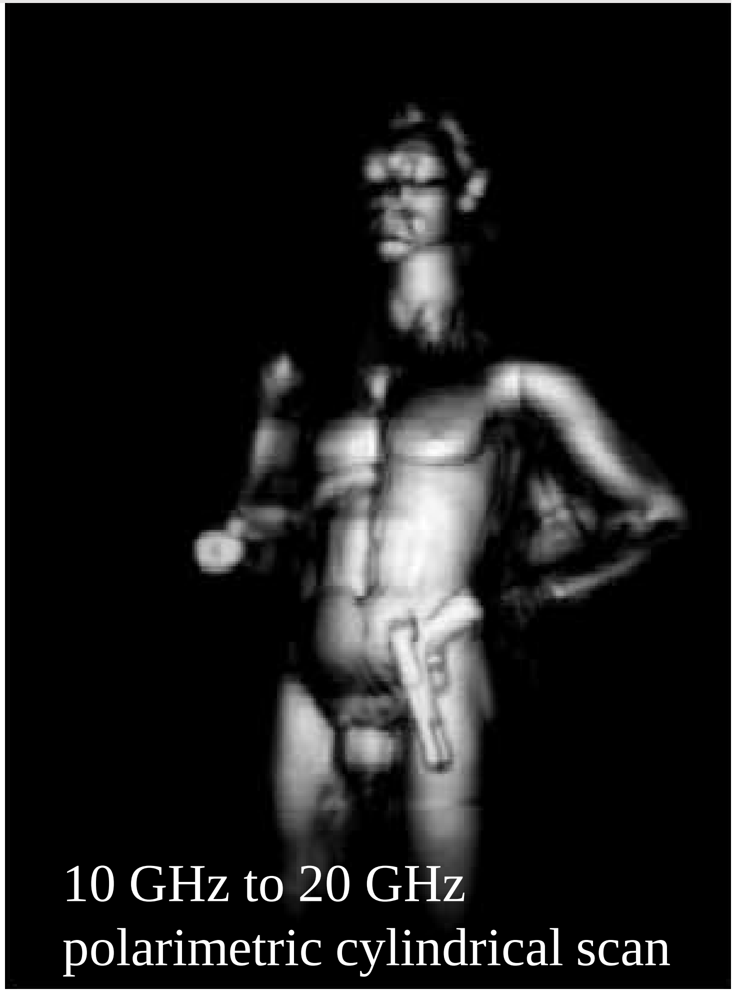

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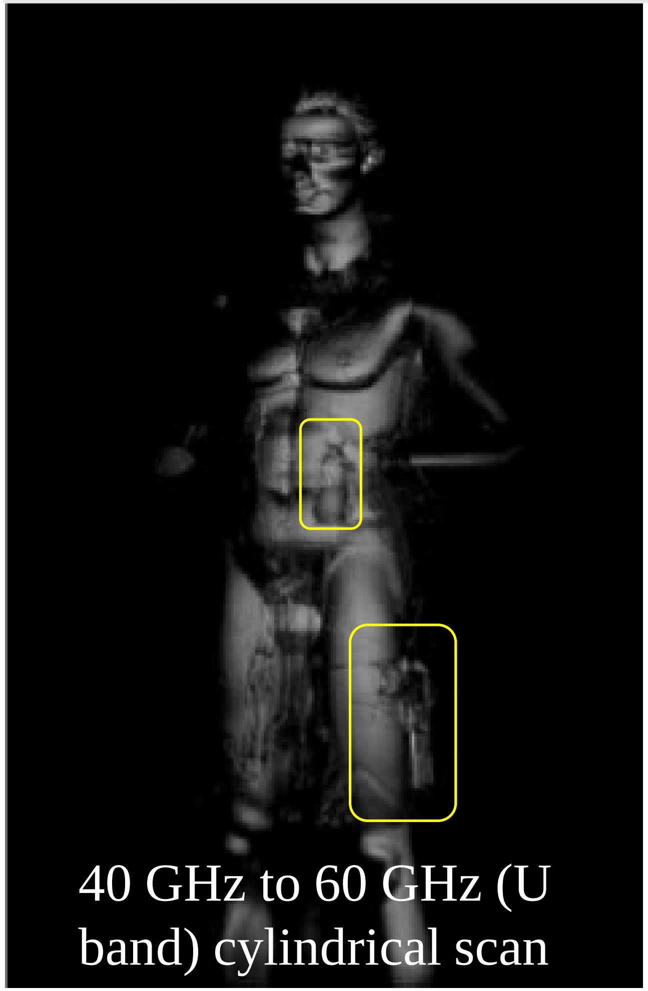

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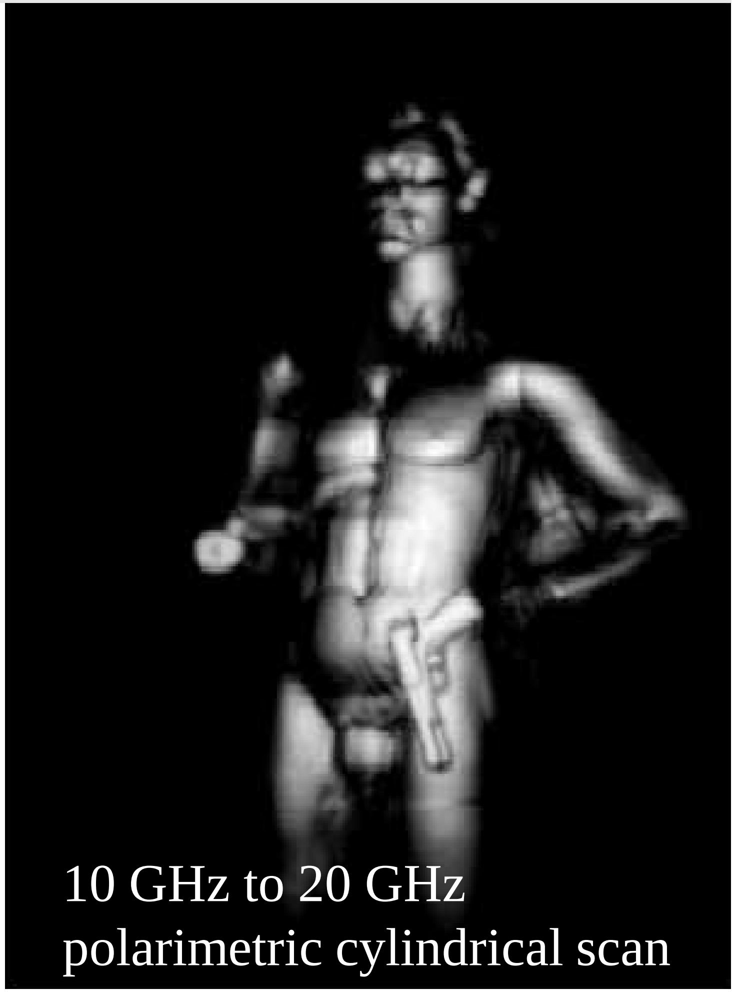

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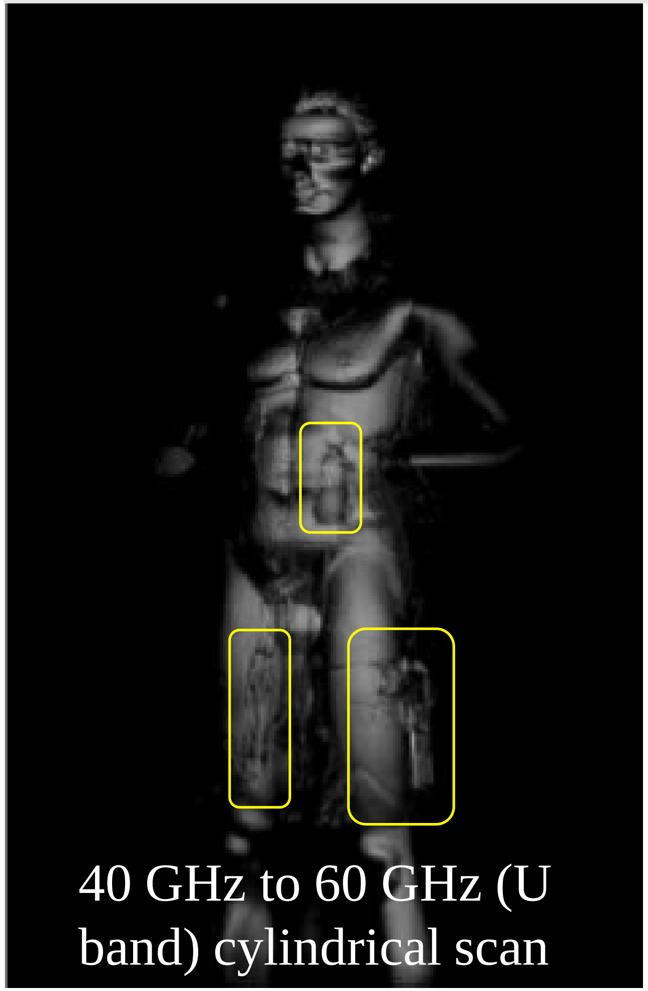
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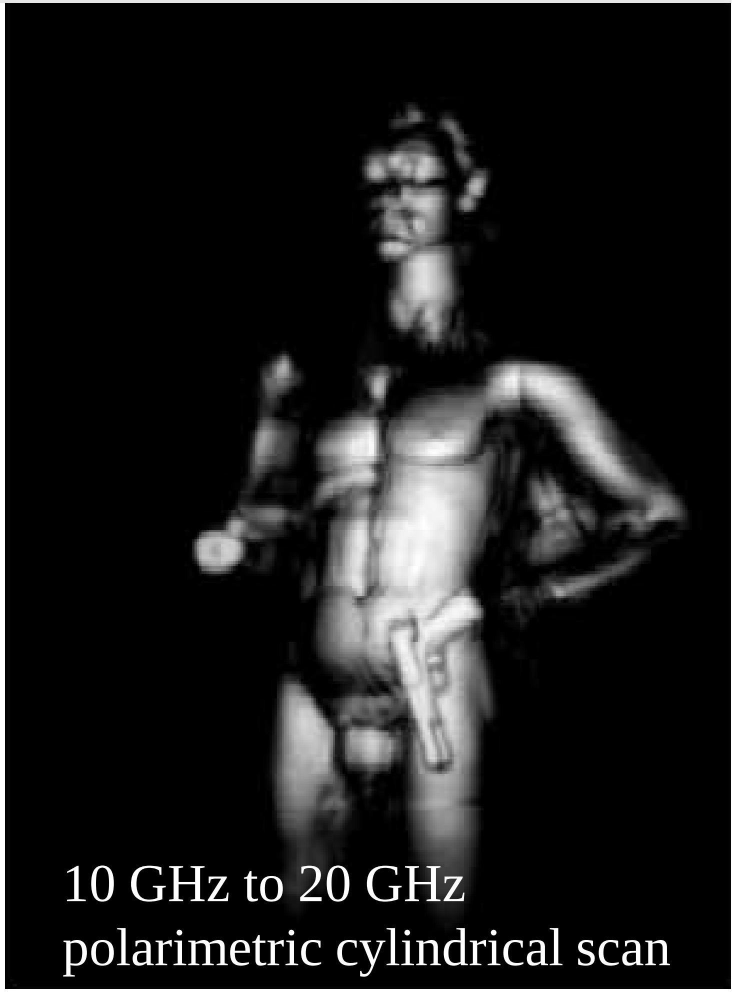

10 GHz to 20 GHz polarimetric cylindrical scan

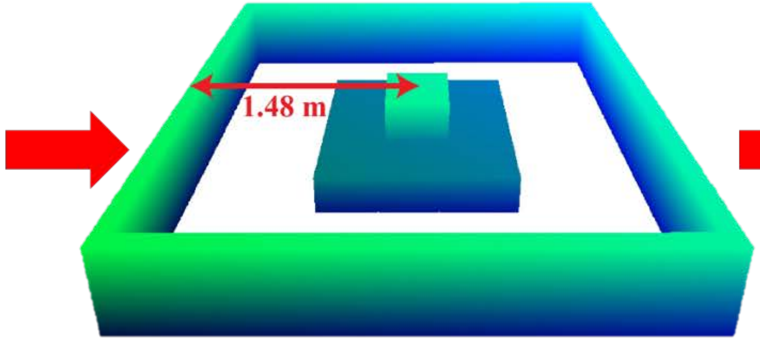
APPLICATIONS: THROUGH-WALL IMAGING

[Depatla *et al.*, "Robotic through-wall imaging," *IEEE A&P Mag.* 2017]

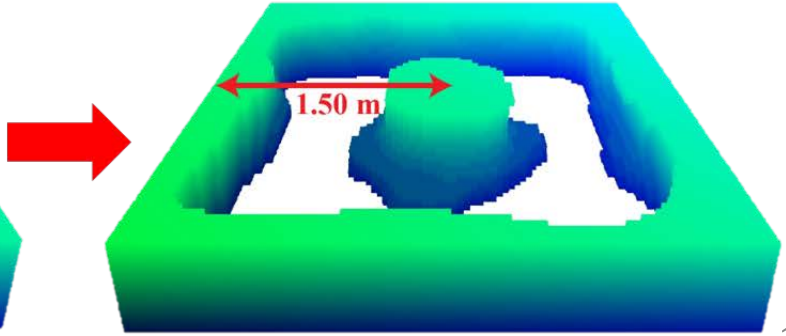
Prof. Mostofi's team at the University of California Santa Barbara

Area of interest – top view

3D binary ground-truth image of the unknown area to be imaged (2.96 m x 2.96 m x 0.4 m)



Our 3D image of the area, based on 3.84 % measurements



APPLICATIONS: MEDICAL IMAGING

[Song *et al.*, "Detectability of breast tumor by a hand-held impulse-radar detector: performance evaluation and pilot clinical study," *Nature Sci. Reports* 2017]

Prof. Kikkawa's team at Hiroshima University, Japan

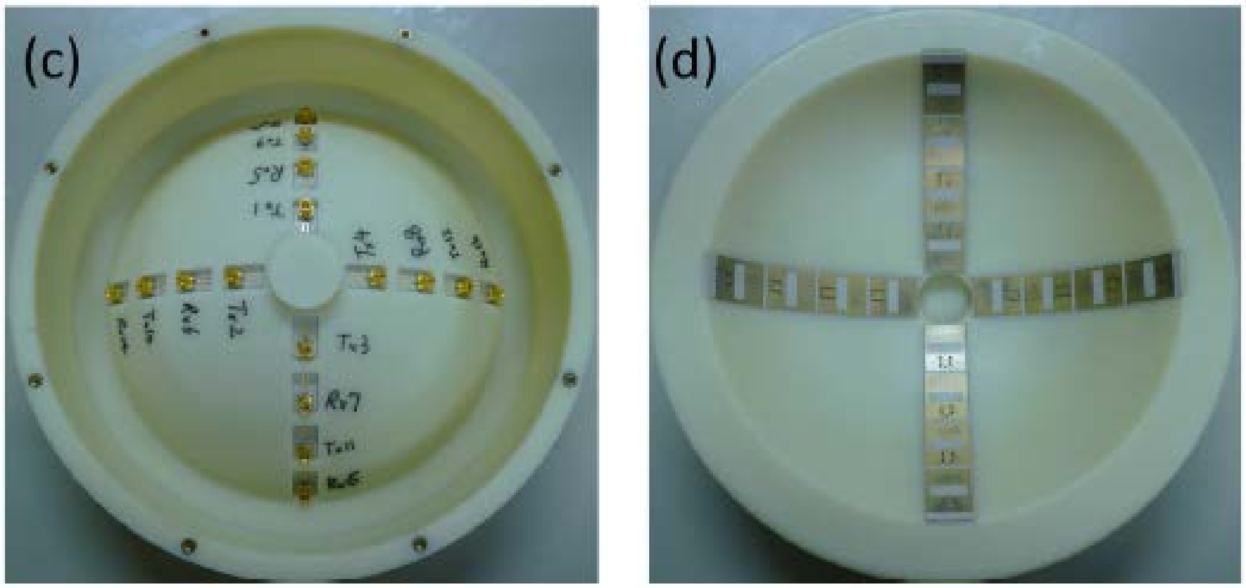


Figure 3. Dome antenna array design. (**a**) The top view of the antenna in *x-y* plane. (**b**) The side view of the antenna in *x-z* plane. (**c**) Top view photograph. (**d**) Bottom view photograph.

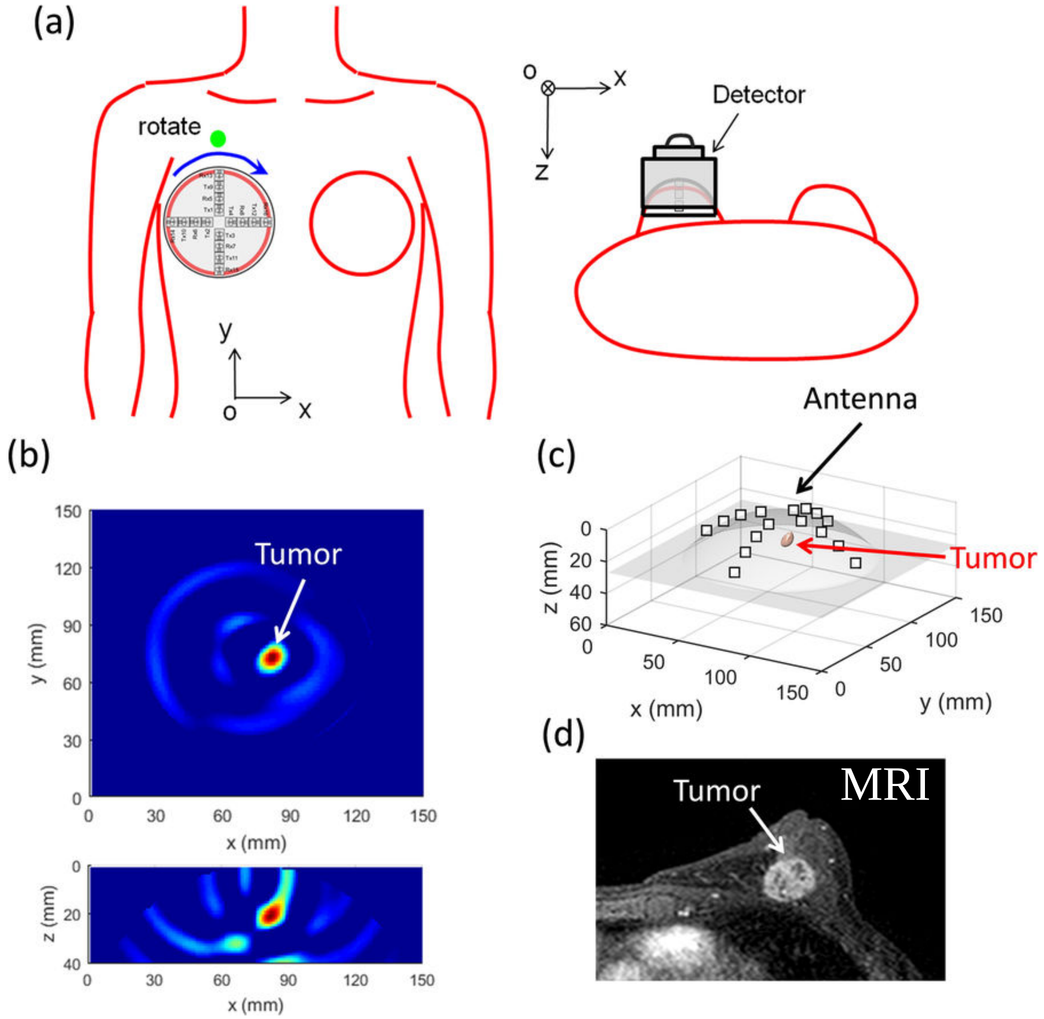
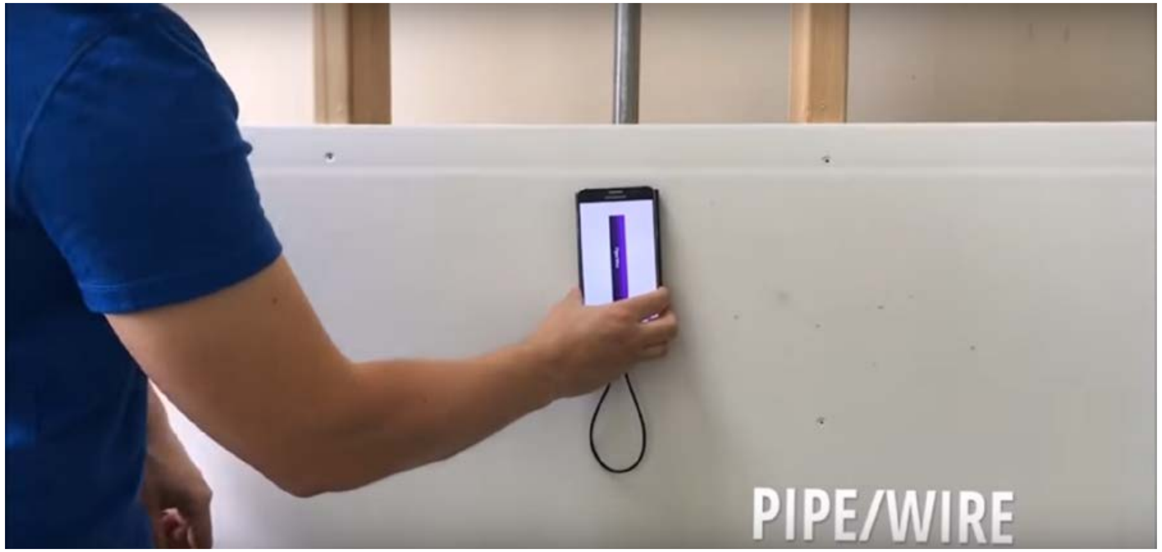


Fig. 7 in Song 2017

MICROWAVE NEAR-FIELD IMAGING: FAST GROWTH COMMERCIALLY

- mm-wave whole-body imagers for airport security inspection (> 30 GHz)
- through-wall and through-floor infrastructure inspection for contractors and home inspectors (UWB, 3 GHz to 8 GHz)
- numerous underground radar applications: detection of pipes, cables, tunnels, etc. (< 3 GHz)

[video credit: https://www.youtube.com/watch?v=_YZEa1hiGO0]

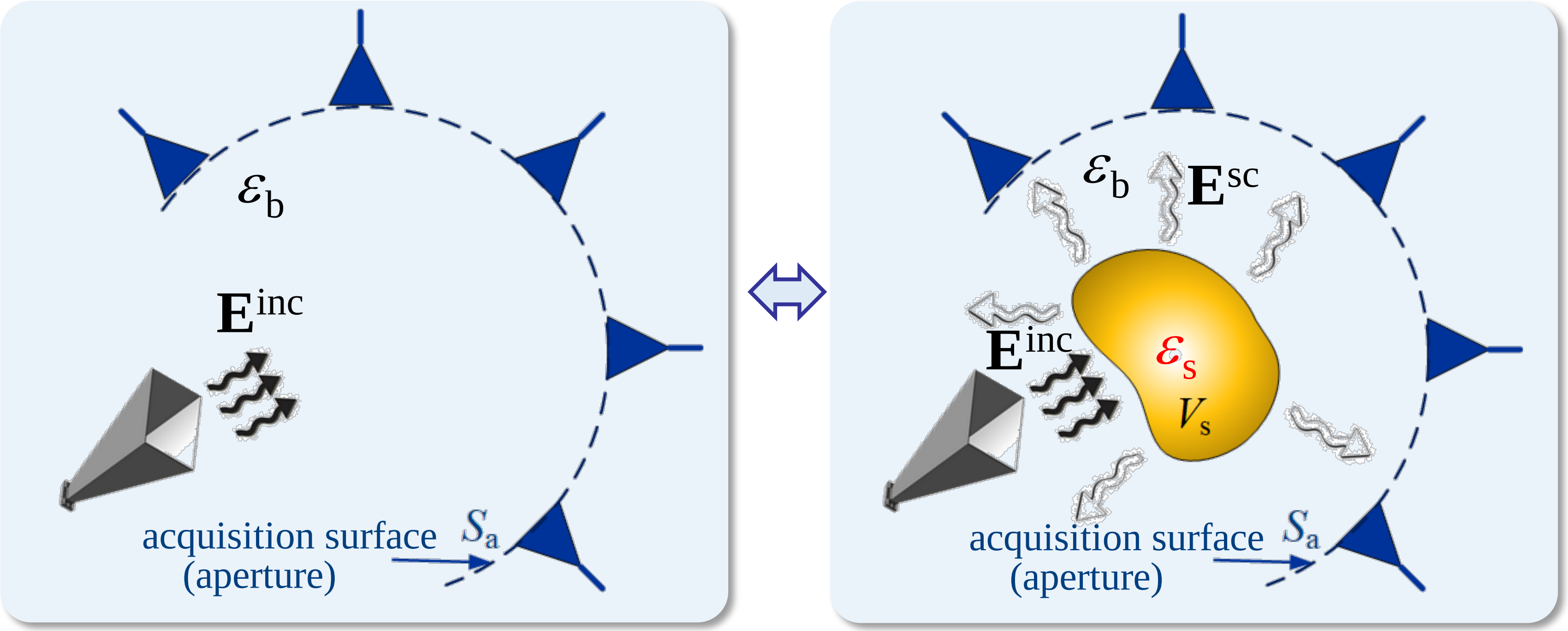


[video credit: https://walabot.com/diy]

MAIN PRINCIPLE OF ACTIVE MICROWAVE IMAGING

- microwave radiation penetrates and interacts with the imaged object
- the wave is modified (amplitude decay, phase delay, etc.) by the *object's EM properties and geometry*
- scattered wave samples are collected and processed to deduce the *object's EM properties and geometry*

baseline
(reference object)
measurement



measurement
of OUT

COMPONENTS OF THE IMAGING PROCESS

MEASURED DATA: *d*

UWB frequency sweep
or UWB pulsed radar

FORWARD MODELS:

$$\underbrace{F(\boldsymbol{x}) = \boldsymbol{d}}_{\text{data equation}} \quad \underbrace{\mathcal{L}_{\mathrm{ME}}\{\boldsymbol{x}, \boldsymbol{E}\} = \boldsymbol{E}}_{\text{state equations}}$$

analytical EM models
EM simulators

INVERSION STRATEGY:

$\boldsymbol{x} = \boldsymbol{F}^{-1}\{\boldsymbol{d}\}$ subject to $\mathcal{L}_{\mathrm{ME}}\{\boldsymbol{x}, \boldsymbol{E}\} = \boldsymbol{E}$

linear and nonlinear solvers
deconvolution and optimization methods
sensitivity analysis

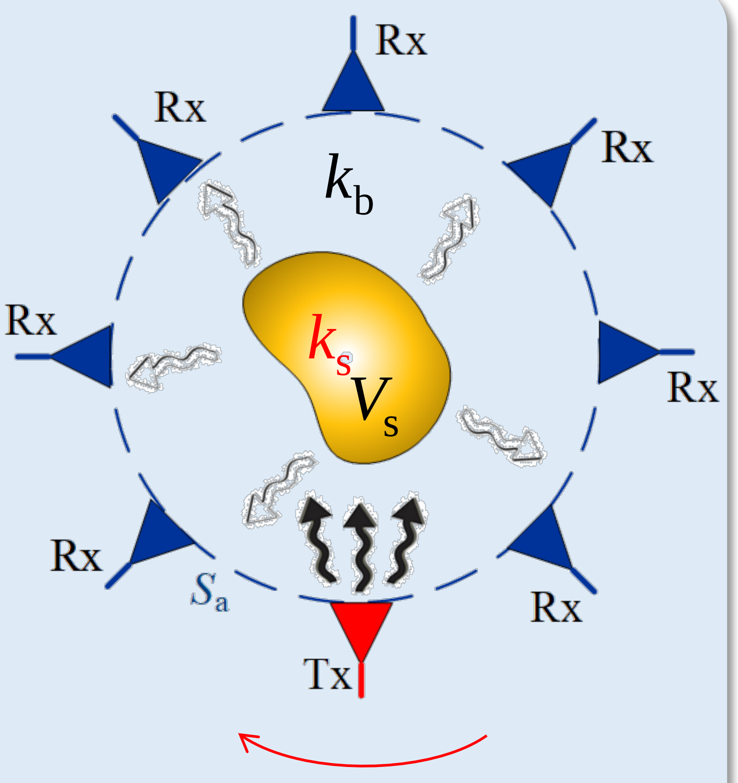
noise analysis & suppression
data filtering
image post-processing

imaging research is an intersection of engineering, math and physics

Main Principle: imaging needs *abundant* and *diverse* data

- spatial data abundance
  - illuminate target from various angles
  - collect scattered signals at various angles/distances
  - scanning is required (acquisition surfaces – planar, cylindrical, spherical)
  - scanning approaches

	mechanical scanning	electronically switched arrays
speed	low	HIGH
complexity	LOW	high
flexibility in adjusting scan parameters	GREAT	limited



$$k_b = \omega\sqrt{\mu_0\varepsilon_b}$$
$$k_s = \sqrt{\mu_0\varepsilon_s}$$

DATA ACQUISITION: SPATIAL SAMPLING

- ***over-sampling*** does not ensure diversity but increases acquisition time
- over-sampling counteracts noise effectively in cross-correlation reconstruction methods
- each sample must add independent information
- linearly dependent data may lead to ill-posed inversion problems

  - stay below but close to the ***maximum spatial sampling step*** – ensures diversity

$$\Delta\zeta \le \Delta\zeta_{\max} \approx \frac{\lambda_{\min}}{4\sin\alpha},\ \zeta \equiv x, y$$

  - ***effective near-field wavelengths*** may be shorter than $\lambda = \dfrac{v_{\mathrm{b}}}{f} = \dfrac{2\pi}{k_{\mathrm{b}}}$

$$\lambda_{\mathrm{eff,min}} = \frac{2\pi}{k_{x,y}^{\max}}$$

$$\tilde{S}(k_x, k_y) = \mathcal{F}_{\mathrm{2D}}\left\{S(x, y)\right\}$$

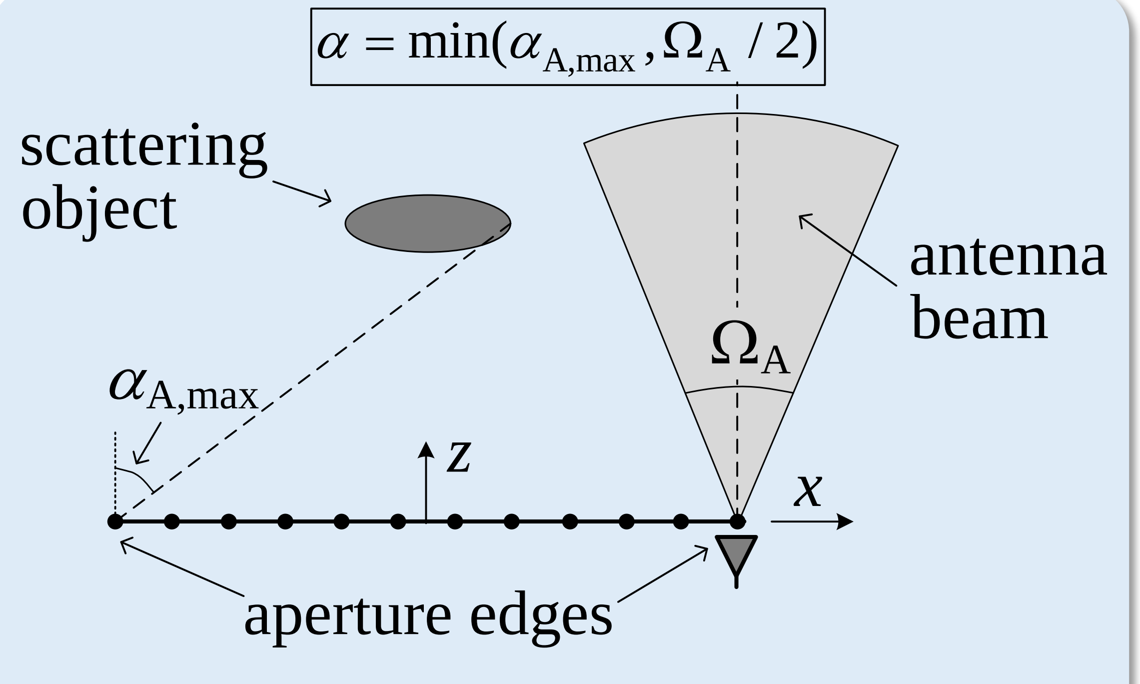


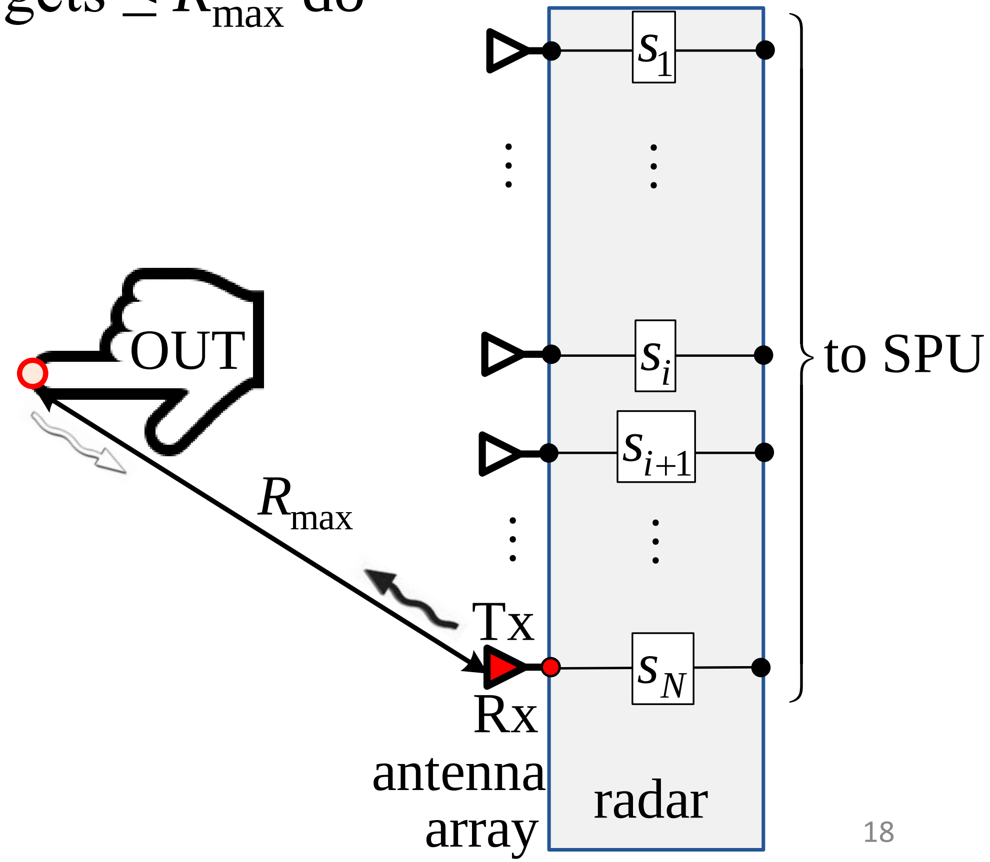
Illustration of Viewing Angle

- frequency data diversity in frequency-domain measurements
  - stay below but close to the ***maximum frequency sampling step***
  - it ensures that back-scattered signals from all targets $\leq R_{\max}$ do not overlap

$$\Delta f \leq \Delta f_{\max} = \frac{1}{2T_{\max}} \approx \frac{v_{\mathrm{b}}}{4R_{\max}}$$

maximum range

maximum observation period



DATA ACQUISITION: TEMPORAL SAMPLING

- temporal data diversity in time-domain (pulsed-radar) measurements
  - ➢ stay below but close to the ***maximum time sampling step***
  - ➢ it ensures that all frequency components of the pulsed signals are fully used (Nyquist)

$$\Delta t \leq \Delta t_{\max} \approx \frac{T_{\min}}{2} = \frac{1}{2\,f_{\max}}$$

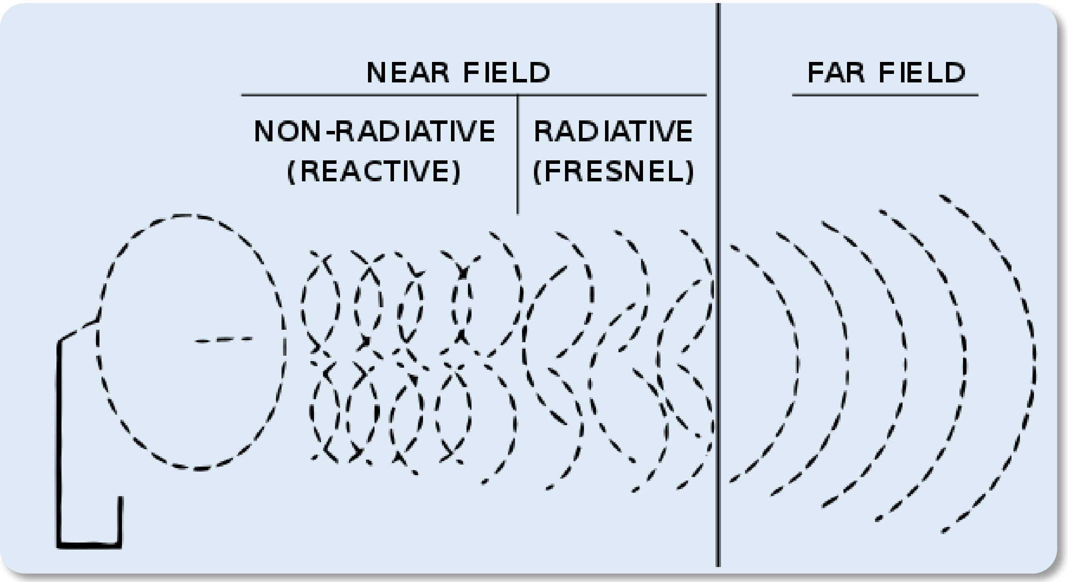
NEAR-FIELD vs. FAR-FIELD IMAGING

any one of the conditions below implies near-field imaging

$$r \le \frac{2D_{\mathrm{A,max}}^2}{\lambda},\ r \le D_{\mathrm{A,max}}$$

$$r \le \frac{2D_{\mathrm{OUT,max}}^2}{\lambda},\ r \le D_{\mathrm{OUT,max}}$$

$$r \le \lambda$$



Goran M Djuknic - commons.wikimedia.org/w/index.php?curid=20417988

- OUT is in Tx/Rx antennas' near field
- antennas are in the OUT's near field
- implication *A*: ***multiple scattering & coupling between antennas and OUT***

$$S_{ik} \ne S_{ik}^{\mathrm{inc}} + S_{ik}^{\mathrm{sc}}$$

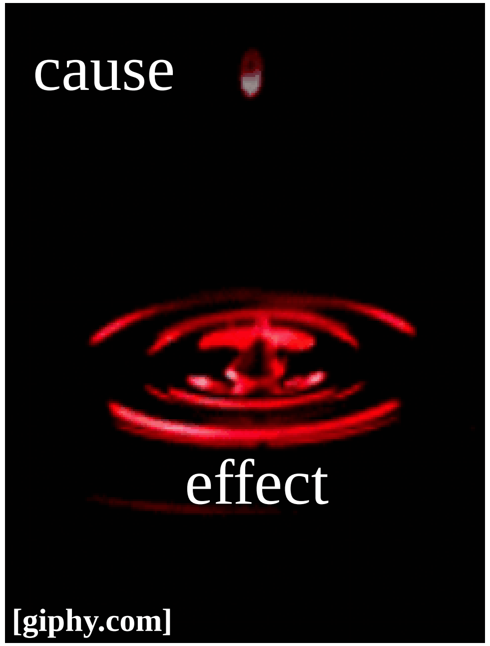
- implication *B*: ***incident antenna fields do not conform to free-space far-zone model***

$$\mathbf{E}^{\mathrm{inc}}(\mathbf{r}') \sim \hat{\mathbf{p}} G(\theta,\varphi) \frac{e^{-\mathrm{i}k_{\mathrm{b}} r}}{r}$$ ⟵ *not valid!*

FORWARD MODELS OF ELECTROMAGNETIC SCATTERING

FORWARD vs. INVERSE PROBLEM

forward problem



- from cause toward effect
- **unique solution**

inverse problem

- from effect toward cause
- **not unique**

FORWARD vs. INVERSE PROBLEM IN EM SCATTERING

EM ANALYSIS

- **cause (known)**
  - excitation
  - boundary conditions
  - medium properties
- **effect (unknown)**
  - scattering parameters
  - radar cross-section
  - antenna far-field pattern
  - etc.

Example: EM simulators – general-purpose forward solvers

INVERSE SCATTERING

- **cause (known)**
  - excitation
  - boundary conditions
- **cause (unknown)**
  - medium properties
- **effect (somewhat known)**
  - scattering parameters
  - radar return

General-purpose inverse solvers do not exist

data equation: maps contrast to the data (field measured outside OUT) $\boxed{\mathbf{r} \notin V_s}$

$$\underbrace{\mathbf{E}^{sc}(\mathbf{r} \in S_a)}_{\text{data}} = \left[\mathbf{E} - \mathbf{E}^{inc}\right]_{\mathbf{r} \in S_a} = \iiint_{V_s} K(\mathbf{r}')\underline{\underline{\mathbf{G}}}_b(\mathbf{r},\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')d\mathbf{r}' \quad \mathcal{F}_D : K \to \boldsymbol{d}$$

$$K(\mathbf{r}') = k_s^2(\mathbf{r}') - k_b^2(\mathbf{r}')$$

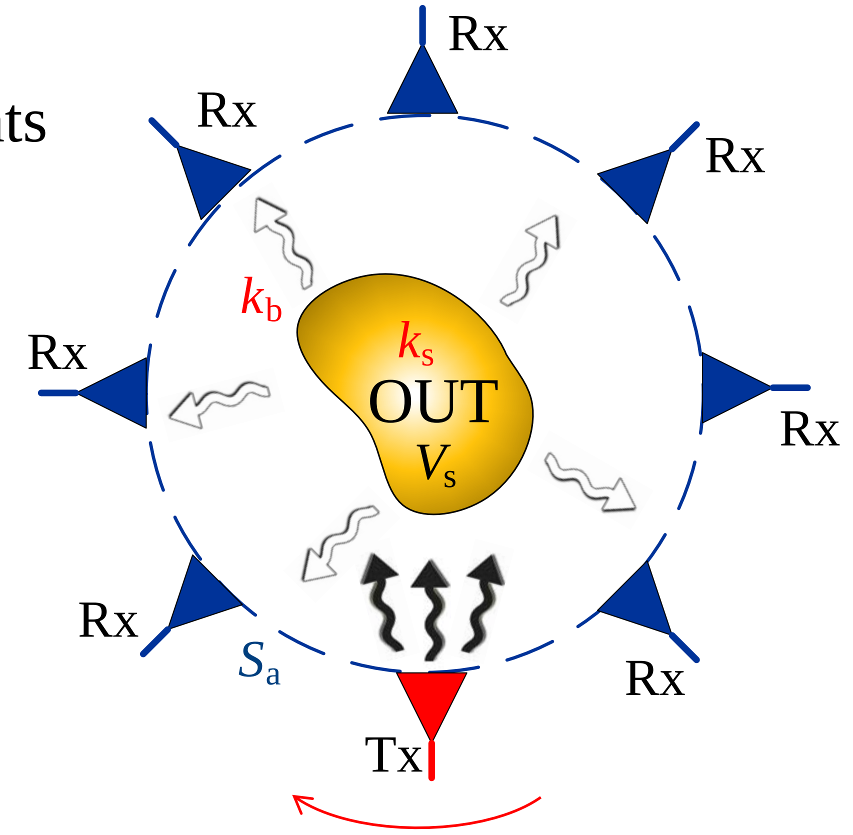
- ensures contrast produces result matching measurements
- *contrast source* concept: $\mathbf{S}(r') = K(\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')$

state equation: maps contrast to field inside OUT (state variables) $\boxed{\mathbf{r} \in V_s}$

$$\mathbf{E}(\mathbf{r} \in V_s) = \iiint_{V_s} K(\mathbf{r}')\underline{\underline{\mathbf{G}}}_b(\mathbf{r},\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')d\mathbf{r}'$$

$$\mathcal{F}_E : K \to \mathbf{E}(\mathbf{r} \in V_s)$$

- ensures contrast source satisfies Maxwell's equations



ROLE OF DATA AND STATE EQUATIONS IN IMAGE RECONSTRUCTION

FORWARD MODELS:

$$\underbrace{F(\mathbf{x}) = \mathbf{d}}_{\text{data equation}} \quad \underbrace{\mathcal{L}_{\mathrm{ME}}\{\mathbf{x}, \boldsymbol{E}\} = \boldsymbol{E}}_{\text{state equations}}$$

?

INVERSION:

$$\underbrace{\mathbf{x} = F^{-1}\{\mathbf{d}\}}_{\substack{\text{solving} \\ \text{data equation}}} \text{ subject to } \mathcal{L}_{\mathrm{ME}}\{\mathbf{x}, \boldsymbol{E}\} = \boldsymbol{E}$$

data equation:

$$\underbrace{\mathbf{E}^{\mathrm{sc}}(\mathbf{r} \in S_{\mathrm{a}})}_{\text{data}} = \iiint_{V_{\mathrm{s}}} K(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{\mathrm{b}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}(\mathbf{r}') d\mathbf{r}'$$

- the unknown is the contrast
- ensures that for a given internal field the forward model matches the data

reconstruction is an interplay of the two equations

state equation:

$$\underbrace{\mathbf{E}(\mathbf{r} \in V_{\mathrm{s}})}_{\text{internal field}} = \iiint_{V_{\mathrm{s}}} K(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{\mathrm{b}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{E}(\mathbf{r}') d\mathbf{r}'$$

- the unknown is the internal field
- ensures that for a given contrast the internal field satisfies Maxwell's eqns.

DATA EQUATION: A CLOSER LOOK

$$\mathbf{E}^{\mathrm{sc}}(\mathbf{r} \in S_{\mathrm{a}}) = \left[\mathbf{E} - \mathbf{E}^{\mathrm{inc}}\right]_{\mathbf{r}\in S_{\mathrm{a}}} = \iiint_{V_{\mathrm{s}}} K(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{\mathrm{b}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{E}(K,\mathbf{r}')d\mathbf{r}'$$

$\Downarrow$

data?

Q1: Can we measure the scattered field?

No, we measure S-parameters or voltage waveforms

Q2: Do we know Green's dyadic $\underline{\underline{\mathbf{G}}}_{\mathrm{b}}(\mathbf{r},\mathbf{r}')$?

No, unless the medium is uniform (or layered) and unbounded

Q3: Do we know the total internal field $\mathbf{E}(K,\mathbf{r}')$?

No, unless we employ Born's approximation $\rightarrow$ BA: $\mathbf{E}(\mathbf{r}') \approx \mathbf{E}^{\mathrm{inc}}(\mathbf{r}')$

Q4: Do we know the incident internal field $\mathbf{E}^{\mathrm{inc}}(\mathbf{r}')$?

No, unless the medium is uniform (or layered) and unbounded

DATA EQUATION FOR SCATTERING (*S*) PARAMETERS

[Nikolova *et al.*, *APS-URSI* 2016][Beaverstone *et al.*, *IEEE Trans. MTT*, 2017]

- scattering from penetrable objects (isotropic scattering is assumed)

$$\underbrace{S_{ik}^{sc}}_{\text{data}} = \underbrace{\frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k}}_{\text{known constants}} \iiint_{V_s} \underbrace{\Delta\varepsilon_r(\mathbf{r}')}_{\text{complex permittivity contrast}} \underbrace{\mathbf{E}_i^{inc}(\mathbf{r}')}_{\text{Green's vector function}} \cdot \underbrace{\mathbf{E}_k(\mathbf{r}')}_{\text{total internal field}} d\mathbf{r}'$$

$$\Delta\varepsilon_r(\mathbf{r}') = \varepsilon_{r,s}(\mathbf{r}') - \varepsilon_{r,b}(\mathbf{r}')$$

data

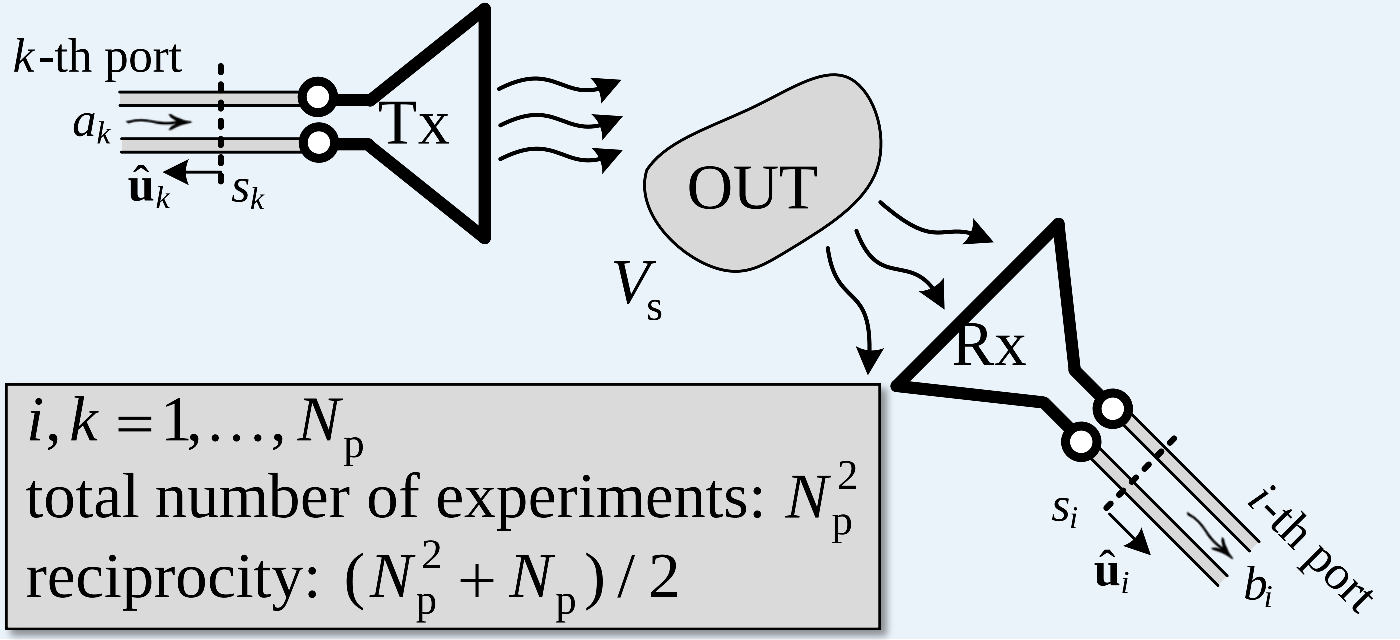
complex permittivity contrast

Green's vector function

total internal field

$\mathbf{E}_i^{inc}(\mathbf{r}')$: incident internal field due to Rx antenna if it were to transmit

$\mathbf{E}_k(\mathbf{r}')$: total internal field due to Tx antenna



$i, k = 1, \ldots, N_p$

total number of experiments: N_p^2

reciprocity: $(N_p^2 + N_p)/2$

DATA EQUATION: BORN'S APPROXIMATION OF TOTAL *INTERNAL* FIELD

$$\boxed{S_{ik}^{\mathrm{sc}} = \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k} \iiint_{V_\mathrm{s}} \Delta\varepsilon_\mathrm{r}(\mathbf{r}')\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}') \cdot \mathbf{E}_k\left(\mathbf{r}';\Delta\varepsilon_\mathrm{r}(\mathbf{r}')\right) d\mathbf{r}'}$$

total internal field?

- the total field $\mathbf{E}_k(\mathbf{r}')$ is generally unknown AND it depends on the contrast: ***data equation is nonlinear in the unknown contrast***
- Born's approximation linearizes the data equation by replacing the unknown ***total internal field*** with the known ***incident internal field*** (Max Born, 1926)

$$\boxed{\mathbf{E}_k(\mathbf{r}') \approx \mathbf{E}_{ik}^{\mathrm{inc}}(\mathbf{r}')} \Rightarrow \boxed{S_{ik}^{\mathrm{sc}} \approx \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k} \iiint_{V_\mathrm{s}} \Delta\varepsilon_\mathrm{r}(\mathbf{r}')\underbrace{\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}') \cdot \mathbf{E}_k^{\mathrm{inc}}(\mathbf{r}')}\, d\mathbf{r}'}$$

data (assumed error-free)

contrast (unknown)

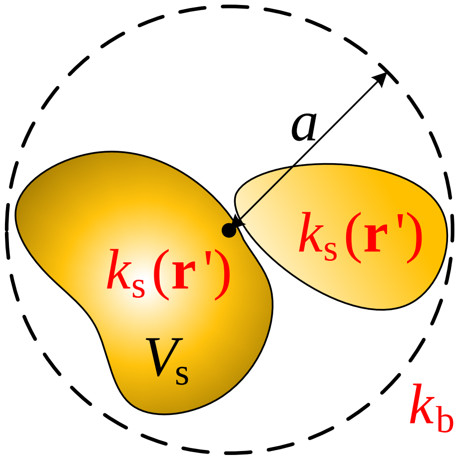
incident fields (assumed known)

LIMITATIONS OF BORN'S APPROXIMATION OF THE TOTAL *INTERNAL* FIELD

- limits both the size and the contrast of the scatterer

$$\boxed{a^2 \left| k_s^2(\mathbf{r}) - k_b^2 \right| \ll 1,\ \mathbf{r} \in V_s}$$

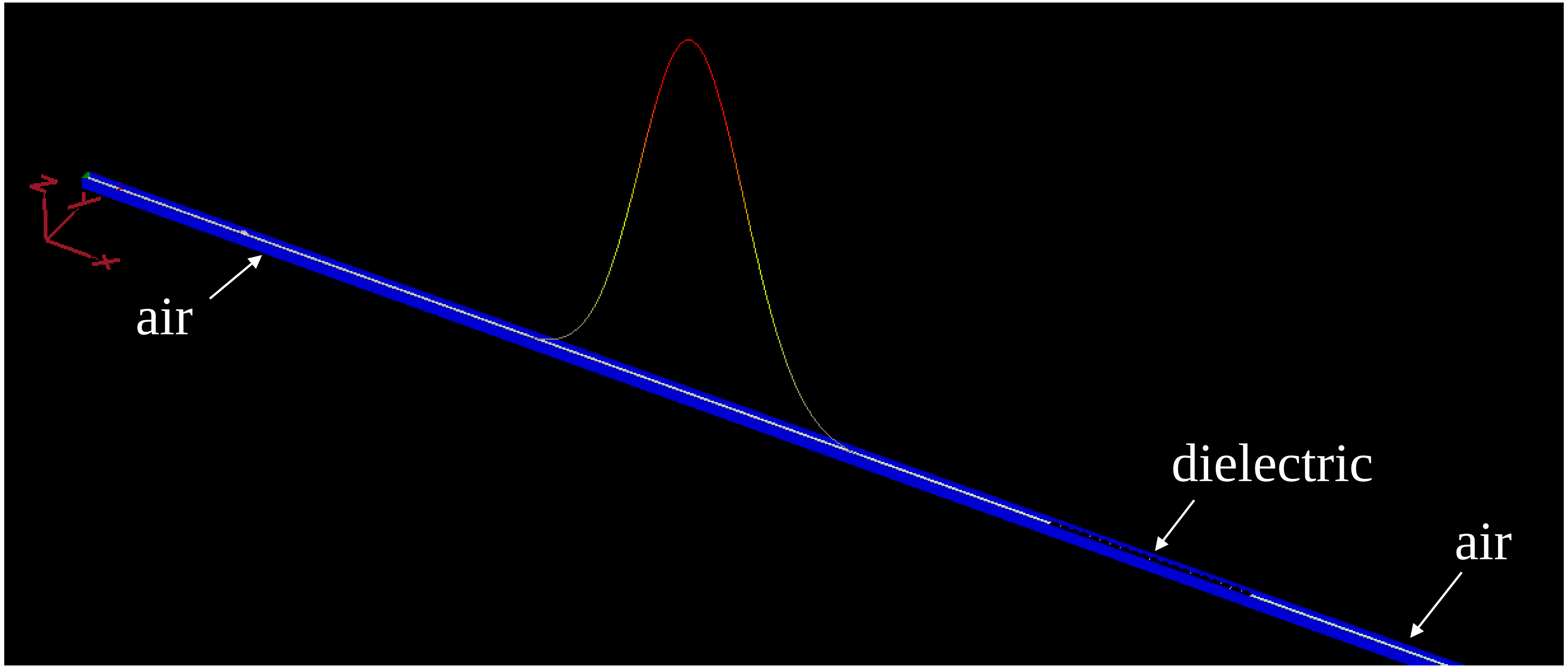
[Nikolova, *Introduction to Microwave Imaging*, 2017]



- if OUT violates the limits: images contain artifacts which reflect differences between $\mathbf{E}_{\mathrm{Tx}}^{\mathrm{inc}}(\mathbf{r}')$ and $\mathbf{E}_{\mathrm{Tx}}(\mathbf{r}')$ rather than contrast

- Born's approximation is underlying all direct inversion methods (real-time imaging)

1-D EXAMPLE: BORN'S APPROXIMATION OF TOTAL *INTERNAL* FIELD

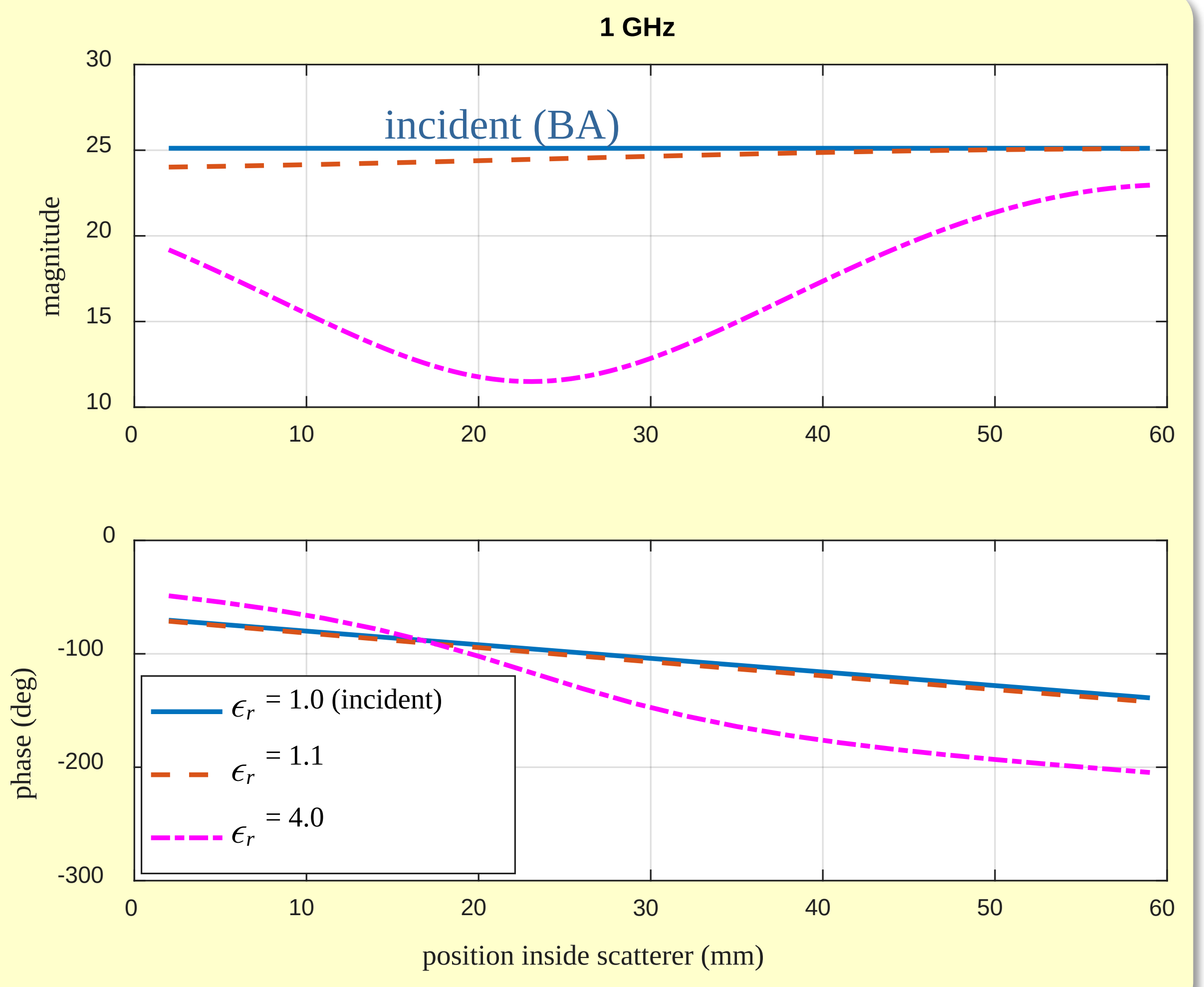


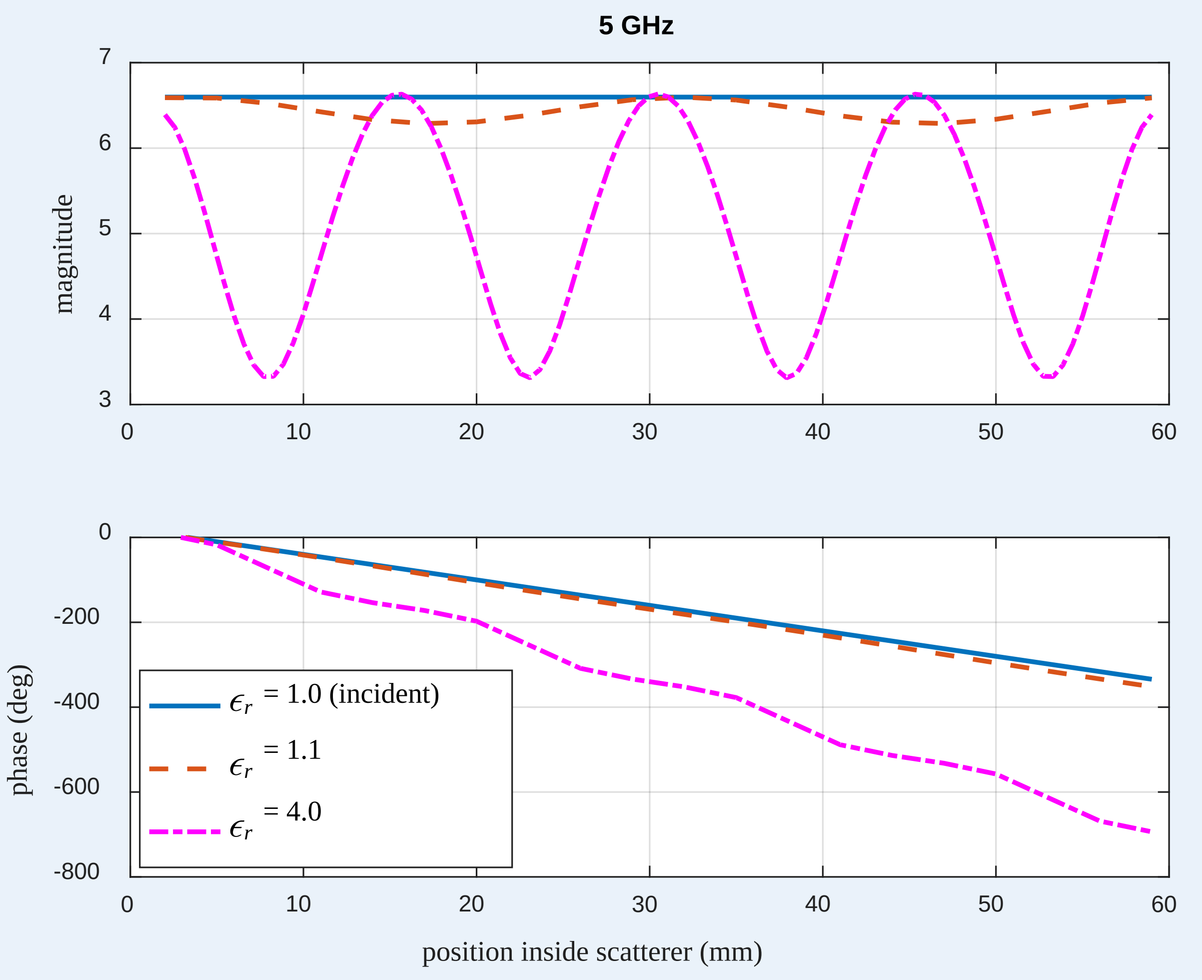
MEFiSTo-3D Nova

- Gaussian pulse bandwidth: 5 GHz at 3-dB level
- 1-D incident wave coming from left
- internal field recorded inside dielectric slab of length L = 6 cm

1-D EXAMPLE: BORN'S APPROXIMATION OF THE TOTAL *INTERNAL* FIELD – 2

comparison of incident wave in air (ε_r = 1.0) with actual internal field in dielectric slabs





1-D EXAMPLE: BORN'S APPROXIMATION OF THE TOTAL *INTERNAL* FIELD – 3

$$a = L/2 = 3\text{ cm}$$
$$\varepsilon_{\text{r,b}} = 1$$
$$\varepsilon_{\text{r,s}} = 1.1,\ 4.0$$
$$\lambda_{\text{b}}^{\text{1GHz}} \approx 30\text{ cm}$$
$$\lambda_{\text{b}}^{\text{5GHz}} \approx 6\text{ cm}$$

- let us evaluate Born's limit in this example – dielectric slab

$$\boxed{a^2\left|k_{\text{s}}^2(\mathbf{r}) - k_{\text{b}}^2\right| \ll 1} \Rightarrow a^2 k_{\text{b}}^2\left(\frac{k_{\text{s}}^2}{k_{\text{b}}^2} - 1\right) = \left(\frac{2\pi a}{\lambda_{\text{b}}}\right)^2\left(\frac{\varepsilon_{\text{r,s}}}{\varepsilon_{\text{r,b}}} - 1\right) \ll 1$$

$$\Rightarrow \begin{array}{l} \left(\varepsilon_{\text{r,s}}^{\text{1GHz}}\right)_{\max} < 3.53 \\ \left(\varepsilon_{\text{r,s}}^{\text{5GHz}}\right)_{\max} < 1.10 \end{array}$$

- BA holds marginally for slab of $\varepsilon_{\text{r,s}} = 1.1$ but error is very large at $\varepsilon_{\text{r,s}} = 4.0$
- BA in the **magnitude** field distribution is more sensitive (than the **phase**) to permittivity contrast because reflections at interfaces are not taken into account – even for $\varepsilon_{\text{r,s}} = 1.1$ magnitude errors are appreciable (esp. at 5 GHz)
- error of the internal-field BA grows with frequency due to increase in scatterer's electrical size a/λ

WHAT IS THE BASIS OF BORN'S APPROXIMATION IN WAVE THEORY?

- Born's approximation is based on a simple linear combination of incident-field and total-field wave equations (Helmholtz equations in the frequency domain)

$$\left|\begin{array}{l}\nabla^2 U + k_s^2 U = 0 \\ \nabla^2 U^{\text{inc}} + k_b^2 U^{\text{inc}} = 0\end{array}\right. \; - \quad \Rightarrow \quad \nabla^2 \underbrace{(U - U^{\text{inc}})}_{U^{\text{sc}}} + k_s^2 U - k_b^2 U^{\text{inc}} = 0$$

$$k_b = \omega\sqrt{\mu_b \varepsilon_b}$$

$$k_s = \omega\sqrt{\mu_s \varepsilon_s}$$

$$\Rightarrow \quad \boxed{\nabla^2 U^{\text{sc}} + k_b^2 U^{\text{sc}} = -K \cdot U, \text{ where } K = k_s^2 - k_b^2}$$

- in scattering from dielectric bodies: $K = \omega^2 \mu_0 \varepsilon_0 (\varepsilon_{r,s} - \varepsilon_{r,b}) = k_0^2 \Delta\varepsilon_r$

- some terminology

$\Delta\varepsilon_r$ – dielectric contrast

K – contrast function

$K \cdot U$ – contrast source

BORN'S APPROXIMATION IN WAVE THEORY – 2

$$\underbrace{\nabla^2 U^{\text{sc}} + k_{\text{b}}^2 U^{\text{sc}}}_{\text{differential operator is that of background}} = -\underbrace{K \cdot U}_{\text{contrast source}}, \text{ where } K = k_{\text{s}}^2 - k_{\text{b}}^2$$

superposition integral $\mathcal{L}_{\text{b}}(KU)$

$$\Rightarrow U(\mathbf{r}) = U^{\text{inc}}(\mathbf{r}) + U^{\text{sc}}(\mathbf{r}) = U^{\text{inc}}(\mathbf{r}) + \iiint_{V_{\text{s}}} G_{\text{b}}(\mathbf{r}, \mathbf{r}') \cdot \underbrace{K(\mathbf{r}')U(\mathbf{r}')}_{\text{contrast source}} dv'$$

background Green's function depends on boundary conditions

- Green's function gives the solution at $\mathbf{r}$ upon point excitation (δ-source) at $\mathbf{r}' \leftarrow \delta(\mathbf{r} - \mathbf{r}')$
- examples of analytical Green's functions for open (unbounded) uniform background medium

$$\text{3D: } G_{\text{b}}(\mathbf{r}, \mathbf{r}') = \frac{e^{-\text{i}k_{\text{b}}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|} \qquad \text{2D: } G_{\text{b}}(\rho, \rho') = \frac{\text{i}}{4} H_0^{(2)}(k_{\text{b}}|\rho - \rho'|)$$

- 0^{th} order Born approximation of the total field

$$\boxed{U_{\mathrm{B}}^{(0)}(\mathbf{r}) = U^{\mathrm{inc}}(\mathbf{r})}$$

this is what we use to linearize the data equations by approximating the total internal field

$$\boxed{S_{ik}^{\mathrm{sc}} = \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k} \iiint_{V_s} \Delta\varepsilon_{\mathrm{r}}(\mathbf{r}')\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}') \cdot \mathbf{E}_k\left(\mathbf{r}'; \Delta\varepsilon_{\mathrm{r}}(\mathbf{r}')\right) d\mathbf{r}'}$$

$$\Rightarrow \boxed{(S_{ik}^{\mathrm{sc}})_{\mathrm{B}} = \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k} \iiint_{V_s} \Delta\varepsilon_{\mathrm{r}}(\mathbf{r}')\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}') \cdot \mathbf{E}_k^{\mathrm{inc}}(\mathbf{r}') d\mathbf{r}'}$$

BORN'S APPROXIMATION AS NEUMANN SERIES EXPANSION – 2

- 1^{st} order Born approximation of the total field

$$U_{\mathrm{B}}^{(1)}(\mathbf{r}) = U_{\mathrm{B}}^{(0)}(\mathbf{r}) + \mathcal{L}(KU_{\mathrm{B}}^{(0)}) = \boxed{U^{\mathrm{inc}}(\mathbf{r}) + \underbrace{\iiint_{V_{\mathrm{s}}} G_{\mathrm{b}}(\mathbf{r},\mathbf{r}') \cdot K(\mathbf{r}') U^{\mathrm{inc}}(\mathbf{r}') dv'}_{\approx U^{\mathrm{sc}}(\mathbf{r})}}$$

  - this is what we use to approximate the total and scattered <u>external</u> fields (or data)

$$U^{\mathrm{OUT}}(\mathbf{r}) = U^{\mathrm{inc}}(\mathbf{r}) + U^{\mathrm{sc}}(\mathbf{r}) \Rightarrow \boxed{U^{\mathrm{sc}}(\mathbf{r}) = U^{\mathrm{OUT}}(\mathbf{r}) - U^{\mathrm{inc}}(\mathbf{r})}$$

- n^{th} order Born approximation of the total field – can be used to obtain iteratively the total <u>internal</u> field

$$\boxed{U_{\mathrm{B}}^{(n)}(\mathbf{r}) = U_{\mathrm{B}}^{(n-1)}(\mathbf{r}) + \mathcal{L}(KU_{\mathrm{B}}^{(n-1)})}$$

  - for known contrast $K(\mathbf{r})$, Born's expansion series converges to the true total field

$$U_{\mathrm{B}}^{(n)}(\mathbf{r}) \to U(\mathbf{r}) \;\text{ if }\; \boxed{a^2 \left| k_{\mathrm{s}}^2 - k_{\mathrm{b}}^2 \right|_{\max} < 1.58}$$

[Nikolova, *Introduction to Microwave Imaging*, 2017]

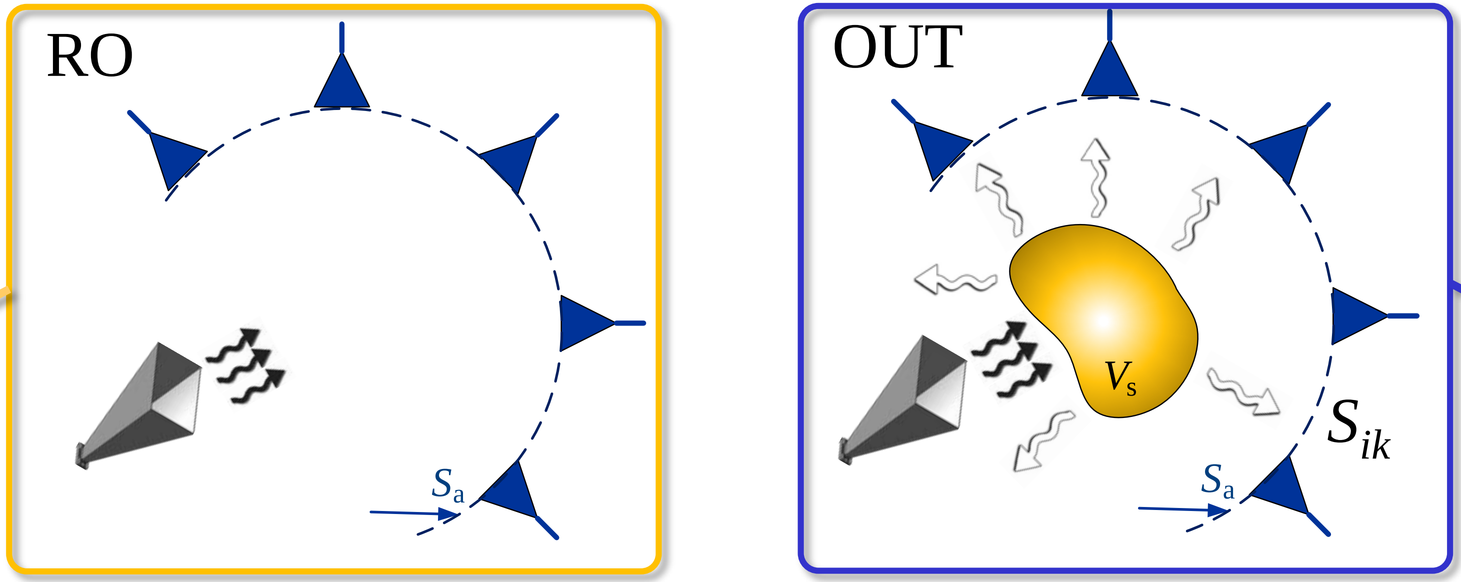
BORN'S APPROXIMATION OF THE DATA (*S*-PARAMETERS)

- Born's approximation of the ***total*** response is an <u>additive correction to the incident one</u>

$$S_{ik} \approx S_{ik}^{\mathrm{inc}} + \left(S_{ik}^{\mathrm{sc}}\right)_{\mathrm{B}} = S_{ik}^{\mathrm{inc}} + \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k} \iiint_{V_{\mathrm{s}}} \Delta\varepsilon_{\mathrm{r}}(\mathbf{r}')\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}')\cdot\mathbf{E}_k^{\mathrm{inc}}(\mathbf{r}')d\mathbf{r}'$$

total data   *incident data*   *Born's approximation of the scattered data*

- acquisition of the incident (*aka* baseline) data: the reference object (RO)
  RO is simply the measurement setup in the absence of an OUT



$S_{ik}^{\mathrm{inc}} \equiv S_{ik}^{\mathrm{RO}}$ $S_{ik} \equiv S_{ik}^{\mathrm{OUT}}$

BORN'S APPROXIMATION OF THE DATA (EXTERNAL FIELD) – SUMMARY

1) Born's superposition model allows to extract the scattered portion of a response

$$S_{ik}^{\mathrm{sc}} = S_{ik}^{\mathrm{OUT}} - S_{ik}^{\mathrm{RO}}$$

➢ requires 2 measurements: RO and OUT

Note: RO is ***not*** a uniform medium – it includes all complexities of the measurement setup

2) 1st order BA supplies a linearized (but approximate) model of scattering

$$\left(S_{ik}^{\mathrm{sc}}\right)_{\mathrm{B}} \approx S_{ik}^{\mathrm{sc}} = S_{ik}^{\mathrm{OUT}} - S_{ik}^{\mathrm{RO}} \approx \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k} \iiint_{V_\mathrm{s}} \Delta\varepsilon_\mathrm{r}(\mathbf{r}')\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}') \cdot \mathbf{E}_k^{\mathrm{inc}}(\mathbf{r}')\,d\mathbf{r}'$$

Note: in reality

$$S_{ik}^{\mathrm{sc}} = \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k} \iiint_{V_\mathrm{s}} \Delta\varepsilon_\mathrm{r}(\mathbf{r}')\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}') \cdot \mathbf{E}_k\left(\mathbf{r}'; \Delta\varepsilon_\mathrm{r}(\mathbf{r}')\right) d\mathbf{r}'$$

LIMITATIONS OF BORN'S APPROXIMATION OF THE DATA

- How accurate is the data approximation with Born's model?

$$\left(S_{ik}^{\mathrm{sc}}\right)_{\mathrm{B}} \approx S_{ik}^{\mathrm{sc}} = S_{ik}^{\mathrm{OUT}} - S_{ik}^{\mathrm{RO}} \approx \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k} \iiint_{V_\mathrm{s}} \Delta\varepsilon_\mathrm{r}(\mathbf{r}')\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}') \cdot \mathbf{E}_k^{\mathrm{inc}}(\mathbf{r}') d\mathbf{r}'$$

in reality: $$S_{ik}^{\mathrm{sc}} = \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k} \iiint_{V_\mathrm{s}} \Delta\varepsilon_\mathrm{r}(\mathbf{r}')\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}') \cdot \mathbf{E}_k\left(\mathbf{r}'; \Delta\varepsilon_\mathrm{r}(\mathbf{r}')\right) d\mathbf{r}'$$

- limit on BA data approximations – less strict compared to that for internal field

$$2a\left|k_\mathrm{s}(\mathbf{r}) - k_\mathrm{b}\right|_{\max} < \pi$$

[Slaney et al., *IEEE Trans. MTT*, 1984]

compare with $\boxed{a^2\left|k_\mathrm{s}^2(\mathbf{r}) - k_\mathrm{b}^2\right| \ll 1}$

1-D EXAMPLE: LIMITATIONS OF BORN'S APPROXIMATION OF THE DATA

- re-visiting the dielectric-slab example for scattered field at external observation points (ports 1 and 2)

- What is the contrast limit now?

$$\boxed{2a\left|k_s(\mathbf{r})-k_b\right|_{\max}<\pi} \Rightarrow \underbrace{2a}_{L}k_b\left(\frac{k_s}{k_b}-1\right)=\left(\frac{2\pi L}{\lambda_b}\right)^2\left(\sqrt{\frac{\varepsilon_{r,s}}{\varepsilon_{r,b}}}-1\right)<\pi$$

$$a = L/2 = 3\text{ cm}$$
$$\varepsilon_{r,b} = 1$$
$$\lambda_b^{1\text{GHz}} \approx 30\text{ cm}$$
$$\lambda_b^{5\text{GHz}} \approx 6\text{ cm}$$

$$\Rightarrow \begin{array}{l}\left(\varepsilon_{r,s}^{1\text{GHz}}\right)_{\max}<12.25\\ \left(\varepsilon_{r,s}^{5\text{GHz}}\right)_{\max}<2.25\end{array}$$

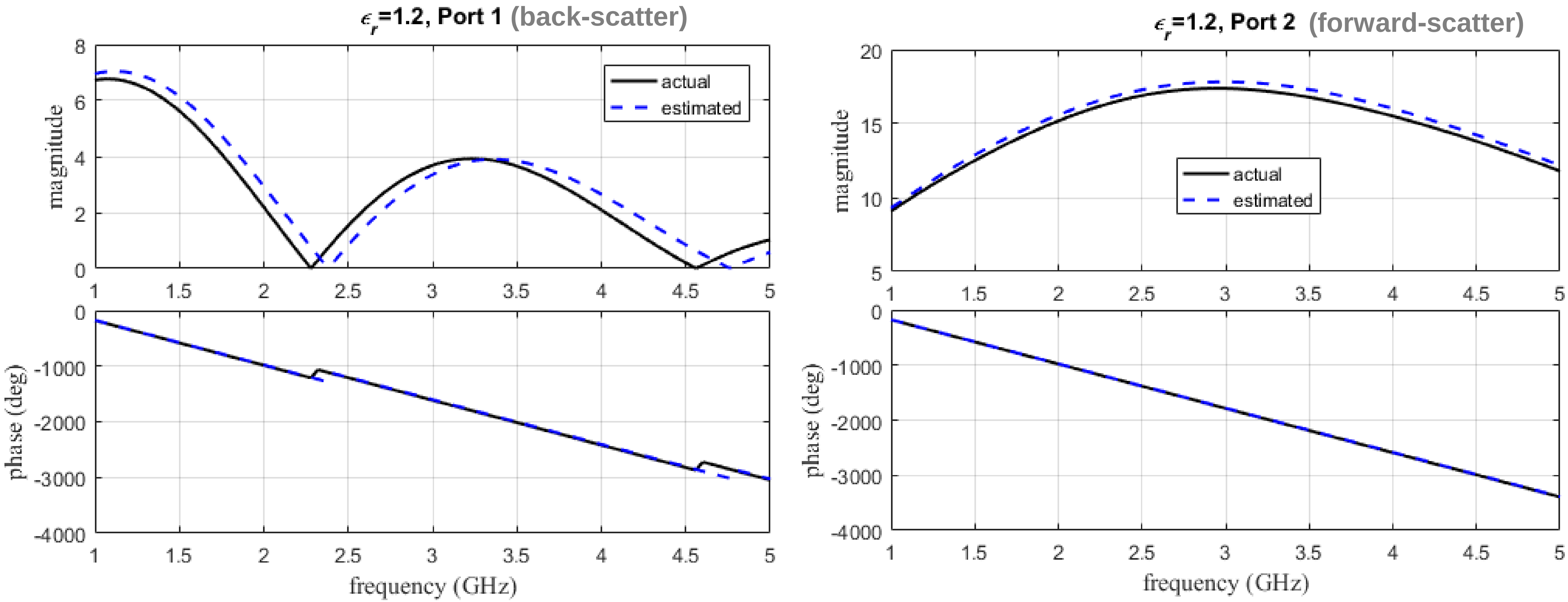
compare with internal-field BA limits

- Be aware! Slaney's limit is derived with transmission measurements in mind!

[Nikolova, *Introduction to Microwave Imaging*, 2017]

1-D EXAMPLE: LIMITATIONS OF BORN'S APPROXIMATION OF THE DATA

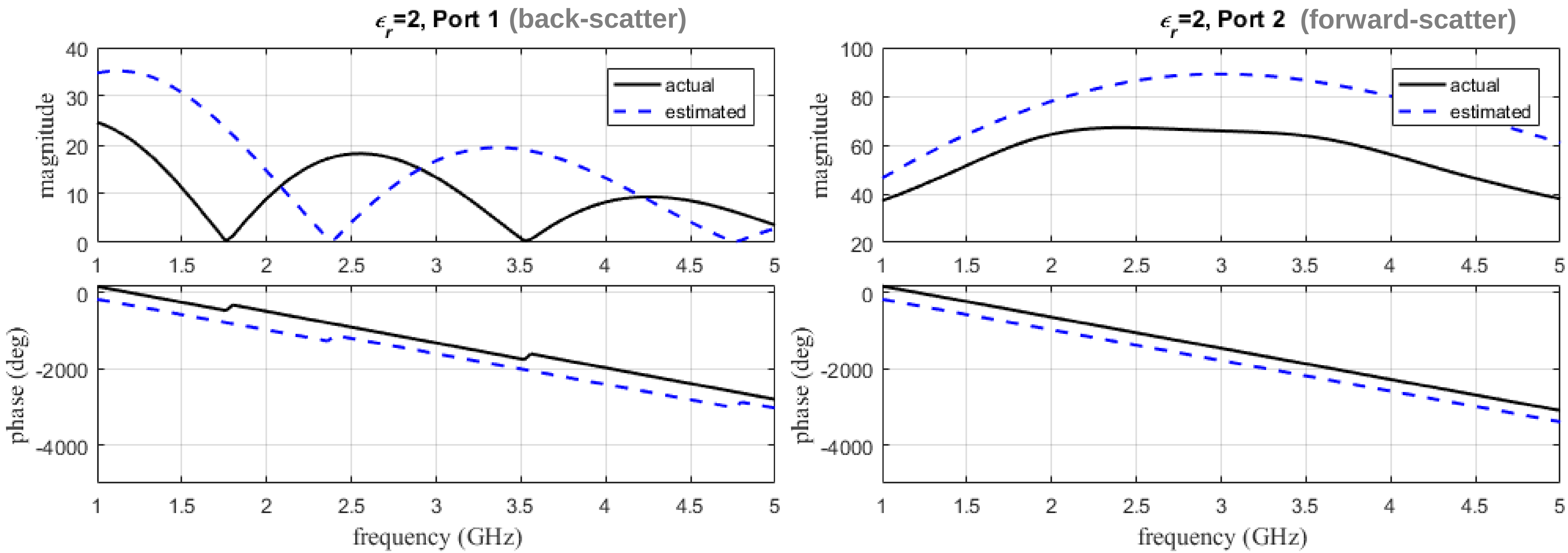
slab relative permittivity = 1.2



60 MM THICK DIELECTRIC SLAB IN AIR

1-D EXAMPLE: LIMITATIONS OF BORN'S APPROXIMATION OF THE DATA

slab relative permittivity = 2



- errors at Port 1 (reflection measurement) are unacceptable, esp. magnitude

RYTOV'S APPROXIMATION OF THE TOTAL FIELD

- Rytov's approximation of the ***total*** field is an exponential correction to the incident field

[S.M. Rytov, *Izv. Akad. Nauk SSSR*, Ser. Fiz 2 (1937)]

- the total field is represented as a complex exponent

$$U(\mathbf{r}) \approx U_{\mathrm{R}}(\mathbf{r}) = \exp\left[\psi_0(\mathbf{r}) + \psi_1(\mathbf{r}) + \psi_2(\mathbf{r}) + \cdots\right]$$

- 0th order Rytov approximation of the total field – used to approximate total **internal** field

$$U_{\mathrm{R}}^{(0)}(\mathbf{r}) = \exp\left[\psi_0(\mathbf{r})\right] = U^{\mathrm{inc}}(\mathbf{r}) \longleftarrow \textit{same as } 0^{th} \textit{ order Born approximation}$$

- 1st order Rytov approximation of the total field – used to approximate total **external** field (data)

$$U_{\mathrm{R}}^{(1)}(\mathbf{r}) = \exp\left[\psi_0(\mathbf{r}) + \psi_1(\mathbf{r})\right] = U^{\mathrm{inc}}(\mathbf{r}) \cdot \exp\left\{\left[U^{\mathrm{inc}}(\mathbf{r})\right]^{-1} \underbrace{\iiint_{V_s} G_{\mathrm{b}}(\mathbf{r},\mathbf{r}') \cdot K(\mathbf{r}') U^{\mathrm{inc}}(\mathbf{r}') dv'}_{U_{\mathrm{B}}^{(1)}(\mathbf{r})}\right\}$$

or

$$\boxed{U_{\mathrm{R}}^{(1)}(\mathbf{r}) = U^{\mathrm{inc}}(\mathbf{r}) \cdot \exp\left[U_{\mathrm{B}}^{(1)}(\mathbf{r}) / U^{\mathrm{inc}}(\mathbf{r})\right]}$$

RYTOV'S APPROXIMATION OF THE SCATTERED-FIELD DATA

- the case of *S*-parameters

$$S_{ik}^{\text{OUT}} = S_{ik}^{\text{RO}} \cdot \exp\left[(S_{ik}^{\text{sc}})_{\text{B}}^{(1)} / S_{ik}^{\text{RO}} \right] \Rightarrow \boxed{S_{ik}^{\text{sc}} \approx (S_{ik}^{\text{sc}})_{\text{R}} = S_{ik}^{\text{RO}} \ln\left(\frac{S_{ik}^{\text{OUT}}}{S_{ik}^{\text{RO}}} \right)}$$

➢ compare with the BA data approximation

$$\boxed{(S_{ik}^{\text{sc}})_{\text{B}} = S_{ik}^{\text{OUT}} - S_{ik}^{\text{RO}}}$$

- limitation of the Rytov's approximation of the data

$$\boxed{\left(k_{\text{s}}^2 - k_{\text{b}}^2\right) / k_{\text{b}}^2 < 1} \quad \text{or} \quad \boxed{\left(\varepsilon_{\text{r,s}} - \varepsilon_{\text{r,b}}\right) / \varepsilon_{\text{r,b}} < 1}$$

➢ no limitation on the size of the scattering object – advantage over Born's approximation

➢ strict limitation on the relative contrast – disadvantage to Born's approximation

EXAMPLE: BORN vs. RYTOV APPROXIMATION OF THE DATA

- re-visiting the dielectric-slab example for scattered field at external observation points (ports 1 and 2)
- What is Rytov's contrast limit?

$$\boxed{\left(\varepsilon_{\mathrm{r,s}} - \varepsilon_{\mathrm{r,b}}\right) / \,\varepsilon_{\mathrm{r,b}} < 1} \;\Rightarrow\; \varepsilon_{\mathrm{r,s}} < 2\varepsilon_{\mathrm{r,b}} = 2$$

- compare with BA limits

$$\left(\varepsilon_{\mathrm{r,s}}^{\mathrm{1GHz}}\right)_{\max} < 12.25$$

$$\left(\varepsilon_{\mathrm{r,s}}^{\mathrm{5GHz}}\right)_{\max} < 2.25$$

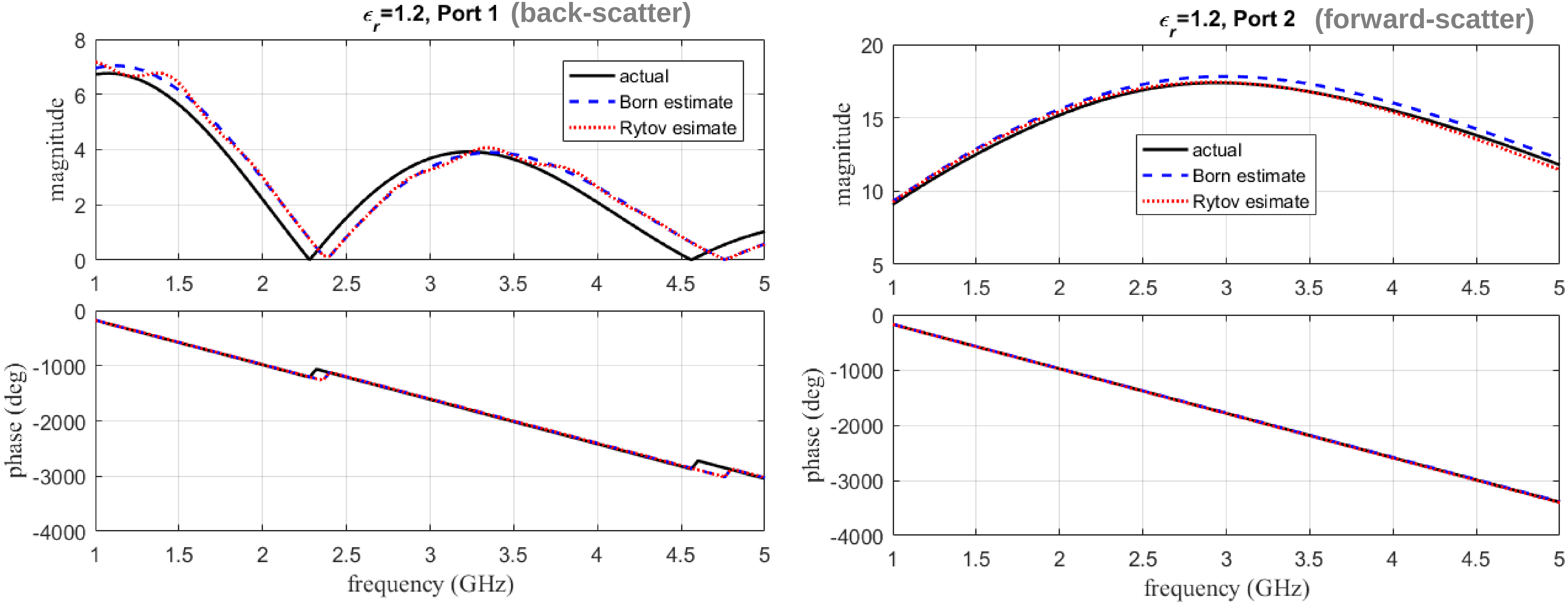
- notice independence of electrical size

EXAMPLE: BORN vs. RYTOV APPROXIMATION OF THE DATA – 2

slab relative permittivity = 1.2

(back-scatter)

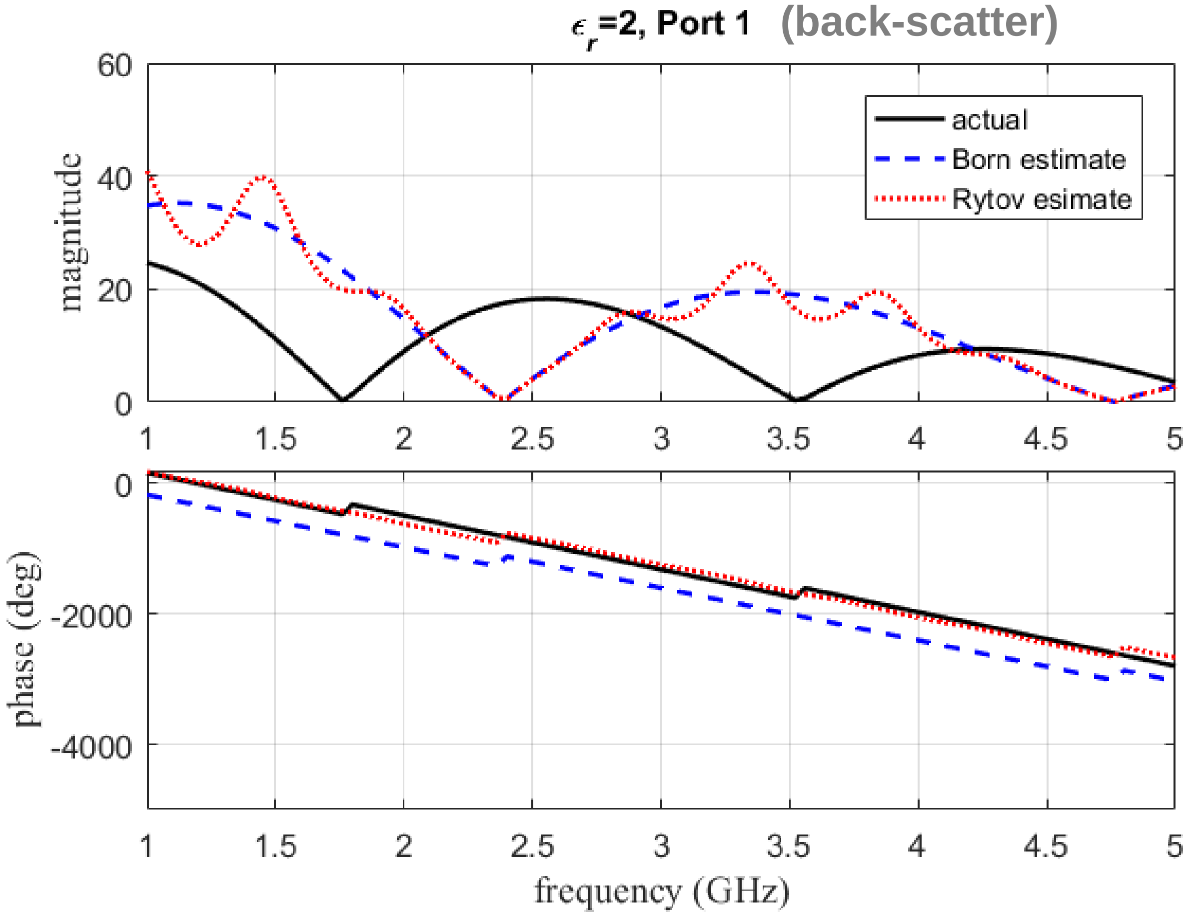
(forward-scatter)

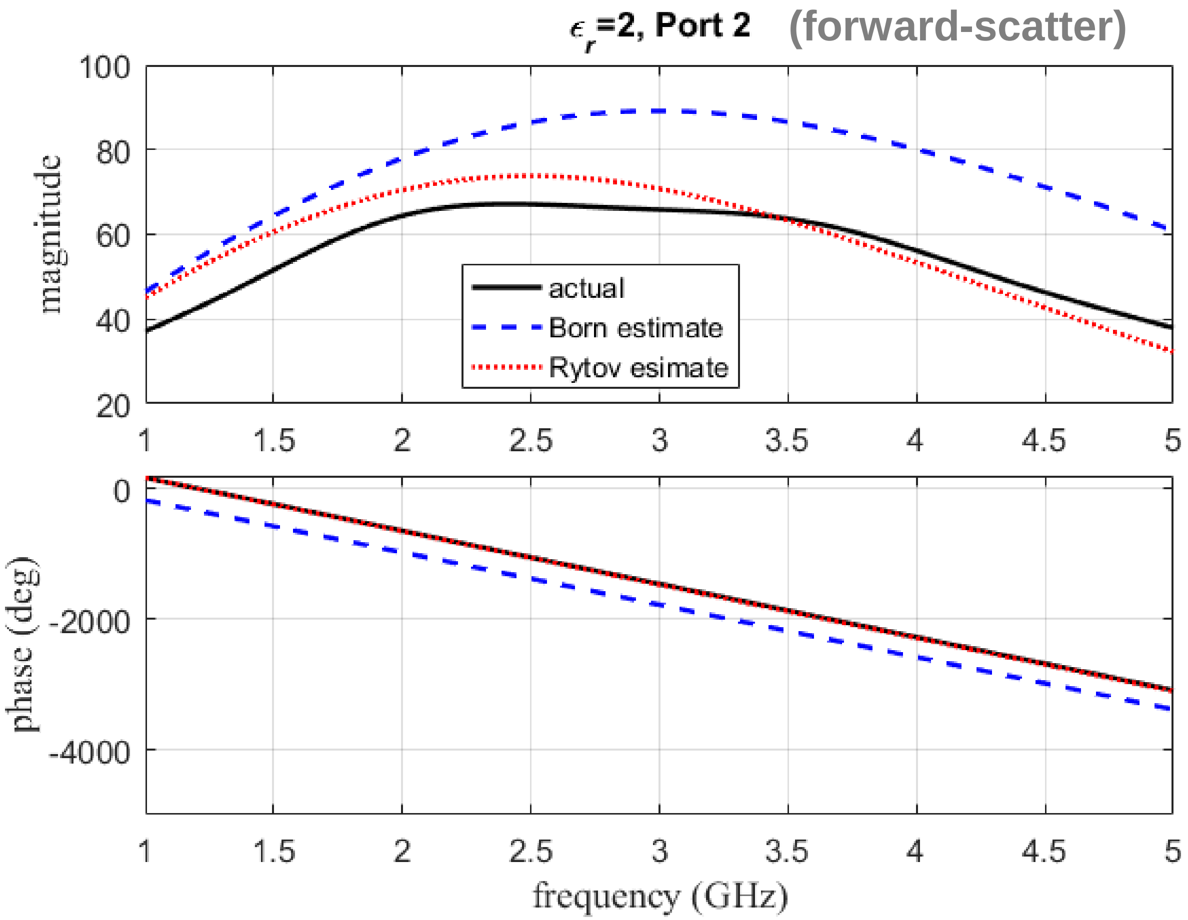


- both approximations perform very well: BA slightly better on back-scatter, RA slightly better on forward-scatter

EXAMPLE: BORN vs. RYTOV APPROXIMATION OF THE DATA – 3

slab relative permittivity = 2.0

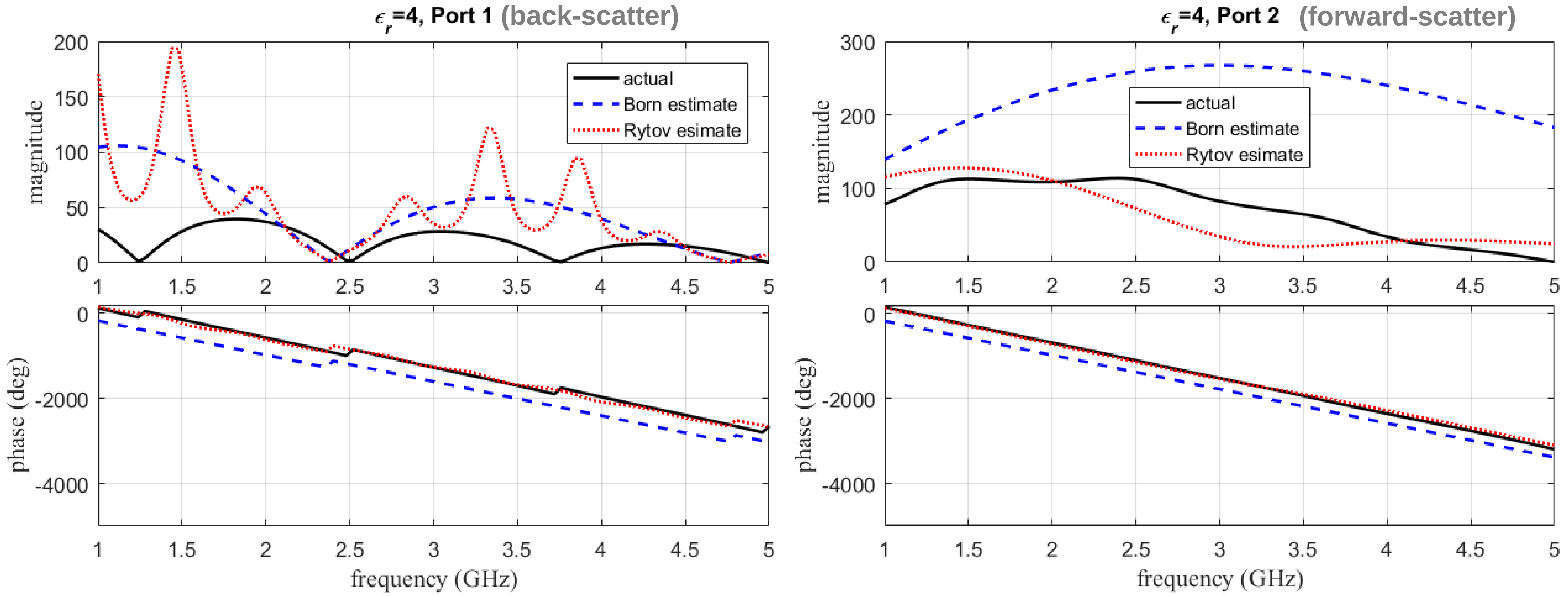




- both approximation show errors in magnitude of back-scatter
- RA better on forward-scatter and in the phase of back-scatter

EXAMPLE: BORN vs. RYTOV APPROXIMATION OF THE DATA – 4

slab relative permittivity = 4.0

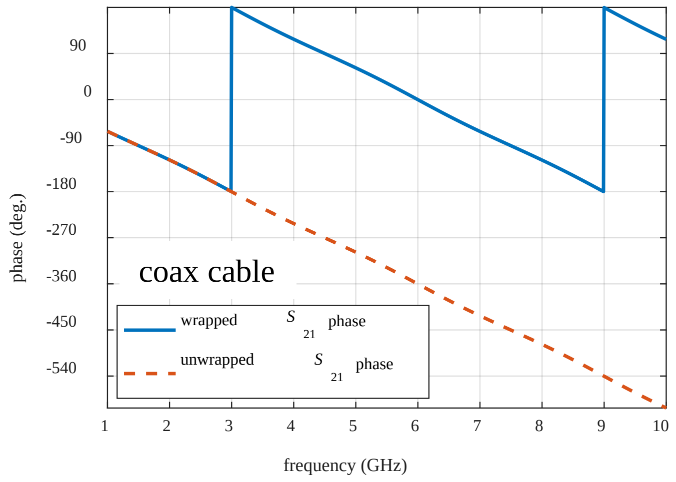


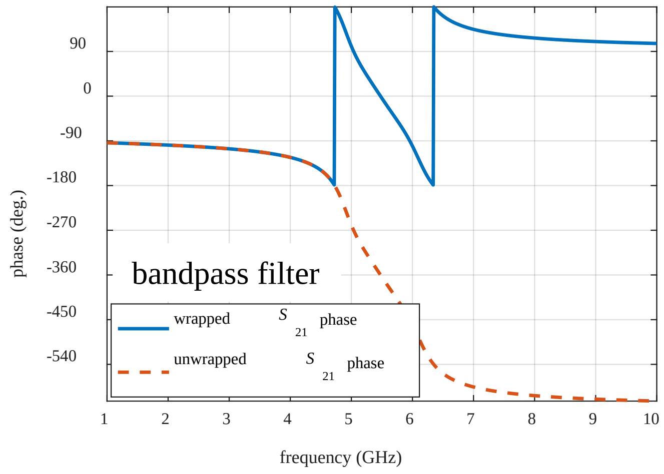
- both approximation show large errors in magnitude of back-scatter
- RA much better on forward-scatter and the phase of both back-scatter

PITFALLS IN THE USE OF RYTOV'S APPROXIMATION

1) Rytov's approximation is prone to errors in unwrapping the phase of the *S*-parameters

$$(S_{ik}^{\mathrm{sc}})_{\mathrm{R}} = S_{ik}^{\mathrm{RO}} \ln\left(\frac{S_{ik}^{\mathrm{OUT}}}{S_{ik}^{\mathrm{RO}}}\right) \Rightarrow \frac{(S_{ik}^{\mathrm{sc}})_{\mathrm{R}}}{S_{ik}^{\mathrm{RO}}} = \left\{\ln\frac{|S_{ik}^{\mathrm{OUT}}|}{|S_{ik}^{\mathrm{RO}}|} + \boxed{\mathrm{i}\left(\angle S_{ik}^{\mathrm{OUT}} - \angle S_{ik}^{\mathrm{RO}}\right)}\right\}$$





phase unwrapping in the frequency domain ensures smooth behavior in this domain

- unwrapped data in frequency does not ensure continuity in space (over the acquisition surface) → spurious large differences between OUT and RO phases corrupt inversion!
- safe to use for object thickness D such that $|k_{\mathrm{s}} - k_{\mathrm{b}}|\,D \ll 2\pi \Rightarrow \left|\sqrt{\varepsilon_{\mathrm{r,s}}} - \sqrt{\varepsilon_{\mathrm{r,b}}}\right| D/\lambda_0 \ll 2\pi$

PITFALLS IN THE USE OF RYTOV'S APPROXIMATION – 2

2) Rytov's approximation is prone to errors when incident field (RO data) is weak

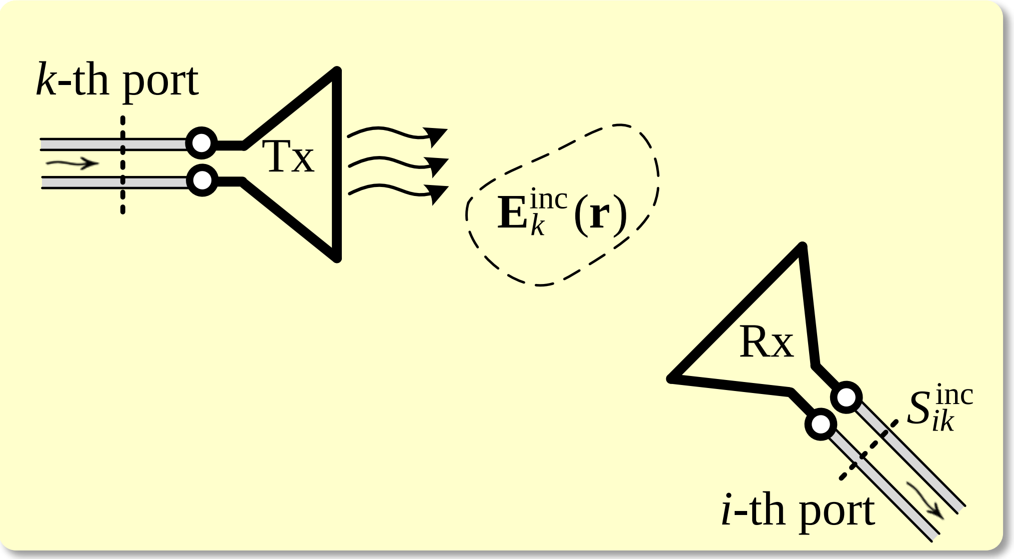
$$(S_{ik}^{\mathrm{sc}})_{\mathrm{R}} = S_{ik}^{\mathrm{RO}} \ln\left(\frac{S_{ik}^{\mathrm{OUT}}}{S_{ik}^{\mathrm{RO}}}\right) \Rightarrow \frac{(S_{ik}^{\mathrm{sc}})_{\mathrm{R}}}{S_{ik}^{\mathrm{RO}}} = \left\{\ln\frac{|S_{ik}^{\mathrm{OUT}}|}{\boxed{|S_{ik}^{\mathrm{RO}}|}} + \mathrm{i}\left(\angle S_{ik}^{\mathrm{OUT}} - \angle S_{ik}^{\mathrm{RO}}\right)\right\}$$

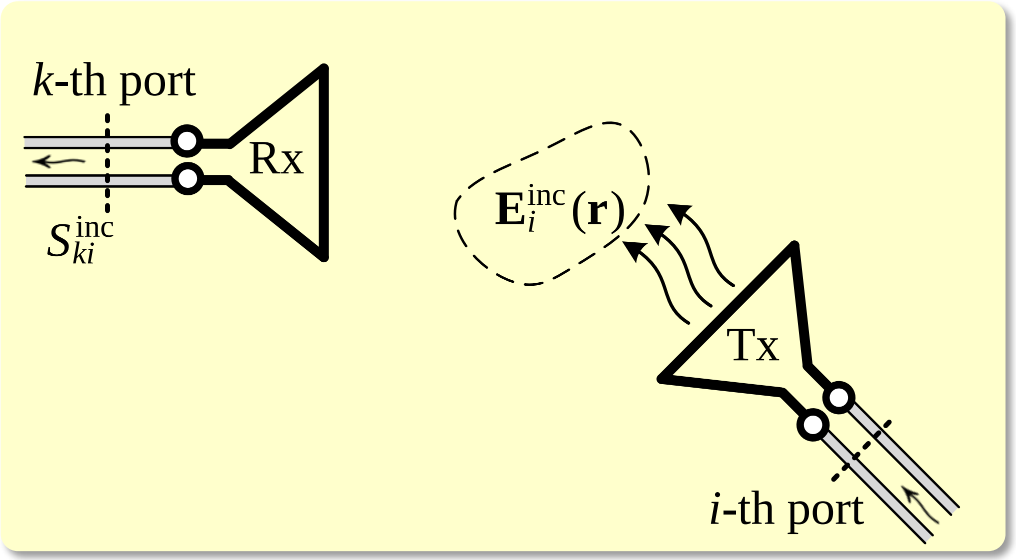
division by zero or noisy signal!

- best use in transmission measurement with significant RO signal strength

THE LINEARIZED DATA EQUATION: A CLOSER LOOK AGAIN!

$$\boxed{\mathbf{E}_k(\mathbf{r}') \approx \mathbf{E}_k^{\mathrm{inc}}(\mathbf{r}')} \quad \Rightarrow \quad \underbrace{S_{ik}^{\mathrm{sc}}}_{\substack{\text{data}\\ \text{assumed noise-free}}} \approx \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k}\iiint_{V_\mathrm{s}} \underbrace{\Delta\varepsilon_\mathrm{r}(\mathbf{r}')}_{\substack{\text{contrast}\\ \text{unknown}}}\, \underbrace{\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}')\cdot\mathbf{E}_k^{\mathrm{inc}}(\mathbf{r}')}_{\substack{\text{incident fields}\\ \text{assumed known}}}\, d\mathbf{r}'$$





Do we really know the incident internal field distributions?

- *NO, unless the RO is uniform (or layered) and unbounded*
- *…and unless V_s is in the far-zone of the antennas*

THE LINEARIZED DATA EQUATION: **ANALYTICAL** INCIDENT FIELD MODELS

- IF V_s is in the far-zone of the antennas and the RO can be assumed uniform & unbounded

  THEN analytical incident-field models exist

plane waves: $\mathbf{E}_{\mathrm{Tx}}^{\mathrm{inc}}(\mathbf{r},\mathbf{r}_{\mathrm{Tx}}) \sim \hat{\mathbf{p}} e^{-\mathrm{i}k_{\mathrm{b}}|\mathbf{r}-\mathbf{r}_{\mathrm{Tx}}|}$

spherical waves: $\mathbf{E}_{\mathrm{Tx}}^{\mathrm{inc}}(\mathbf{r},\mathbf{r}_{\mathrm{Tx}}) \sim \hat{\mathbf{p}} \dfrac{e^{-\mathrm{i}k_{\mathrm{b}}|\mathbf{r}-\mathbf{r}_{\mathrm{Tx}}|}}{|\mathbf{r}-\mathbf{r}_{\mathrm{Tx}}|}$

cylindrical waves: $\mathbf{E}_{\mathrm{Tx}}^{\mathrm{inc}}(\mathbf{r},\mathbf{r}_{\mathrm{Tx}}) \sim \hat{\mathbf{p}} H_0^{(2)}(k_{\mathrm{b}}\rho),\ \rho = \sqrt{(x-x_{\mathrm{Tx}})^2 + (y-y_{\mathrm{Tx}})^2}$

- antenna far-field pattern $F(\theta,\phi)$ improves incident-field model, for example
  [Amineh *et al.*, *IEEE AWPL*, 2012]

$$\mathbf{E}_{\mathrm{Tx}}^{\mathrm{inc}}(r,\theta,\phi) \sim \hat{\mathbf{p}} F(\theta,\phi) \frac{e^{-\mathrm{i}k_{\mathrm{b}} r}}{r}$$

THE LINEARIZED DATA EQUATION: **SIMULATED** INCIDENT FIELD MODELS

- simulated incident fields are often used in near-field imaging where the analytical far-zone models do not apply
- incident-field distributions often suffer from modeling errors

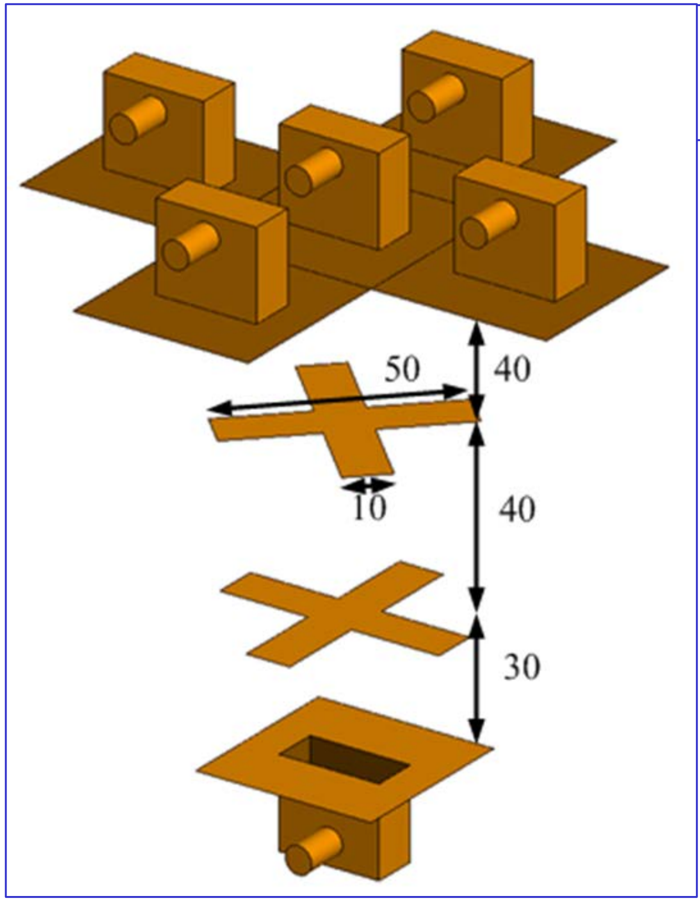
  [Amineh *et al.*, *Trans. AP*, 2011; *Int. J. Biomed. Imaging*, 2012][Li *et al.*, *Inverse Problems*, 2010]
  [Tu *et al.*, *Inverse Problems*, 2015]

- modeling errors increase with: (i) decreasing the stand-off distance between the antennas and the OUT, (ii) the complexity of the measurement setup
- errors in incident fields corrupt the ***resolvent kernel*** of the data equation

$$S_{ik}^{\mathrm{sc}} \approx \frac{\mathrm{i}\omega\varepsilon_0}{2a_i a_k} \iiint_{V_s} \Delta\varepsilon_{\mathrm{r}}(\mathbf{r}') \underbrace{\mathbf{E}_i^{\mathrm{inc}}(\mathbf{r}') \cdot \mathbf{E}_k^{\mathrm{inc}}(\mathbf{r}')}_{\substack{\text{resolvent kernel} \\ \mathcal{K}_{ik}(\mathbf{r}')}} d\mathbf{r}'$$

EXAMPLE: SIMULATED vs. MEASURED INCIDENT FIELD MODELS

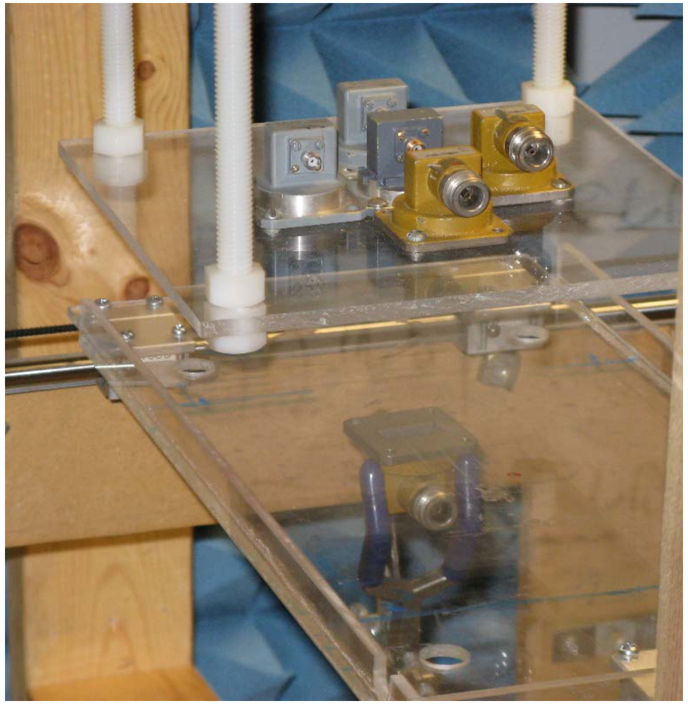
[Amineh *et al.*, *Trans. IM*, 2015]

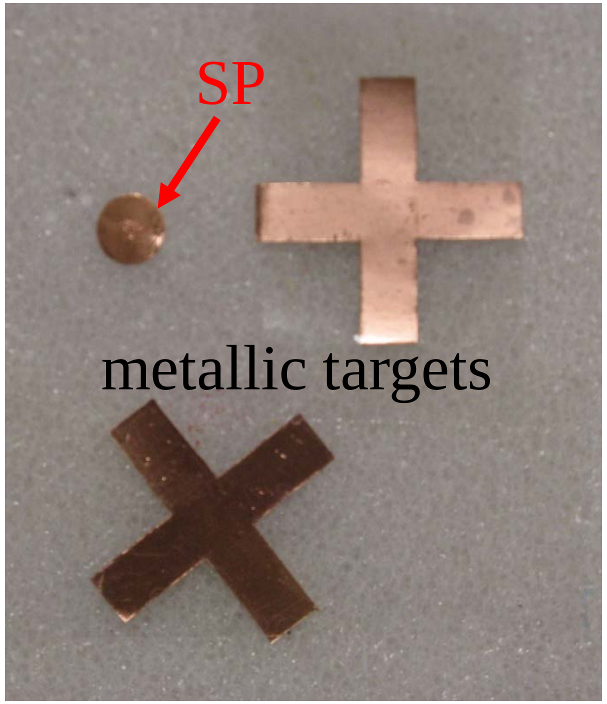


$\Delta x = \Delta y = 5\,\text{mm}$
$\Delta f = 250\,\text{MHz}$

X-band (WR90) open-end waveguides ($f_c \approx 6.56\,\text{GHz}$)

f (GHz)	λ (mm)	D_{far} (mm)
3	100	12.5
8.2	37	34
20	15	83

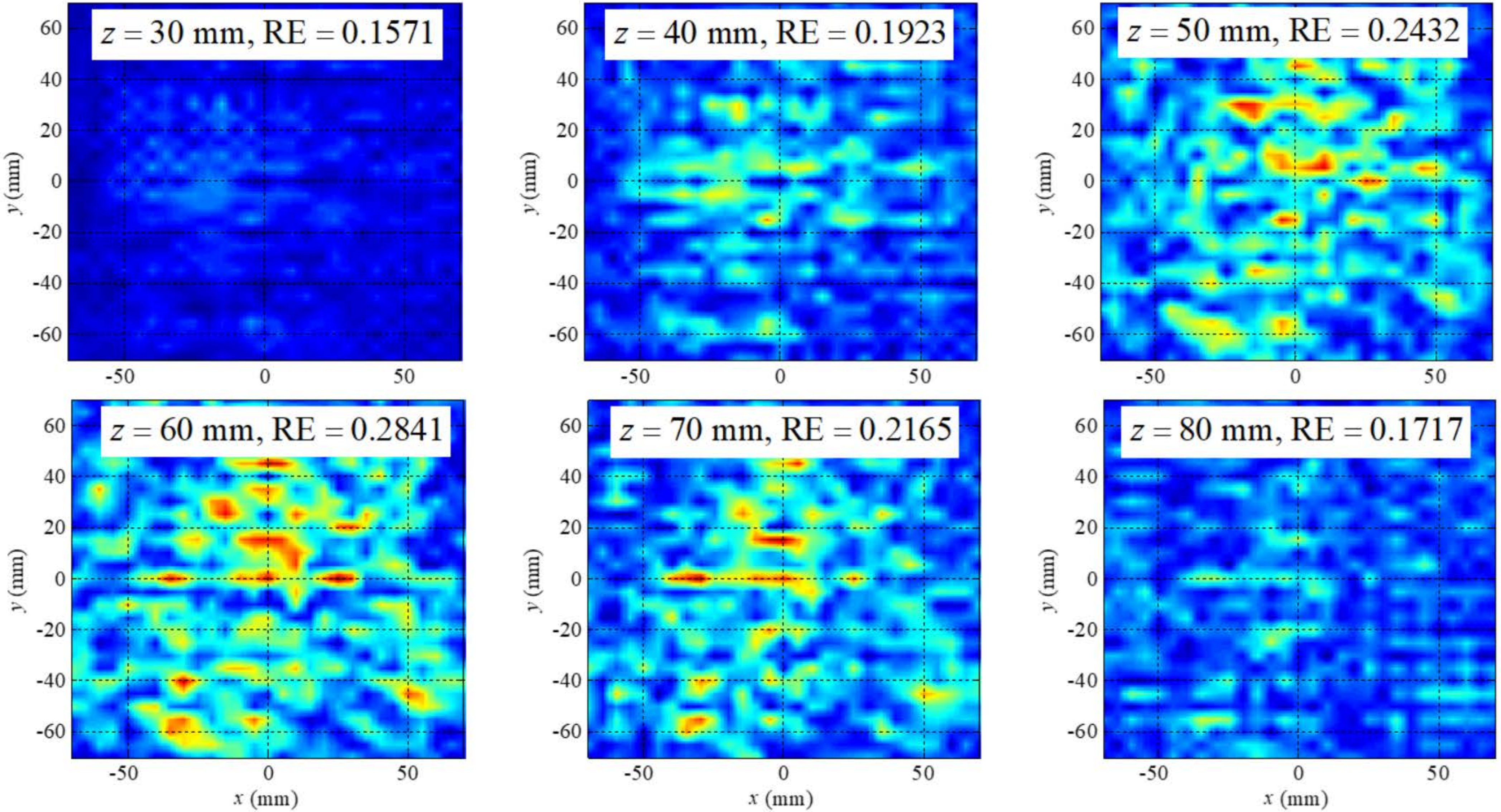




[photo credit: Justin McCombe]

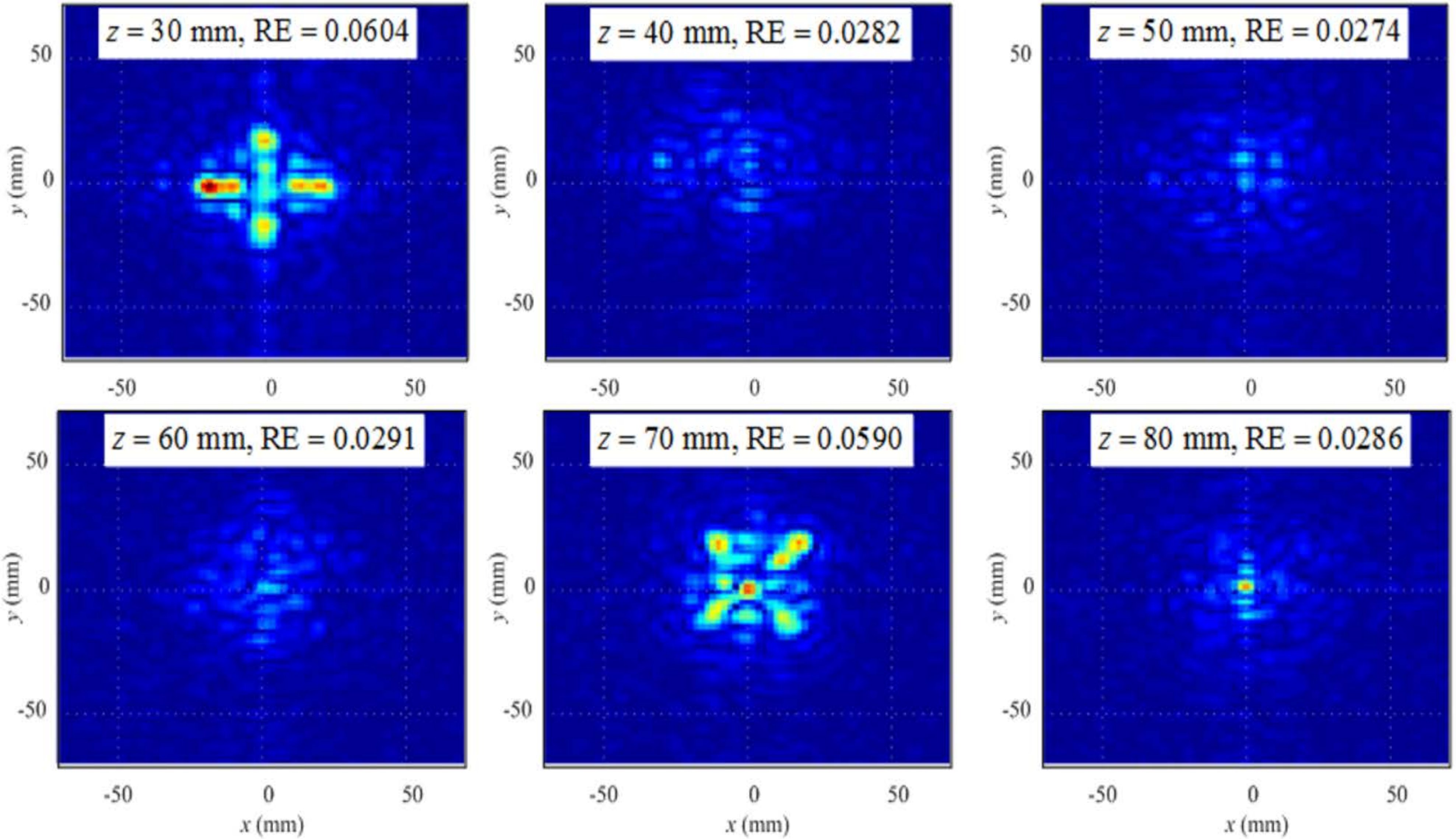
METALLIC TARGETS IN AIR – RESULTS WITH SIMULATED KERNEL

[Amineh *et al.*, *Trans. IM*, 2015]



METALLIC TARGETS IN AIR – RESULTS WITH MEASURED KERNEL

[Amineh *et al.*, *Trans. IM*, 2015]



ANALYTICAL, SIMULATED & MEASURED RESOLVENT KERNELS

- analytical and simulated kernels of the data equation are often inadequate for measurements in the antenna near zone
- measured kernels provide accurate system specific data equation for the reconstruction process
- **we discuss how to obtain the data-equation kernel through measurements in our next lecture**

- the forward-model choice is a compromise between two opposing requirements: ***speed*** and ***accuracy***

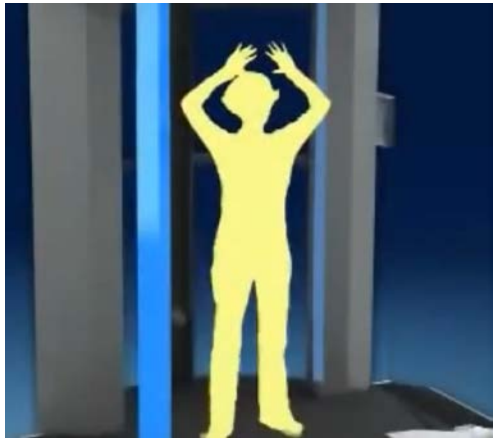
Example 1: whole-body imagers for concealed weapon detection

➢ we can get away with analytical kernels in the data equation

[Sheen *et al*., *Appl. Opt*., 2010]

plane-wave kernel approximation for reflection data:

$$\mathcal{K}(x', y', z'; x, y, z; \omega) = \overline{\mathbf{E}}^{\text{inc}}_{\text{Rx}} \cdot \underbrace{\overline{\mathbf{E}}^{\text{inc}}_{\text{Rx}}}_{\overline{\mathbf{E}}^{\text{inc}}_{\text{Tx}} = \overline{\mathbf{E}}^{\text{inc}}_{\text{Rx}}} \sim e^{-\text{i}2k_{\text{b}}\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$$

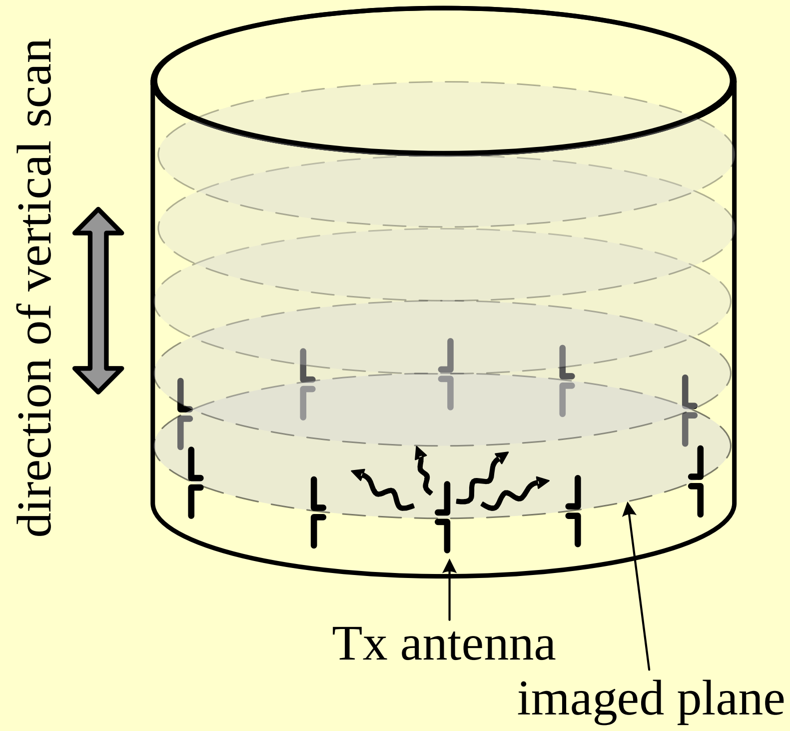


[video credit: https://www.youtube.com/watch?v=_YZEa1hiGO0]

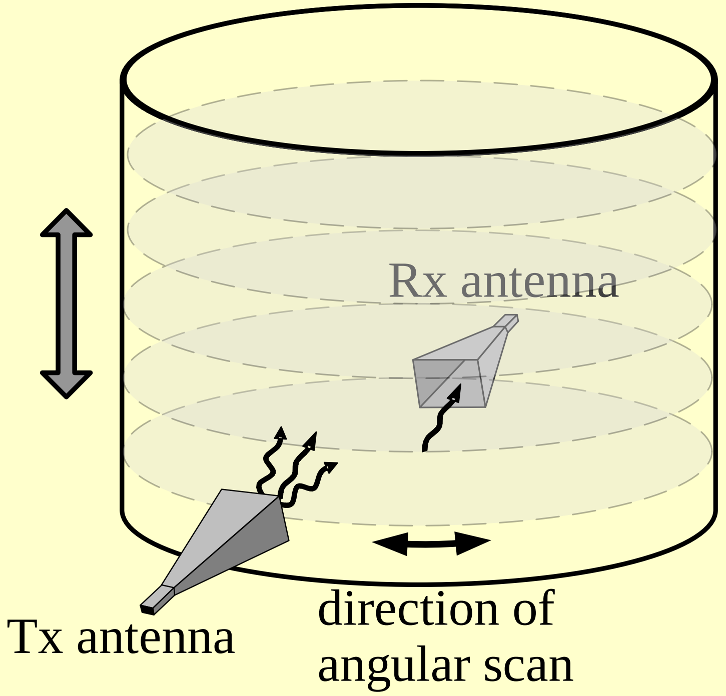
➢ we assume that Born's approximation of the total internal field satisfies the state equation – no need to solve the state equation

$$\overline{\mathbf{E}}^{\text{inc}}_{\text{Tx}} = \overline{\mathbf{E}}^{\text{inc}}_{\text{Rx}} \approx \overline{\mathbf{E}}_{\text{Tx}}$$

- tomography is an imaging procedure where a 3-D image of an object is obtained by a series of 2-D image reconstructions (one slice at a time)



electronically switched angular scan with mechanical scan in the vertical direction



mechanical scans in both the angular and the vertical directions

APPROXIMATIONS IN THE FORWARD MODEL OF IMAGING: TOMOGRAPHY – 2

- microwave tomography is common in tissue imaging where Born's approximation of the internal field is inaccurate and the ***state equation must be solved***
- it achieves better reconstruction speed by solving many 2-D inversions instead of one 3-D inversion
- it solves ***iteratively*** both the data and state equations (using EM simulations) where hundreds of simulations may be required – EM simulation speed is critical!
  - ➢ arithmetic-operation count for linear systems of equations: $O(N^3)$, sparse $O(N \log_{10} N)$
- microwave tomography assumes a ***plane of field symmetry at the imaged slice***

$$\boxed{\frac{\partial \mathbf{E}}{\partial z} = 0}$$

  - ➢ this amounts to TM_z field

$$E_x = E_y = 0,\ E_z \neq 0$$
$$\mathrm{i}\omega\mu_{\mathrm{b}} H_x = -\partial E_z / \partial y$$
$$\mathrm{i}\omega\mu_{\mathrm{b}} H_y = \partial E_z / \partial x$$

⇨

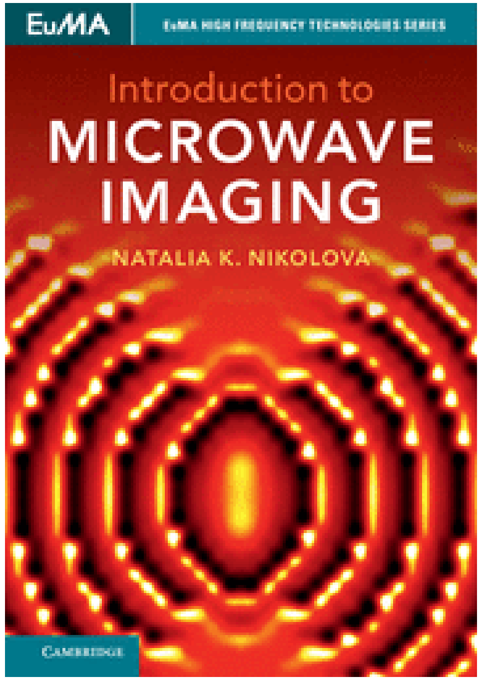
2-D scalar problem!

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \omega^2 \mu_{\mathrm{b}} \varepsilon_{\mathrm{s}} E_z = 0,\ z = const.$$

APPROXIMATIONS IN THE FORWARD MODEL OF IMAGING: TOMOGRAPHY – 3

- to achieve the desired symmetry, careful design of the antennas and the imaging setup is required
- symmetry condition holds strictly only in incident-field measurements (RO data)
- sources of error: OUT corrupts the symmetry assumption

SUMMARY OF DAY ONE



- forward models are an essential component of the imaging process – they reflect our understanding of the relationship between measured data and reconstructed EM properties
- we need 2 forward models
  - ➢ data equations: relate measured data to contrast
  - ➢ state equations: relate total internal field to contrast
- there are 2 approximations that we can use to linearize the data equation: Born's and Rytov's approximations
- for best accuracy, the data equation must be expressed in terms of the measured data (e.g., *S*-parameters) instead of the **E**-field
- analytical representations of the total internal field are strictly limited to far-zone measurements in uniform reflection-free environment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THANK YOU!