Lecture 6: Friis Transmission Equation and Radar Range Equation

(*Friis equation. EIRP. Maximum range of a wireless link. Radar cross section. Radar equation. Maximum range of a radar.*)

1. Friis Transmission Equation

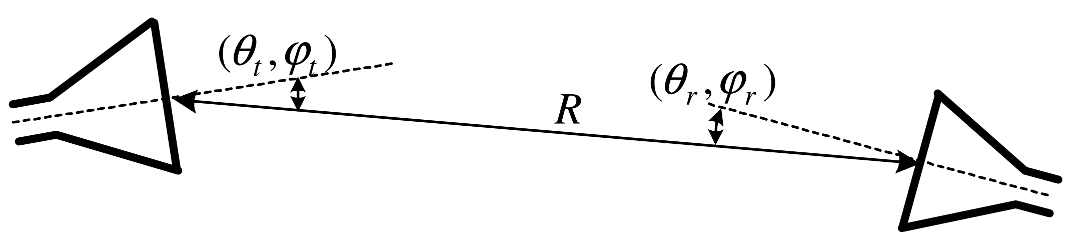
Friis transmission equation is essential in the analysis and design of wireless communication systems. It relates the power fed to the transmitting (Tx) antenna and the power received by the receiving (Rx) antenna when the two antennas are separated by a sufficiently large distance ($R >> 2D_{\max}^2 / \lambda$), i.e., they are in each other's far zones. We derive the Friis equation next.

A Tx antenna produces power density $W_t(\theta_t, \varphi_t)$ in the direction (θ_t, φ_t). This power density depends on the Tx antenna gain in the given direction $G(\theta_t, \varphi_t)$, on the power of the transmitter P_t fed to it, and on the distance R between the antenna and the observation point as

$$W_t = \frac{P_t}{4\pi R^2} G_t(\theta_t, \varphi_t) = \frac{P_t}{4\pi R^2} e_t D_t(\theta_t, \varphi_t). \tag{6.1}$$

Here, e_t denotes the radiation efficiency of the Tx antenna and D_t is its directivity in the direction (θ_t, φ_t). The power P_r at the terminals of the receiving antenna can be expressed via its effective area A_r and W_t:

$$P_r = A_r W_t. \tag{6.2}$$



To include polarization efficiency and dissipation in the receiving antenna, we add the radiation efficiency of the receiving antenna e_r and the PLF:

$$P_r = e_r \cdot \text{PLF} \cdot A_r W_t = A_r W_t e_r \, |\hat{\mathbf{p}}_t^* \cdot \hat{\mathbf{p}}_r|^2, \tag{6.3}$$

$$\Rightarrow P_r = \underbrace{D_r(\theta_r, \varphi_r) \cdot \frac{\lambda^2}{4\pi}}_{A_r} \cdot W_t e_r \, |\hat{\mathbf{p}}_t^* \cdot \hat{\mathbf{p}}_r|^2. \tag{6.4}$$

Here, D_r is the directivity or the receiving antenna. In the PLF, the polarization vectors of the Tx and receiving antennas, $\hat{\mathbf{p}}_t$ and $\hat{\mathbf{p}}_r$, are evaluated in their respective coordinate systems; therefore, one of them has to be conjugated.

The signal is incident upon the receiving antenna from a direction (θ_r, φ_r), which is defined in the coordinate system of the receiving antenna:

$$\Rightarrow P_r = D_r(\theta_r, \varphi_r) \cdot \frac{\lambda^2}{4\pi} \cdot \underbrace{\frac{P_t}{4\pi R^2} e_t D_t(\theta_t, \varphi_t)}_{W_t} \cdot e_r \, |\hat{\mathbf{p}}_t^* \cdot \hat{\mathbf{p}}_r|^2 . \tag{6.5}$$

The ratio of the received to the transmitted power is obtained as

$$\frac{P_r}{P_t} = e_t e_r \, |\hat{\mathbf{p}}_t^* \cdot \hat{\mathbf{p}}_r|^2 \left(\frac{\lambda}{4\pi R} \right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r) . \tag{6.6}$$

If the impedance-mismatch loss factor is included in both the receiving and the Tx antenna systems, the above ratio becomes

$$\frac{P_r}{P_t} = (1 - |\Gamma_t|^2)(1 - |\Gamma_r|^2) e_t e_r \, |\hat{\mathbf{p}}_t^* \cdot \hat{\mathbf{p}}_r|^2 \left(\frac{\lambda}{4\pi R} \right)^2 D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r) . \tag{6.7}$$

The above equations are variations of Friis' transmission equation, which is widely used in the design of wireless systems as well as in antenna measurements.

For the case of impedance-matched and polarization-matched Tx and receiving antennas, Friis equation reduces to

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R} \right)^2 G_t(\theta_t, \varphi_t) G_r(\theta_r, \varphi_r) . \tag{6.8}$$

The factor $(\lambda / 4\pi R)^2$ is called the ***free-space loss factor***. It reflects two effects: 1) the decrease in the power density due to the spherical spread of the wave through the term $1/(4\pi R^2)$, and 2) the effective aperture dependence on the wavelength as $\lambda^2 / (4\pi)$.

2. Effective Isotropically Radiated Power (EIRP)

From the Friis equation (6.8), it is seen that to estimate the received power P_r we need the product of the transmitter power P_t and the Tx antenna gain G_t. If the transmission line introduces losses in addition to those of the antenna system, these need to be accounted for. This is why often a transmission system is characterized by its ***effective isotropically radiated power (EIRP)***:

$$EIRP = P_t G_t e_{\mathrm{TL}}, \text{ W} \tag{6.9}$$

where e_{TL} is the loss efficiency of the transmission line connecting the transmitter to the antenna. Usually, the EIRP is given in dB, in which case (6.9) becomes

$$EIRP_{dB} = P_{t dB} + G_{t dBi} + e_{TL,dB}. \quad (6.10)$$

Bearing in mind that $P_{in,t} = e_{TL} P_t$ and $G_t = 4\pi U_{max,t} / P_{in,t}$, the EIRP from (6.9) can also be written as

$$EIRP = 4\pi U_{max,t}, \text{ W} \quad (6.11)$$

where $U_{max,t}$ is the maximum transmitted radiation intensity. It is now clear that the ***EIRP is a fictitious amount of power that an isotropic radiator would have to emit in order to produce the peak power density observed in the direction of the maximum radiation***. As such, and as evident from (6.9), the EIRP is greater than the actual power an antenna needs in order to achieve a given amount of radiation intensity in its direction of maximum radiation.

The EIRP is often used to specify the radiation limits on wireless communication or radar devices in EM interference regulations.

3. Maximum Range of a Wireless Link

Friis' transmission equation is frequently used to calculate the ***maximum range*** at which a wireless link can operate. For that, we need to know the nominal power of the transmitter P_t, all the parameters of the Tx and receiving antenna systems (polarization, gain, losses, impedance mismatch), and the minimum power at which the receiver can operate reliably $P_{r\,min}$. Then,

$$R_{max}^2 = (1-|\Gamma_t|^2)(1-|\Gamma_r|^2) e_t e_r |\hat{\mathbf{p}}_t^* \cdot \hat{\mathbf{p}}_r|^2 \left(\frac{\lambda}{4\pi}\right)^2 \left(\frac{P_t}{P_{r\,min}}\right) D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r). \quad (6.12)$$

The minimum power $P_{r\,min}$ at which the receiver can operate reliably is determined by the minimum signal-to-noise ratio (SNR) at which information can be recovered. In turn, the SNR depends critically on the level of noise present in the wireless channel. There are various sources of noise, but we are concerned with the noise of the antenna itself. This topic is considered in the next lecture.

4. Radar Cross-section (RCS) or Echo Area

The RCS is a far-field characteristic of a radar target, which creates an echo by scattering (reflecting) the radar EM wave.

The RCS of a target σ is the equivalent area capturing that amount of power, which, when scattered isotropically, produces at the receiver power-flux density, which is equal to that produced by the actual target:

$$\sigma = \lim_{R_{\mathrm{Rx}} \to \infty} \left[4\pi R_{\mathrm{Rx}}^2 \frac{W_s}{W_i} \right], \text{ m}^2. \tag{6.13}$$

Here,

R_{Rx} is the distance from the target to the Rx antenna, m;
W_i is the incident power density, W/m^2;
W_s is the scattered power density at the receiver, W/m^2.

To understand better the above definition, we can re-write (6.13) in an equivalent form:

$$\frac{\sigma W_i}{4\pi R_{\mathrm{Rx}}^2} = W_s \sim \frac{1}{R_{\mathrm{Rx}}^2}, \ R_{\mathrm{Rx}} \to \infty. \tag{6.14}$$

The product σW_i represents the equivalent intercepted power, which is assumed to be scattered (re-radiated) isotropically to create a fictitious spherical wave, the power density W_s of which is received by the radar. Note that W_s decreases with distance from target as $1/R_{\mathrm{Rx}}^2$ in the far zone. The quantity σW_i is independent of R_{Rx} because σ describes the target itself whereas W_i describes the incident-wave power flux density at the target, which depends only on the distance to the Tx antenna, not the Rx one. W_s is the scattered power density produced by the scatterer (the target) and measured by the radar's Rx antenna.

In general, the RCS differs from any of the physical cross-sections of the actual scatterer. However, it is representative of the reflection properties of the target. It depends very much on the angle of incidence, on the angle of observation, on the shape and size of the scatterer, on the EM properties of the materials that it is built of, and on the wavelength. The RCS of targets is similar to the concept of effective aperture of antennas. A radar targets can be viewed as a secondary source (radiator), which gets its power from the illumination due to the radar's Tx antenna.

Large RCSs result from large metal content in the structure of the object (e.g., trucks and jumbo jet airliners have large RCS, $\sigma > 100$ m^2). The RCS increases also due to sharp metallic or dielectric edges and corners. The reduction of the RCS is desired for stealth military aircraft (or ships) meant to be invisible to radars. This is achieved by careful *shaping* and *coating* (with special materials) of the outer surface of the airplane. The materials are mostly designed to absorb EM waves at the radar frequencies (usually S and X bands). Layered structures can also cancel the backscatter in a particular bandwidth. Shaping aims mostly at directing the backscattered wave at a direction different from the direction of incidence. Thus, in the case of a monostatic radar system, the scattered wave is directed away from the radar. The *Stealth* aircraft has RCS smaller than 10^{-4} m^2, which makes it comparable to the RCS of a penny.

5. Radar Range Equation

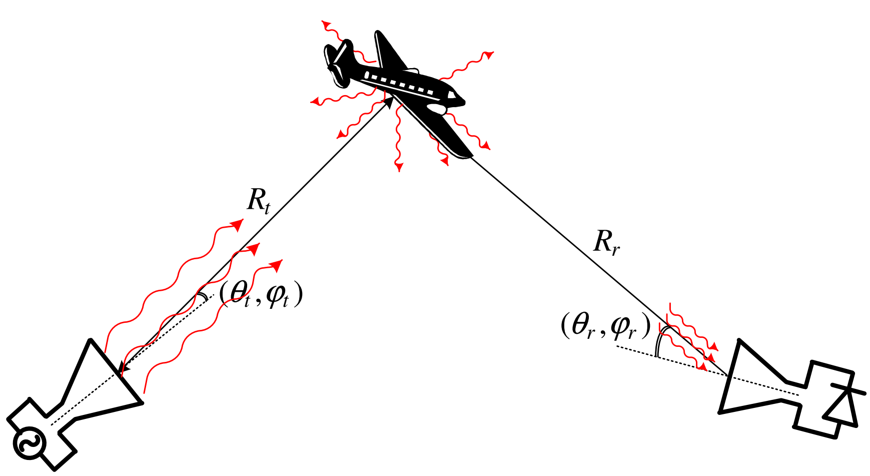
The radar range equation (RRE) gives the ratio of the transmitted power (fed to the Tx antenna) to the received power, after it has been scattered (re-radiated) by a target of RCS σ.

In general, the radar's transmitting (Tx) and receiving (Rx) antennas may be located at different positions as shown in the figure below. This is the ***bistatic radar*** scenario.

Often, one antenna is used to both transmit and to receive. This case is referred to as ***monostatic radar***, and the signals monostatic radars work with are referred to as ***backscattering***. Bear in mind that the RCS of a target may vary considerably as the location of the Tx and Rx antennas change.

Let us express the power-flux density of the Tx wave at the target as

$$W_t = \frac{P_t G_t(\theta_t, \varphi_t)}{4\pi R_t^2} = \frac{P_t e_t D_t(\theta_t, \varphi_t)}{4\pi R_t^2}, \text{ W/m}^2. \tag{6.15}$$



The target is represented by its RCS σ, which is used to calculate the captured power $P_c = \sigma W_t$ (W), which, when scattered isotropically, gives the power density measured by the receiving antenna. The density of the scattered power at the Rx antenna is

$$W_r = \frac{P_c}{4\pi R_r^2} = \frac{\sigma W_t}{4\pi R_r^2} = e_t \sigma \frac{P_t D_t(\theta_t, \varphi_t)}{(4\pi R_t R_r)^2}. \tag{6.16}$$

The power delivered to the receiver is then

$$P_r = e_r \cdot A_r \cdot W_r = e_r \cdot \left(\frac{\lambda^2}{4\pi}\right) D_r(\theta_r, \varphi_r) \cdot e_t \sigma \frac{P_t D_t(\theta_t, \varphi_t)}{(4\pi R_t R_r)^2}. \tag{6.17}$$

Re-arranging and including impedance mismatch losses as well as polarization losses, yields the complete radar range equation:

$$\frac{P_r}{P_t} = (1-|\Gamma_t|^2)(1-|\Gamma_r|^2)\,|\hat{\mathbf{p}}_t^* \cdot \hat{\mathbf{p}}_r|^2\, \sigma \left(\frac{\lambda}{4\pi R_t R_r}\right)^2 \frac{G_t(\theta_t, \varphi_t) G_r(\theta_r, \varphi_r)}{4\pi}. \tag{6.18}$$

For polarization matched antennas aligned for maximum directional radiation and reception,

$$\frac{P_r}{P_t} = \sigma \left(\frac{\lambda}{4\pi R_t R_r}\right)^2 \frac{G_{t0} G_{r0}}{4\pi}. \tag{6.19}$$

The radar range equation is used to calculate the ***maximum range of a radar system*** if the RCSs of targets are approximately known. Conversely, in a fixed range scenario, where R_t and R_r are known, (6.19) is used to calculate the ***RCS σ of a target*** from the known transmitted power P_t and the measured scattered-signal power P_r.

For radar range calculations, as in the case of Friis' transmission equation, in addition to the RCS, we need to know the parameters of the Tx and Rx antennas, as well as the minimum received power at which the receiver operates reliably. Then,

$$(R_t R_r)^2_{\max} = e_t e_r (1-|\Gamma_t|^2)(1-|\Gamma_r|^2)|\hat{\boldsymbol{\rho}}_t^* \cdot \hat{\boldsymbol{\rho}}_r|^2 \cdot \frac{P_t}{P_{r\min}} \sigma \left(\frac{\lambda}{4\pi}\right)^2 \frac{D_t(\theta_t, \varphi_t) D_r(\theta_r, \varphi_r)}{4\pi}. \tag{6.20}$$

Finally, we note that the above RCS and radar-range calculations are only basic. The subjects of radar system design and EM scattering are huge research areas themselves and are not going to be considered in detail in this course.