1. Introduction

Microstrip antennas (MSA) received considerable attention in the 1970’s, although the first designs and theoretical models appeared in the 1950’s. They are suitable for many mobile applications: handheld devices, aircraft, satellite, missile, etc. The MSA are low profile, mechanically robust, inexpensive to manufacture, compatible with MMIC designs and relatively light and compact. They are quite versatile in terms of resonant frequencies, polarization, pattern and impedance. They allow for additional tuning elements like pins or varactor diodes between the patch and the ground plane.

Some of the limitations and disadvantages of the MSA are:

- relatively low efficiency (due to dielectric and conductor losses)
- low power
- spurious feed radiation (surface waves, strips, etc.)
- narrow frequency bandwidth (at most a couple of percent)
- relatively high level of cross polarization radiation

MSA are applicable in the GHz range ($f > 0.5$ GHz). For lower frequencies their dimensions become too large.
2. Construction and Geometry

Generally the MSA are thin metallic patches of various shapes etched on dielectric substrates of thickness $h$, which usually is from $0.003 \lambda_0$ to $0.05 \lambda_0$. The substrate is usually grounded at the opposite side.

The dimensions of the patch are usually in the range from $\lambda_0/3$ to $\lambda_0/2$. The dielectric constant of the substrate $\varepsilon_r$ is usually in the range from 2.2 to 12. The most common designs use relatively thick substrates with lower $\varepsilon_r$ because they provide better efficiency and larger bandwidth. On the other hand, this implies larger dimensions of the antennas. The choice of the substrate is limited by the RF or microwave circuit coupled to the antenna, which has to be built on the same board. The microwave circuit together with the antenna is usually manufactured by photo-etching technology.
Types of microstrip radiators:
(a) single radiating patches

(b) single slot radiator

The feeding microstrip line is beneath (etched on the other side of the substrate) – see dash-line.
(c) microstrip traveling wave antennas (MTWA)

Comb MTWA

Meander Line Type MTWA

Rectangular Loop Type MTWA

Franklin – Type MTWA

The open end of the long TEM line is terminated in a matched resistive load.
(d) microstrip antenna arrays
3. Feeding Methods

1) Microstrip feed – easy to fabricate, simple to match by controlling the inset position and relatively simple to model. However, as the substrate thickness increases, surface waves and spurious feed radiation increase.

2) Coaxial probe feed – easy to fabricate, low spurious radiation; difficult to model accurately; narrow bandwidth of impedance matching.
3) Aperture coupling (no contact), microstrip feed line and radiating patch are on both sides of the ground plane, the coupling aperture is in the ground plane – low spurious radiation, easy to model; difficult to match, narrow bandwidth.

4) Proximity coupling (no contact), microstrip feed line and radiating patch are on the same side of the ground plane – largest bandwidth (up to 13%), relatively simple to model, has low spurious radiation.
More examples of microstrip and coaxial probe feeds:
4. Surface Waves

Surface waves can be excited at the dielectric-to-air interface. They give rise to end-fire radiation. In addition, they can lead to unwanted coupling between array elements. The phase velocity of the surface waves is strongly dependent on the dielectric constant $\varepsilon_r$ and the thickness $h$ of the substrate. The excitation of surface waves in a dielectric slab backed by a ground plane has been well studied (Collin, *Field Theory of Guided Waves*). The lowest-order TM mode, TM$_0$, has no cut-off frequency. The cut-off frequencies for the higher-order modes (TM$_n$ and TE$_n$) are given by

\[ f_c^{(n)} = \frac{n \cdot c}{4h\sqrt{\varepsilon_r - 1}}, \quad n = 1, 2, \ldots, \]  

where $c$ is the speed of light in vacuum. The cut-off frequencies for the TE$_n$ modes are given by the odd $n = 1, 3, 5, \ldots$, and the cut-off frequencies for the TM$_n$ modes are given by the even $n$. For the TE$_1$ mode, the calculated values of $h / \lambda^{(1)}_c$ are $[\lambda^{(1)}_c = c / f_c^{(1)}, \quad h / \lambda^{(1)}_c = n / (4\sqrt{\varepsilon_r - 1})]$:

a) 0.217 for duroid ($\varepsilon_r = 2.32$),

b) 0.0833 for alumina ($\varepsilon_r = 10$).

Thus, the lowest-order TE$_1$ mode is excited at 41 GHz for 1.6 mm thick duroid substrate, and at about 39 GHz for 0.635 mm thick alumina substrate. The substrate thickness is chosen so that the ratio $h / \lambda_0$ is well below $h / \lambda^{(1)}_c$ ($\lambda_0$ is the free-space wavelength at the operating frequency), i.e., [3]

\[ h < \frac{c}{4f_u \sqrt{\varepsilon_r - 1}}, \]  

where $f_u$ is the highest frequency in the band of operation. Note that $h$ should be chosen as high as possible under the constraint of (2), so that maximum radiation efficiency is achieved. Also, $h$ has to conform to the commercially available substrates. Another practical formula for $h$ is given in [2]:

\[ h \leq \frac{0.3c}{2\pi f_u \sqrt{\varepsilon_r}}. \]  

The TM$_0$ mode has no cut-off frequency and is always present to some extent. The surface TM$_0$ wave excitation becomes appreciable when $h/\lambda > 0.09$ ($\varepsilon_r \approx 2.3$) and when $h/\lambda > 0.03$ ($\varepsilon_r \approx 10$). Generally, to suppress the TM$_0$ mode,
the dielectric constant should be lower and the substrate height should be smaller. Unfortunately, decreasing $\varepsilon_r$ increases the antenna size, while decreasing $h$ leads to smaller antenna efficiency and narrower frequency band.

5. Criteria for Substrate Selection

1) surface-wave excitation
2) dispersion of the dielectric constant and loss tangent of the substrate
3) copper loss
4) anisotropy in the substrate
5) effects of temperature, humidity, and aging
6) mechanical requirements: conformability, machinability, solderability, weight, elasticity, etc.
7) cost

The first 3 factors are of special concern in the millimeter-wave range ($f \geq 30$ GHz).

**Electrical Properties of Commonly Used Substrate Materials for Microstrip Antennas**

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric Constant</th>
<th>Loss Tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unreinforced PTFE, Cuflon</td>
<td>2.1</td>
<td>0.0004</td>
</tr>
<tr>
<td>Reinforced PTFE, RT Duroid 5880</td>
<td>2.20 (1.5%)</td>
<td>0.0009</td>
</tr>
<tr>
<td>Fused Quartz</td>
<td>3.78</td>
<td>0.0001</td>
</tr>
<tr>
<td>96% Alumina</td>
<td>9.40 (5%)</td>
<td>0.0010</td>
</tr>
<tr>
<td>99.5% Alumina</td>
<td>9.80 (5%)</td>
<td>0.0001</td>
</tr>
<tr>
<td>Sapphire</td>
<td>9.4, 1.6</td>
<td>0.0001</td>
</tr>
<tr>
<td>Semi-Insulating GaAs</td>
<td>12.9</td>
<td>0.0020</td>
</tr>
</tbody>
</table>
## Non-electrical Properties of Commonly Used Substrate Materials for Microstrip Antennas

<table>
<thead>
<tr>
<th>Properties</th>
<th>PTFE</th>
<th>Fused Quartz</th>
<th>Alumina</th>
<th>Sapphire</th>
<th>GaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>temperature range (°C)</td>
<td>-55 – 260</td>
<td>&lt; +1100</td>
<td>&lt; +1600</td>
<td>-24 – 370</td>
<td>-55 – 260</td>
</tr>
<tr>
<td>Thermal conductivity (W/cm·K)</td>
<td>0.0026</td>
<td>0.017</td>
<td>0.35 to 0.37</td>
<td>0.42</td>
<td>0.46</td>
</tr>
<tr>
<td>coefficient of thermal expansion (ppm/K)</td>
<td>16.0 to 108.0</td>
<td>0.55</td>
<td>6.30 to 6.40</td>
<td>6.00</td>
<td>5.70</td>
</tr>
<tr>
<td>Temperature coefficient of dielectric constant (ppm/K)</td>
<td>+350.0 to +480.0</td>
<td>+13.0</td>
<td>+136.0</td>
<td>+110 to +140</td>
<td>-</td>
</tr>
<tr>
<td>minimum thickness (mil)</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Machinability</td>
<td>Good</td>
<td>Very poor</td>
<td>Very poor</td>
<td>Poor</td>
<td>Poor</td>
</tr>
<tr>
<td>Solderability</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>Dimensional Stability</td>
<td>Poor for unreinforced, very good for others</td>
<td>Good</td>
<td>Excellent</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>Cost</td>
<td>Very low</td>
<td>High</td>
<td>Low</td>
<td>-</td>
<td>Very high</td>
</tr>
</tbody>
</table>
6. Dispersion Effects in the Substrate

The dependence of the dielectric constant $\varepsilon_r$ and the loss tangent on the frequency is referred to as frequency dispersion. For frequencies up to 100 GHz (the typical range for printed antennas is < 30 GHz), the dispersion of $\varepsilon_r$ is practically negligible. The losses, however, display noticeable changes with frequency. In general, the loss increases with frequency.

7. Dielectric Loss and Copper Loss

The loss in the feed lines and the patches themselves are usually computed with formulas, which were first derived for microstrip transmission lines, i.e., the patch is treated as a wide piece of a microstrip line.

a) Dielectric loss (in dB per unit length, length is in the units used for $\lambda_0$)

$$\alpha_d = 27.3 \cdot \frac{\varepsilon_r}{\sqrt{\varepsilon_{\text{reff}}(f)}} \cdot \frac{\left[\varepsilon_{\text{reff}}(f) - 1\right]}{(\varepsilon_r - 1)} \cdot \frac{\tan \delta}{\lambda_0}$$  \hspace{1cm} (4)

b) Copper loss (in dB per unit length)

$$\alpha_c = \left\{ \begin{array}{ll}
1.38 \cdot \frac{R_s'}{hZ_0} \cdot \frac{32 - \left(\frac{W'}{h}\right)^2}{32 + \left(\frac{W'}{h}\right)^2} \cdot \Lambda, & \text{for } \frac{W}{h} \leq 1 \\
6.1 \times 10^{-5} \frac{R_s'Z_0\varepsilon_{\text{reff}}(f)}{h} \cdot \left[ \frac{W'}{h} + \frac{0.667 \frac{W'}{h}}{\frac{W'}{h} + 1.444} \right] \cdot \Lambda, & \text{for } \frac{W}{h} \geq 1
\end{array} \right.$$  \hspace{1cm} (5)
In the above equations:

- $\varepsilon_{\text{eff}}(f)$ is the effective dielectric constant (generally, dispersive). Its quasi-static (low frequency) expression [2] is

$$\varepsilon_{\text{eff}}(0) = \begin{cases} \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \cdot \left(1 + 12 \frac{h}{W}\right)^{-1/2}, & \text{for } W / h > 1 \\ \frac{\varepsilon_r + 1}{2} - \frac{\varepsilon_r - 1}{2} \cdot \left(1 + 12 \frac{h}{W}\right)^{-1/2} + 0.04 \left(1 - \frac{W}{h}\right)^2, & \text{for } W / h \leq 1 \end{cases}$$

(6)

Alternative expression for the quasi-static approximation of $\varepsilon_{\text{eff}}$ can be found in [5].

The quasi-static expressions need a dispersion correction for frequencies higher than 8 GHz. One possible correction is based on an empirical formula for the dispersive phase velocity in a microstrip line [5]. We first compute a normalized frequency (normalized with respect to the cut-off of the TE$_1$ mode):

$$\bar{f} = \frac{f}{f_c^{(1)}} = \frac{4h \sqrt{\varepsilon_r - 1}}{\lambda_0}.$$  

(7)

Then, the dispersive phase velocity is calculated as

$$v_p = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_{\text{eff}}(0)}} \cdot \frac{\bar{f}^2 \sqrt{\varepsilon_{\text{eff}}(0)} + \sqrt{\varepsilon_r}}{\bar{f}^2 + 1}.$$  

(8)

Finally,

$$\varepsilon_{\text{eff}}(f) = \left(c / v_p \right)^2.$$  

(9)

For alternative formulas, refer to [5].

- $Z_0$ is the characteristic impedance of the microstrip line (generally, dispersive):
\[ Z_0 = \begin{cases} 
\frac{120\pi \sqrt{\varepsilon_{\text{ref}}}}{W/h + 1.393 + 0.667 \ln \left( \frac{W}{h} + 1.444 \right)}, & \text{for } \frac{W}{h} \geq 1 \\
\frac{60}{\sqrt{\varepsilon_{\text{ref}}}} \cdot \ln \left( \frac{8h}{W} + 0.25 \frac{W}{h} \right), & \text{for } \frac{W}{h} \leq 1
\end{cases} \]  
(10)

- \( \Lambda \) is a constant dependent on the strip thickness \( t \):

\[ \Lambda = \begin{cases} 
1 + \frac{h}{W'} \left[ 1 + \frac{1.25t}{\pi W} + \frac{1.25}{\pi} \ln \left( \frac{4\pi W}{t} \right) \right], & \text{for } \frac{W}{h} \leq \frac{1}{2\pi} \\
1 + \frac{h}{W'} \left[ 1 - \frac{1.25t}{\pi W} + \frac{1.25}{\pi} \ln \left( \frac{2t}{t} \right) \right], & \text{for } \frac{W}{h} \geq \frac{1}{2\pi}
\end{cases} \]  
(11)

- \( W' \) is the effective strip width:

\[ \frac{W'}{h} = \begin{cases} 
\frac{W}{h} + \frac{1.25t}{\pi h} \left[ 1 + \ln \left( \frac{4\pi W}{t} \right) \right], & \text{for } \frac{W'}{h} \leq \frac{1}{2\pi} \\
\frac{W}{h} + \frac{1.25t}{\pi h} \left[ 1 + \ln \left( \frac{2h}{t} \right) \right], & \text{for } \frac{W'}{h} \geq \frac{1}{2\pi}
\end{cases} \]  
(12)

- \( R'_s \) is the effective surface resistance of the conductor:

\[ R'_s = R_s \left\{ 1 + \frac{2}{\pi} \arctan \left[ 1.4 \left( \frac{\Delta}{\delta} \right)^2 \right] \right\}, \Omega \]  
(13)

where \( R_s = \sqrt{\pi f \mu / \sigma} \) is the high-frequency surface resistance of the conductor. \( R_s \) relates to the skin-depth \( \delta \) as \( R_s = (\delta \sigma)^{-1} \). For a uniform surface current distribution over a conducting rod of length \( l \) and perimeter of its cross-section \( P \), the resultant resistance is

\[ R_{hf} = R_s \cdot l / P, \Omega. \]  
(14)

Finally, the total loss is the sum of the conduction and dielectric losses:

\[ \alpha_t = \alpha_d + \alpha_c. \]  
(15)
### Surface Resistance and Skin-depth of Commonly Used Conductors

<table>
<thead>
<tr>
<th>Metal</th>
<th>Rs [Ohm/square x10^{-7}]</th>
<th>Skin-depth at 2 GHz [µm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>2.5</td>
<td>6.1x10^{-7}S/m</td>
</tr>
<tr>
<td>Copper</td>
<td>2.6</td>
<td>5.8x10^{-7}S/m</td>
</tr>
<tr>
<td>Gold</td>
<td>3.0</td>
<td>4.1x10^{-7}S/m</td>
</tr>
<tr>
<td>Aluminum</td>
<td>3.3</td>
<td>3.5 x10^{-7}S/m</td>
</tr>
</tbody>
</table>

### Some References


