Chapter5 Solution

1.

$$s(t) = A \sum_{-\infty}^{\infty} J_n(mf) \cos(\omega_c + n\omega_0)t$$

Set $J_0(mf) = 0$

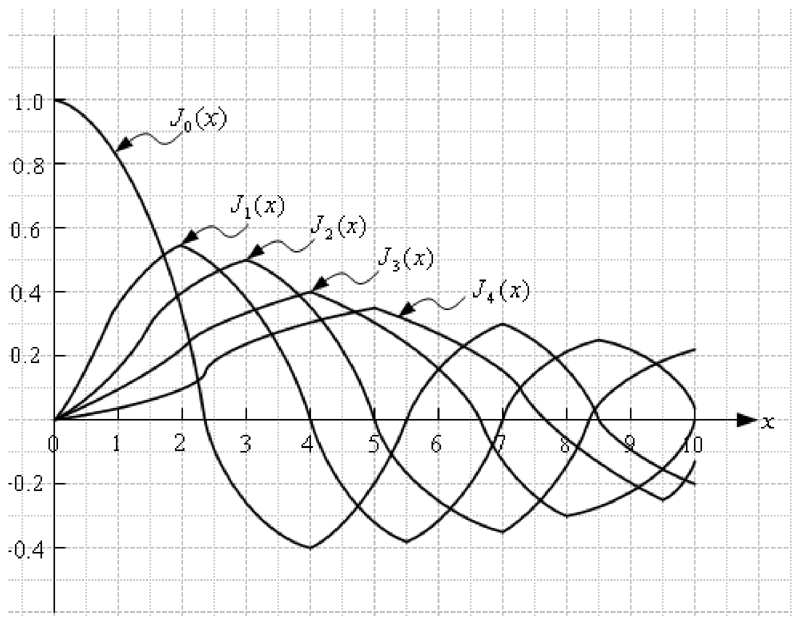


Figure 1:

The first two values of mf for no carriers power are at:

$$\begin{cases} mf \simeq 2.55 \\ mf \simeq 5.18 \end{cases}$$

2. (a) $J_1(mf) = 0$ when $mf \simeq 3.8$

$$J_0(mf \simeq 3.8) \simeq -0.4$$

The average carrier power is:

$$P_c = \frac{J_0^2(3.8)}{J_0^2(0)} \times 100W = 16W$$

(b) The average power in all the remaining sideband is:

$$P_s = P_T - P_c = 100W - 16W = 84W$$

(c)

$$J_2(mf = 3.8) \simeq 0.41$$

Therefore The average power in the second order sidebands is

$$2 \times \frac{J_2^2(3.8)}{J_0^2(0)} \times 100W \simeq 34W$$

3. (a) The unit square pulse has a transform of the form

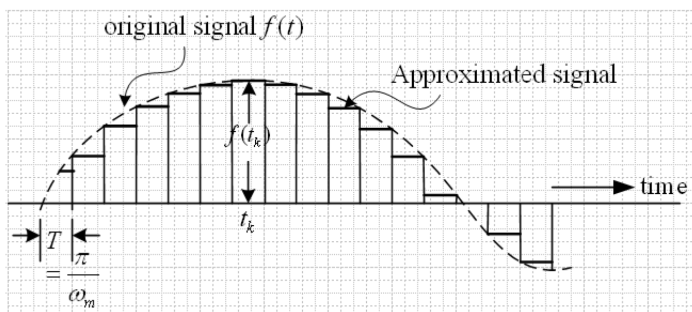
$$[\sin(\frac{\omega}{2\omega_m})\pi]/[\frac{\omega}{2\omega_m}\pi]$$

The sinusoidal wave has a Fourier transform of

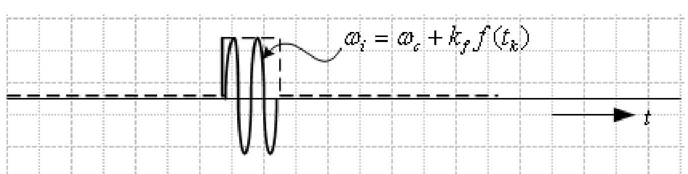
$$\delta(\omega - \omega_i) + \delta(\omega + \omega_i)$$

The multiplication of the two function in time domain is equivalent to the convolution of their transforms in frequency domain

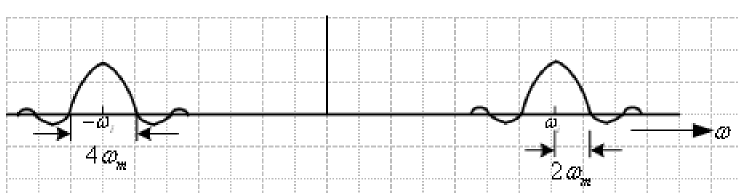
$$\left[\frac{\sin(\frac{\omega-\omega_i}{2\omega_m})\pi}{(\frac{\omega-\omega_i}{2\omega_m})\pi} + \frac{\sin(\frac{\omega+\omega_i}{2\omega_m})\pi}{(\frac{\omega+\omega_i}{2\omega_m})\pi}\right]$$

2a.png


(a)

2b.png


(b)

2c.png


(c)

The spectrum lies within $\omega_i - 2\omega_m$ to $\omega_i + 2\omega_m$ (a)

$$\omega_i = \omega_c + k_f f(t_k) \text{ at } t = t_k$$

The highest and lowest instantaneous frequency is

$$\omega_i = \omega_c \pm k_f|f(t)|_{max}(b)$$

Substitute (b) to (a), the spectrum lies within

$$\omega_c - k_f|f(t)|_{max} - 2\omega_m \rightarrow \omega_c + k_f|f(t)|_{max} + 2\omega_m$$

$$\begin{aligned}\Rightarrow W &\simeq 2k_f|f(t)|_{max} + 4\omega_m \\ &= 2(\Delta\omega + 2\omega_m)\end{aligned}$$

(b) Bandwidth

$$W \simeq 2(\Delta\omega + 2\omega_m) = 2 \times (5KHz + 2 \times 2KHz) = 18KHz$$

$$\begin{aligned}\Delta\omega_1 &= k_f|f(t)|_{max1} \\ \Delta\omega_2 &= k_f|f(t)|_{max2}\end{aligned}$$

$$\frac{\Delta\omega_1}{\Delta\omega_2} = \frac{|f(t)|_{max1}}{|f(t)|_{max2}} \Rightarrow \Delta\omega_2 = \frac{|f(t)|_{max1}}{|f(t)|_{max2}} \cdot \Delta\omega_1 = 3\Delta\omega_1$$

Therefore the new bandwidth is

$$\omega \simeq 2(15KHz + 2 \times 1KHz) = 34KHz$$

4. (a)

$$P = \frac{A^2}{2} = 50Hz$$

(b)

$$\begin{aligned}\omega_i &= 2\pi \times 10^6 + 2000\pi \times 0.1 \cos 2000\pi t \\ \Delta\omega &= 200\pi \\ \Delta f &= 100Hz\end{aligned}$$

(c) Since $B = 1000 > \Delta f$ this is a narrowband case and the bandwidth B_{EM} is

$$B_{EM} = 2(\Delta f + B) = 2(100 + 1000) = 2200Hz$$

5. (a)

$$\begin{aligned}\omega_0 = \frac{1}{\sqrt{LC}} &= \frac{[v_b + v_0 + f(t)]^{1/4}}{\sqrt{LK_0}} \\ &= \frac{\left[1 + \frac{f(t)}{v_b+v_0}\right]^{1/4} \cdot (v_b + v_0)^{1/4}}{\sqrt{LK_0}} \\ &= \sqrt{\frac{\sqrt{v_b + v_0}}{LK_0}} \cdot \left[1 + \frac{f(t)}{v_b + v_0}\right]^{1/4}\end{aligned}$$

(b) By direct calculation $\{1+[f(t)/(v_b+v_0)]\}^{1/4}$ departs from the linear approximation $1+[f(t)/4(v_b+v_0)]$ by 1% when $-0.286 \leq f(t)/(v_b+v_0) \leq 0.376$
Hence

$$\frac{|f(t)|_{max}}{v_b+v_0} \leq 0.286$$

6. (a)

$$|H(f)| = \left|\frac{1}{1+\frac{jf}{f_2}}\right| = \frac{1}{\sqrt{1+(\frac{f}{f_2})^2}} = \frac{1}{\sqrt{1+u^2}}\Bigg|_{u=\frac{f}{f_2}}$$

$$\frac{d^2|H(f)|}{du^2} = -\frac{d}{du}\frac{u}{(1+u^2)^{3/2}} = -\frac{(1+u^2)^{3/2}-3u^2\sqrt{1+u^2}}{(1+u^2)^3} = 0$$

Therefore

$$u = \frac{f_c}{f_2} = \frac{1}{\sqrt{2}}$$

$$f_2 = 1.414 f_c$$

(b)

$$\frac{d|H(f)|}{dt}\Bigg|_{f=f_c} = -\frac{\frac{f_c}{f_2}/f_2}{[1+(f_c/f_2)^2]^{3/2}}$$

$$= \frac{0.5/(\sqrt{2}\times 10^6)}{(1+0.5)^{3/2}}$$

$$= 1.92\times 10^{-7} volts/Hz$$

7.

$$S_v(\mu) = \alpha p(v)$$

Where $\mu = f - f_c$ and α is a constant of proportionality.
But

$$\mu(t) = \frac{k_f v(t)}{2\pi} \Rightarrow v(t) = \frac{2\pi\mu(t)}{k_f}$$

Therefore

$$S_v(\mu) = \begin{cases} \frac{2\pi\alpha\mu}{k_f} e^{\frac{-(2\pi\mu)^2}{2k_f^2}} & \mu \geq 0 \\ 0 & \mu < 0 \end{cases}$$

Replacing μ by $f - f_c$

$$S_v(f) = \begin{cases} \frac{\pi\alpha(|f|-f_c)}{k_f} e^{\frac{-2\pi^2(|f|-f_c)^2}{k_f^2}} & |f| > f_c \\ 0 & |f| < f_c \end{cases}$$

To find α

$$\int_{-\infty}^{\infty} S_v(f)df = \frac{A^2}{2}$$

$$\begin{aligned}\int_{-\infty}^{\infty} S_v(f)df &= 2\alpha \int_{f_c}^{\infty} \frac{\pi(f-f_c)}{k_f} e^{\frac{-2\pi^2(f-f_c)^2}{k_f^2}} df \\ &= \frac{\alpha}{2\pi}\int_0^{\infty} \frac{x}{k_f} e^{\frac{-x^2}{2k_f^2}} dx \\ &= \frac{\alpha k_f}{2\pi} \\ &= \frac{A^2}{2}\end{aligned}$$

Where

$$\begin{aligned} x &= 2\pi(f-f_c) \\ df &= \frac{dx}{2\pi} \end{aligned}$$

$$\therefore \alpha = \frac{\pi A^2}{k_f}$$

Hence

$$S_v(f) = \begin{cases} \frac{\pi^2 A^2(|f|-f_c)}{k_f^2} e^{\frac{-2\pi^2(|f|-f_c)^2}{k_f^2}} & |f| \geq f_c \\ 0 & |f| < f_c \end{cases}$$

8.

$$S_v(\mu) = \alpha p(v)$$

Where $\mu = f - f_c$ and α is a constant of proportionality.
But

$$\mu(t) = \frac{k_f v(t)}{2\pi} \Rightarrow v(t) = \frac{2\pi\mu(t)}{k_f}$$

$$\begin{aligned} S_v(\mu) &= \frac{1}{2}\alpha e^{-|\frac{2\pi\mu(t)}{k_f}|} \\ S_v(f) &= \frac{\alpha}{4} e^{\frac{-2\pi|f-f_c|}{k_f}} + \frac{\alpha}{4} e^{\frac{-2\pi|f+f_c|}{k_f}} \end{aligned}$$

To find α

$$\int_{-\infty}^{\infty} S_v(f)df = \int_{-\infty}^{\infty} S_v(\mu)d\mu$$
$$= \alpha \int_0^{\infty} e^{\frac{-2\pi\mu}{k_f}} d\mu$$
$$= \frac{\alpha}{2\pi} \int_0^{\infty} e^{\frac{-x}{k_f}} dx$$
$$= \frac{\alpha k_f}{2\pi}$$
$$= \frac{A^2}{2}$$
$$\therefore \alpha = \frac{\pi A^2}{k_f}$$

$$S_v(f) = \frac{\pi A^2}{4k_f} e^{\frac{-2\pi|f-f_c|}{k_f}} + \frac{\pi A^2}{4k_f} e^{\frac{-2\pi|f+f_c|}{k_f}}$$

9. (a) Refer to figure 2

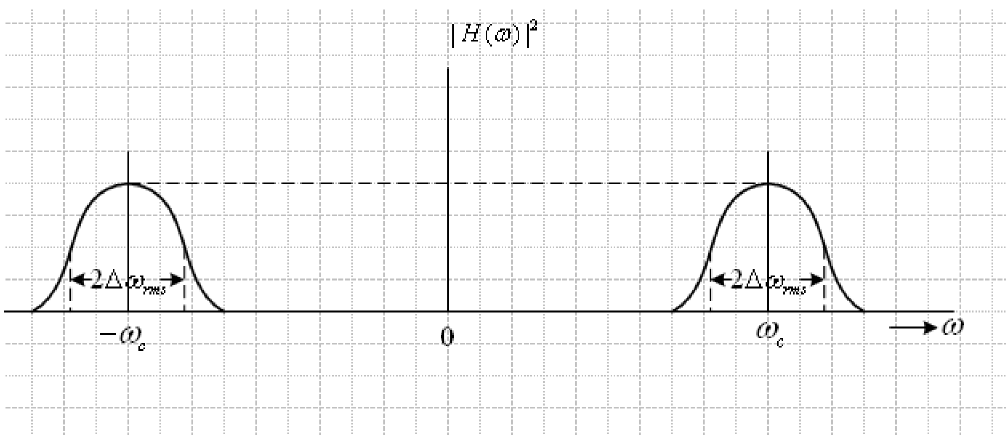


Figure 2:

(b) To find the 3-dB bandwidth of the filter, we use the equation

$$e^{-v_0^2/2B^2} = \frac{1}{2}$$
$$(v_0 = f_{3dB} - f_c)$$

Therefore

$$
\begin{aligned}
B_{3dB} &= 2V_0 \\
lne^{-v_0^2/2B^2} &= ln\frac{1}{2} \\
\frac{-v_0^2}{2B^2} &= ln\frac{1}{2} \\
-v_0^2 &= 2B^2 ln\frac{1}{2} \\
v_0 &= \sqrt{2ln2}B \\
2v_0 &= 2\sqrt{2ln2}B \\
B_{3dB} &= 2v_0
\end{aligned}
$$

(c) We want that the 3dB bandwidth B_{3dB} to pass 98% of the total signal power

$$
\begin{aligned}
\therefore B3dB &= 4.6\Delta f_{rms} \\
\therefore 2B\sqrt{2ln2} &= 4.6\Delta f_{rms} \\
B &= \frac{2.3}{\sqrt{2ln2}}\Delta f_{rms}
\end{aligned}
$$

(d)

$$
W = B_{3dB} = 2v_0 = 2B\sqrt{2ln2}
$$

10. The FM signal has two modulating baseband signals $v_1(t)$ and $v_2(t)$, Therefore the resulting FM signal is given by

$$
v(t) = A\cos[2\pi f_c t + k_1 \int v_1(t)dt + k_2 \int v_2(t)dt]
$$

Where $A = 2volts$ and $f_c = 1MHz$
frequency deviation Δf_1 for the first baseband signal is given by

$$
\begin{aligned}
\Delta f_1 &= \frac{k_1}{2\pi} v_1(t)|_{v_1=1volt} = \frac{k_1}{2\pi} = 3 \times 10^3 Hz \\
\therefore k_1 &= 2\pi \times 3 \times 10^3 = 6\pi \times 10^3 Hz/volt
\end{aligned}
$$

frequency deviation Δf_2 for the second baseband signal is given by

$$
\begin{aligned}
\Delta f_2 &= \frac{k_2}{2\pi} v_2(t)|_{v_2=1volt} = \frac{k_2}{2\pi} = -4 \times 10^3 Hz \\
\therefore k_2 &= -8\pi \times 10^3 Hz/volt
\end{aligned}
$$

$$
\begin{aligned}
\Delta f_(rms)^2 &= E\left[\left[\frac{1}{2\pi}k_1 v_1(t) + \frac{1}{2\pi}k_2 v_2(t)\right]^2\right] \\
&= \frac{1}{(2\pi)^2}\left[k_1^2 E[v_1^2(t)] + k_2^2 E[v_2^2(t)]\right]
\end{aligned}
$$

But

$$
\begin{aligned}
E[v_1^2(t)] &= E[v_2^2(t)] = 1volt^2 \\
\therefore \Delta f_{rms}^2 &= \frac{1}{4\pi^2}(k_1^2 + k_2^2) \\
&= \frac{1}{4\pi^2}(36\pi^2 \times 10^6 + 64\pi^2 \times 10^6) \\
&= 25 \times 10^6 Hz^2
\end{aligned}
$$

Also the spectral power density of an FM signal with gaussian baseband is given by

$$
S_v(f) = \frac{A^2}{4\sqrt{2\pi}\Delta f_{rms}}\left[e^{-(f-f_c)^2/2\Delta f_{rms}^2} + e^{-(f+f_c)^2/2\Delta f_{rms}^2}\right]
$$

Substituting the values of f_c and $\Delta f_(rms)$

$$
S_v(f) = \frac{1}{5\sqrt{2\pi} \times 10^3}\left[e^{-(f-10^6)^2/5\times 10^7} + e^{-(f+10^6)^2/5\times 10^7}\right]
$$