Chapter6 Solution

1. The largest alias terms are those $\omega_1 = \omega_s/2$

$$|F(\omega)|^2 = \frac{1}{w^2 + a^2}; \qquad \omega_{-3_{dB}} = a \qquad (1)$$

$$\frac{a^2}{\omega_1^2 + a^2} = \frac{1}{10}; \qquad \omega_1 = 3a = \qquad (2)$$

According to equation (1) and equation (2) we get $\frac{\omega_s}{\omega_{-3dB}} = 6$

2. (a) The minimum value of the sampling frequency

$$\omega = 2 * \{\text{maximum frequency component in f(t)}\} = 2 * 3\omega_0 = 6\omega_0$$

The minimum band-width of the low pass filter is 0 to $3\omega_0$

(b) PAM waveform obtained by sampling f(t)at $5\omega_0$

$$\begin{aligned}
f_{PAM} &= [f(t)p(t)]_{\omega=5\omega_0} \\
&= [4\cos\omega_0 t + 2\cos 2\omega_0 + 2\cos 3\omega_0 t][\frac{2}{T}(\frac{1}{2} + \cos 5\omega_0 t + \cos 10\omega_0 t + ...)] \\
&= \frac{1}{T}(4\cos\omega_0 t + 2\cos 2\omega_0 t + 2\cos 3\omega_0 t) \\
&+ \frac{2}{T}(4\cos\omega_0 t\cos 5\omega_0 t + 2\cos 2\omega_0 t\cos 5\omega_0 t + 2\cos 3\omega_0 t\cos 5\omega_0 t) \\
&+ \frac{2}{T}(4\cos\omega_0 t\cos 10\omega_0 t + 2\cos 2\omega_0 t\cos 10\omega_0 t + 2\cos 3\omega_0 t\cos 10\omega_0 t) + ...
\end{aligned}$$

and expressing product terms in the form

$$\cos x\cos y = 1/2[\cos(x+y) + \cos(x-y)]$$

We have

$$\begin{aligned}
f_{PAM} = \frac{f(t)}{T} + \frac{2}{T}&[2(\cos 6\omega_0 t + \cos 4\omega_0 t) + (\cos 7\omega_0 t + \cos 3\omega_0 t) \\
&+ (\cos 8\omega_0 t + \cos 2\omega_0 t) + 2(\cos 11\omega_0 t + \cos 9\omega_0 t) + (\cos 12\omega_0 t + \cos 8\omega_0 t) \\
&+ (\cos 13\omega_0 t + \cos 7\omega_0 t) + ...]
\end{aligned}$$

Now since the low pass filter will pass components of $\omega < 3\omega_0$, the output waveform is

$$\begin{aligned}
f_0(t) &= \frac{1}{T}[f(t) + 2(\cos 3\omega_0 t + \cos 2\omega_0 t)] \\
&= \frac{1}{T}[4\cos\omega_0 t + 4\cos 2\omega_0 t + 4\cos 3\omega_0 t]
\end{aligned}$$

3. (a)

$$\begin{aligned}
T &= (2f_m)^{-1}; \\
T_x &= T/2; \\
B_x &\geq (2T_x)^{-1} = 2f_m \\
\Rightarrow f_m &\leq B/2 = 5\text{kHz}
\end{aligned}$$

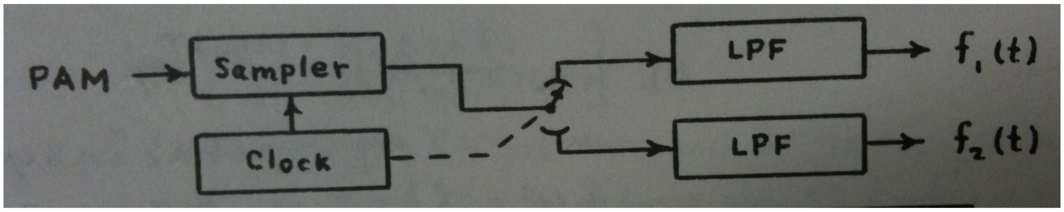

Figure 1:

(b)

4. (a)

$$(8\text{kHz})(25) = 200\text{kHz}$$

(b)

$$T_x = (200kHz)^{-1} = 5\mu\text{sec}$$

(c)

$$B_x \geq (2T_x)^{-1} = 100\text{kHz}$$
$$B_{DSB} = 2(100\text{kHz}) = 200\text{kHz}$$

5. Minimum sampling rate is larger of:

$$[2(3)\text{or}2(5/2)] = 6\text{kHz}$$

Minimum commutator clock rate is:

$$4(6\text{kHz}) = 24\text{kHz}$$

$$T_x \geq (2B_x)^{-1} = (40\text{kHz})^{-1}$$

Therefore max commutator clock rate is 40kHz

6. For the period $0 < t < \tau$ the output voltage in channel 1

$$v_{o1} = v(1 - e^{-t/RC})$$
$$= v\text{at } t = \tau, \text{ since } \tau \gg RC$$

For the guard time period $\tau < t < \tau + \tau_G$

$$v_{o1} = ve^{-(t-\tau)/RC}$$
$$= ve^{-\tau_G/RC} t = \tau + \tau_G$$

The output extending from v_{o1} overlapping into channel 2,

$$v_{o2} = (ve^{-\tau_G/RC})e^{-(t-\tau-\tau_G)/RC}$$

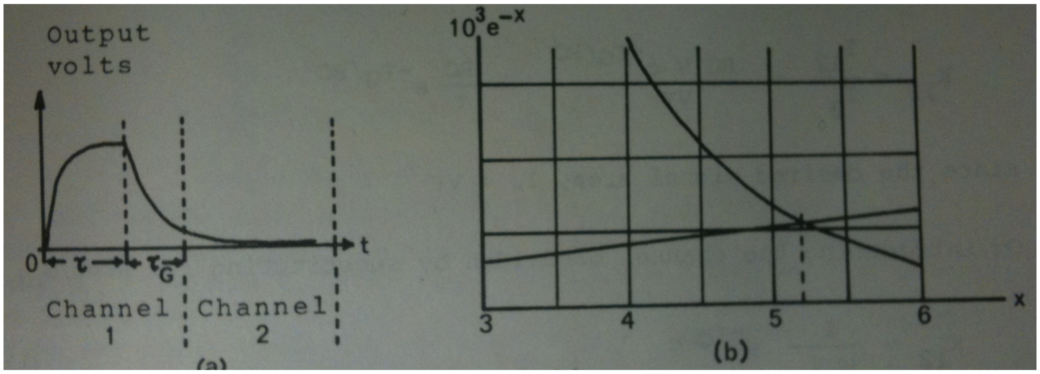

Figure 2:

and the time integral of this voltage

$$\begin{aligned} I_{12} &= \int_{\tau+\tau_G}^{2(\tau+\tau_G)} v_{o2} dt \\ &= (ve^{-\tau_G/RC}) * -RC \left[e^{-(t-\tau-\tau_G)/RC}\right]_{\tau+\tau_G)}^{2(\tau+\tau_G)} \\ &= RCve^{-\tau_G/RC}[1 - e^{-(\tau+\tau_G)/RC}] \\ &\simeq RCve^{-\tau_G/RC} \text{ when } \tau + \tau_G \gg RC \end{aligned}$$

The cross talk factor

$$K_{12} = \frac{I_{12}}{I_2} = \frac{RCve^{-\tau_G/RC}}{V\tau} = \frac{RC}{\tau} e^{-\tau_G/RC}$$

since the desired signal area $I_2 = V\tau$
on introducing the channel bandwidth by substituting for $RC = 1/\omega_c$,

$$k_{12} = \frac{1}{\omega_c \tau} e^{-\tau_G \omega_c}$$

For the 24 channel system sampled at 8kHz (i.e sampling period $125\mu s$)the time slot for each channel is

$$\tau + \tau_G = 125/24 = 5.21\mu s$$

and as $\tau = 4\mu s$ $\tau_G = 1.21\mu s$
If the cross talk is to $70dB(10^7)$ down, then k_{12} which is a voltage ratio is set at $\sqrt{10^{-7}} = 3.162 * 10^{-4}$
Equation (1) provides the link relating k_{12} to bandwidth.
Rewriting (1) in the form

$$\left[\frac{k_{12}\tau}{\tau_G}\right] x = e^{-x}$$

Where

$$\begin{aligned} k_{12}\tau/\tau_G &= 3.162 * 10^{-4} * 4/1.21 = 1.045 * 10^{-3} \\ x &= \tau_G \omega_c \end{aligned}$$

and solving (2) graphically we have $x = 5.2$ So

$$\omega_c = 5.2/1.21 * 10^{-6} = 4.30 \text{ Mrad/s}$$

and the channel bandwidth $f_c = 684\text{kHz}$