Chapter 7 Solution

Note: In this chapter ν stands for frequency in Hz, i.e., $\omega = 2\pi\nu$.

1. (a) Since the total power in $n_c(t)$ and $n_s(t)$ is equal to the total power in $n(t)$ and $n_c(t)$ and $n_s(t)$ are the two low pass components of $n(t)$, then we have:

$$S_{nc}(\nu) = S_{ns}(\nu) = S_n(\nu - \nu_c) + S_n(\nu + \nu_c) \qquad |\nu| \leq \nu_m$$

$$S_{nc}(\nu) = S_{ns}(\nu) = 10^{-9}\text{Watt/Hz} \qquad |\nu| \leq \nu_m$$

(b) When the noise $n(t)$ is demodulated by $cos\omega_c t$ the output is

$$n_o^{'}(t) = n(t)cos\omega_c t \leftrightarrow \frac{1}{2}[N(\nu - \nu_c) + N(\nu + \nu_c)]$$

After filtering

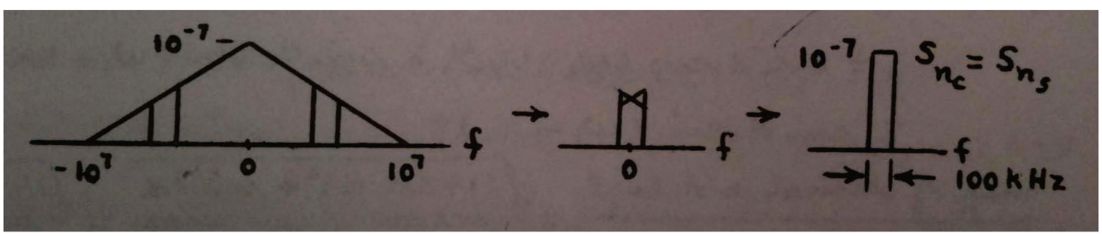
$$S_{no}(\nu) = \frac{1}{4} * 10^{-9}\text{Watt/Hz} \qquad |\nu| \leq 3\text{kHz}$$

But this is amplified by a voltage gain of 10. Therefore the total power is

$$2\int_0^3 \frac{10^{-9} * 10^2}{4} d\nu = 1.5 * 10^{-4}\text{Watts}$$

2.

$$\overline{n_c^2(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{nc}(\omega)d\omega = (10^{-7})(10^5) = 0.01v^2 = \overline{n_s^2(t)}$$



3. (a)

$$\text{DSB:} \qquad N_i = 2(6000)(10^{-9}) = 12\mu W \Rightarrow S_i/N_i = 10^{-3}/12 * 10^{-6} = 83.3$$
$$\text{SSB:} \qquad N_i = 2(3000)(10^{-9}) = 6\mu W \Rightarrow S_i/N_i = 10^{-3}/6 * 10^{-6} = 166.7$$

(b)

$$\text{DSB:} \qquad S_o/N_o = 2S_i/N_i = 166.7$$
$$\text{SSB:} \qquad S_o/N_o = S_i/N_i = 166.7$$

(c) DSB:

$$N_i = (2*10^3)\int_{10^6-3k}^{10^6+3k} d\nu/\nu = 2000 * ln\nu|_{10^6-3k}^{10^6+3k} = 12\mu W$$

Similarly N_i for $SSB = 6.000\mu$W. Note that the white noise assumption is very good one here as a result of the narrow bandwidth even though the noise power spectral density is not flat.

4. From the notes, the output signal power is :

$$S_o = E[f^2(t)] = \overline{m^2(t)} = \int_{-\nu_m}^{\nu_m} \frac{a|\nu|}{2\nu_m} d\nu$$
$$= 2\int_0^{\nu_m} \frac{a\nu}{2\nu_m} d\nu = \frac{2a}{\nu_m}\frac{\nu_m^2}{2} = \frac{a\nu_m}{2}$$

The output noise power is

$$N_o = \int_{-\nu_m}^{\nu_m} \frac{\eta}{2} d\nu = 2*\frac{\eta}{2}\int_0^{\nu_m} d\nu = \eta\nu_m$$

Therefore

$$\frac{S_o}{N_o} = \frac{a}{2\eta}$$

5. (a)

$$S = \frac{A^2}{2} + \frac{E[f^2(t)]}{2} = \frac{1}{2}(3^2) + (\frac{1}{2})(\frac{1}{2})(1^2) = 4.75\text{W}$$
$$N = E[n_i^2(t)] = 2*10^4*10^{-6} = 2*10^{-2}\text{W}$$

Therefore

$$\frac{S}{N} = 237.5$$

(b)

$$N_i = 2*100*10^{-6} = 2*10^{-4}$$
$$S_i = (\frac{1}{2})(\frac{1}{2})^2 = \frac{1}{8}\text{W}$$

Therefore

$$\frac{S_i}{N_i} = 625$$

(c) By synchronous detector the output is: $e = \frac{1}{2}[A + f(t) + n_c(t)]$

$$S_o = \overline{[(\frac{1}{2})(\frac{1}{2})\cos 1000\pi t]^2} = \frac{1}{32}$$
$$n_o(t) = \{[n_c(t)\cos\omega_c t - n_s(t)\sin\omega_c t]\cos\omega_c t\}_{LP}$$
$$N_o = \frac{1}{4}\overline{n_c^2(t)} = \frac{1}{4}\overline{n^2(t)} = 1/4(2*10^{-4})(50/100) = \frac{1}{4}*10^{-4}$$
$$\frac{S_o}{N_o} = 1250$$

6. (a)

$$s(t) = A(1+e^{j\theta}) = Ae^{j\theta/2}(e^{j\theta/2}+e^{-j\theta/2})$$
$$|s(t)| = 2A\cos(\theta/2)$$
$$|\cos(179^o/2)| = 8.73*10^{-3} = -20.6dB$$

minimum required dynamic range is then $20.6dB$

(b)

$$l = [(100+s)^2+4]^{1/2}$$
$$l^2 = (100+s)^2+4$$
$$2ldl = 2(100+s)ds$$
$$ds = \frac{ldl}{100+s} \approx \frac{[100^2+2^2]^1/2}{100}dl \approx dl = \frac{\lambda}{2} = 1.5\text{m}$$

7. (a) DSB-SC: input of $f(t)\cos\omega_c t$ results in $\frac{1}{2}f(t)$

$$P_{DSB} = \overline{[f(t)\cos\omega_c t]^2} = \frac{1}{2}\overline{f^2(t)}$$

SSB: input of $f(t)\cos\omega_c t + \hat{f(t)}\sin\omega_c t$ produces $\frac{1}{2}f(t)$

$$P_{SSB} = \overline{[f(t)\cos\omega_c t + \hat{f(t)}\sin\omega_c t]^2} = \frac{1}{2}\overline{f^2(t)} + \frac{1}{2}\overline{f^2(t)} = \overline{f^2(t)}$$

or $P_{SSB} = 2P_{DSB}$ for same received signal strength

(b) Transmitted sideband power is the same as the SSB-SC also requires P watts in this case.

8. Let the input signal to noise ratio in the FM system be $(\frac{S_i}{N_i})_{FM}$
From notes the signal to noise ratio at the output of the FM system is:

$$(\frac{S_o}{N_o})_{FM} = \frac{6k_f^2\overline{m^2(t)}\Delta\omega}{\omega_m^3}(\frac{S_i}{N_i})_{FM}$$
$$= \frac{6k_f^2\overline{m^2(t)}}{\omega_m^2}\frac{\Delta\omega}{\omega_m}(\frac{S_i}{N_i})_{FM}$$
$$= \frac{6\Delta\omega_{rms}^2\beta}{\omega_m^2}(\frac{S_i}{N_i})$$

However, for a SSB system, we have

$$(\frac{S_o}{N_o})_{SSB} = (\frac{S_i}{N_i})_{SSB}$$

If the output signal to noise ratio in the two system are the same. then

$$(\frac{S_i}{N_i})_{SSB} = \frac{6\Delta\omega_{rms}^2\beta}{\omega_m^2}(\frac{S_i}{N_i})_{FM} = 72*10^3(\frac{S_i}{N_i})_{FM}$$

Or

$$(\frac{S_i}{N_i})_{SSB}/(\frac{S_i}{N_i})_{FM} = 48.6\text{dB}$$

9. For FM process:

$$\left(\frac{S_o/N_o}{S_i/N_i}\right)_{FM} = \frac{6k_f^2 E[f^2(t)]\Delta\omega}{\omega_m^3}$$
$$= \frac{\frac{6m_f^2\omega_o^2}{A^2}\frac{A^2}{2}m_f\omega_o}{\omega_m^3}$$
$$= \frac{3m_f^2\omega_o^3}{\omega_m^3}$$

Where

$$k_f = \frac{m_f\omega_o}{A}$$
$$E[f^2(t)] = \frac{A^2}{2}$$
$$\Delta\omega = m_f\omega_o$$

For AM process:

$$\left(\frac{S_o/N_o}{S_i/N_i}\right)_{AM} = \frac{2}{3}$$

Therefore

$$\left[\frac{(S_o/N_o)_{FM}}{(S_i/N_i)_{AM}}\right] = \left(\frac{S_o/N_o}{S_i/N_i}\right)_{FM} \Bigg/ \left(\frac{S_o/N_o}{S_i/N_i}\right)_{AM} = \frac{9m_f^2\omega_o^3}{2\omega_m^3}$$

When $\omega_m = \omega_o$

$$\left[\frac{(S_o/N_o)_{FM}}{(S_i/N_i)_{AM}}\right] = \frac{9m_f^2}{2}$$

10. (a)

$$N_c = 2(0.25 * 10^{-14})(2 * 10^4) = 10^{-10}$$
$$S_c = (10^4/m^2)N_c = 10^{-6}/m^2$$
$$P_T = 10^10(10^{-6}/m^2) = 10^4/m^2$$
$$\text{for} \quad m = 0.707 \qquad P_T = 20kW$$
$$\text{for} \quad m = 1.00 \qquad P_T = 10kW$$

(b) For $\Delta\nu = 10$kH, $\beta = 1$

$$B \approx (2 * 10^4)(1+1) = 4 * 10^4$$
$$N_c = 2(0.25 * 10^{-14})(4 * 10^4) = 2 * 10^{-10}$$
$$S_c = \frac{N_c}{3\beta^2}(\frac{S_o}{N_o}) = \frac{2 * 10^{-10}}{3(1^2)}(10^4) = 6.67 * 10^{-7}$$
$$P_T = 10^{10}S_c = 6.67\text{kW}$$

For $\Delta\nu = 50$kH, $\beta = 5$

$$B \approx (2 * 10^4)(1+5) = 12 * 10^4$$
$$N_c = 2(0.25 * 10^{-14})(12 * 10^4) = 6 * 10^{-10}$$
$$P_T = \frac{(10^{10})(6 * 10^{-10})(10^4)}{3(5)^2} = 800\text{W}$$

11. Since $\nu_1 \ll \nu_m$, total power in the signal is given by

$$\int_{-\nu_m}^{\nu_m} S_m(\nu)d\nu = \int_{-\nu_m}^{\nu_m} \frac{\alpha_o}{[1+(\nu/\nu_1)^2]}d\nu \approx \int_{-\infty}^{\infty} \frac{\alpha_o}{[1+(\nu/\nu_1)^2]}d\nu = \alpha_o\nu_1\pi$$

Also the total power of the signal after pre-emphasis is given by

$$\int_{-\nu_m}^{\nu_m} |H_P(\nu)|^2 S_m(\nu)d\nu = \int_{-\nu_m}^{\nu_m} \frac{\alpha_o K^2\nu^2}{[1+(\nu/\nu_1)^2]}df \approx \alpha_o K^2\nu_1^2(2\nu_m - \pi\nu_1)$$

If the pre-emphasis is not to increase the bandwidth then

$$\int_{-\nu_m}^{\nu_m} |H_P(\nu)|^2 S_m(\nu)d\nu \le \int_{-\nu_m}^{\nu_m} S_m(\nu)d\nu$$

Therefore

$$\alpha_o K^2\nu_1^2(2\nu_m - \pi\nu_1) \le \alpha_o\nu_1\pi$$
$$K^2 \le \frac{\pi}{\nu_1(2\nu_m - \pi\nu_1)}$$

12. (a) Let $H_2(\omega)$ be the transfer function of the de-emphasis filter. Then we require:

$$S_n(\omega)|H_2(\omega)|^2 = a$$
$$|H_2(\omega)|^2 = \frac{a}{e^{10^{-4}|\omega|}} = ae^{-10^{-4}|\omega|}$$
$$\text{or} \qquad |H_2(\omega)| = K_2 e^{-10^{-4}|\omega|/2}$$

For no net signal distortion, the required pre-emphasis is :

$$|H_1(\omega)| = K_1 e^{10^{-4}|\omega|/2}$$

(b) Signal to noise ratio improvement = noise power without de-emphasis / noise power with de-emphasis

$$\frac{\int_0^{2\pi*10^4} exp(10^{-4})\omega d\omega}{\int_0^{2\pi*10^4} d\omega} = \frac{e^{2\pi}-1}{2\pi} = 85.1$$

13. (a)

$$3\beta^2 = 100 \Rightarrow \beta = 5.77$$
$$\Delta\nu = \beta\nu_m = (5.77)(13*10^3) = 75.1\text{kHz}$$

(b) Note: Γ is defined by $N_o/N_o^{'}$ in equation 7.43, $f_1 = 2.1$ is given in the book

$$3\beta^2\Gamma = 100 \Rightarrow \beta^2\Gamma = 33.3$$

$$\frac{\nu_m}{\nu_1} = \frac{13}{2.1} = 6.19$$
$$\Gamma = 16.54$$

Therefore

$$\beta^2 = \frac{33.3}{16.54} = 2.01$$
$$\Delta\nu = \beta\nu_m = \sqrt{2.01}(13*10^3) = 18.4\text{kHz}$$

14. (a) After the synchronous detector , the signal and the noise both are reduced by a factor of two (and thus the power spectra by a factor of four). In compose with the mono channel, we multiply by four. thus if the stereo carrier frequency is ω_s, the spectral densities of the mono channel noise and the stereo channel noise are:

$$S_n(\omega) = \omega^2$$
$$S_{n'}(\omega) = (\omega + \omega_s)^2 + (\omega - \omega_s)^2$$

$$\begin{aligned}\frac{\text{Stereo}}{\text{mono}} &= \frac{\int_0^{\omega_m} S_{n'}(\omega)dw}{\int_0^{\omega_m} S_n(\omega)dw} \\ &= 2 + 6(\frac{\omega_s}{\omega_m})^2 \\ &= 2 + 6(\frac{38}{15})^2 \\ &= 40.5\end{aligned}$$

(b)

$$|H(\omega)|^2 = \frac{1}{[1 + (\omega/\omega_1)^2]}$$

$$\begin{aligned}\frac{\text{stereo}}{\text{mono}} &= \frac{\int_0^{\omega_m} S_{n'}(\omega)|H(\omega)|^2dw}{\int_0^{\omega_m} S_n(\omega)|H(\omega)|^2dw} \\ &= 2 + \frac{2\omega_s^2 \tan^{-1}(\omega_m/\omega_1)}{\omega_1\omega_m - \omega_1^2 \tan^{-1}(\omega_m/\omega_1)} \\ &= 2 + \frac{2(38)^2 \tan^{-1}(15/2.1}{(2.1)(15) - 2.1^2 \tan^{-1}(15/2.1)} \\ &= 166.2\end{aligned}$$

15. (a)

$$\int_{-\infty}^{\infty} p(s)ds = \int_{-4}^{4} Ke^{-|s|}ds = 2K \int_0^4 e^{-s}ds = 2K(1 - e^{-4}) = 1$$
$$\Rightarrow K \approx 0.51$$

(b) The step size $\Delta = 8/4 = 2$ and the quantization levels $s_1 = -3$, $s_2 = -1$,$s_3 = 1$, $s_4 = 3$

(c) Total variance of the quantization error is equal to the sum of the variance of the quantization error at each step of quantization, i.e

$$\begin{aligned}E(e^2) &= \sum_{i=1}^{4} \int_{s_i - \frac{\Delta}{2}}^{s_i + \frac{\Delta}{2}} (s - s_i)^2 p(s)ds \\ &= 2K \int_0^2 (s-1)^2 e^{-s}ds + 2K \int_2^4 (s-3)^2 e^{-s}ds \\ &= 2K(1 - 5e^{-2}) + 2K(e^{-2} - 5e^{-4}) \simeq 0.374\end{aligned}$$

(d) If p(f)is constant in each level

$$E(e^2) = \frac{\Delta^2}{12}$$

16. (a) let the noise power due to distortion of filtering be N_D, thus

$$\begin{aligned} N_D &= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_m(\nu)d\nu - \frac{1}{2\pi}\int_{-\nu_m}^{\nu_m} S_m(\nu)d\nu \\ &= \frac{1}{2\pi}\int_{-\infty}^{-\nu_m} S_m(\nu)d\nu + \frac{1}{2\pi}\int_{\nu_m}^{\infty} S_m(\nu)d\nu \\ &= 2 * \frac{1}{2\pi}\int_{\nu_m}^{\infty} S_m(\nu)d\nu \\ &= \frac{1}{\pi}\int_{\nu_m}^{\infty} S_m(\nu)d\nu \end{aligned}$$

(b)

$$S_m(\nu) = A_o e^{-|\nu|/\nu_1}$$

$$N_D = 2\int_{\nu_m}^{\infty} A_o e^{-\nu/\nu_1} d\nu = \frac{1}{\pi} A_0 \nu_1 e^{-\nu_m/\nu_1}$$

(c) Total output signal to noise ratio is:

$$\frac{S_o}{N_D + N_q} = \frac{\sigma_m^2}{\frac{1}{\pi} A_o \nu_1 e^{-\nu_m/\nu_1} + s^2/12}$$

Where s is the step size of quantization