EE3TR4 **Introduction to Communication Systems** 14 March, 2014

Supplementary Mid-Term Test *Prof. K.M. Wong*

DURATION OF TEST: 2 hours
No book or notes allowed
Only Cassio FX991 calculators are permitted

1. a) Given that $\lim_{k\to\infty} \frac{k}{\pi}\frac{\sin kx}{kx} = \delta(x)$, evaluate the Fourier transform of the exponential $e^{j\omega_0 t}$. (5%)

   b) Fig. 1 shows a time function $\delta_T(t)$ consisting of a sequence of equidistant impulses separated by T and each of unit strength. Find the Fourier transform of $\delta_T(t)$. (5%)

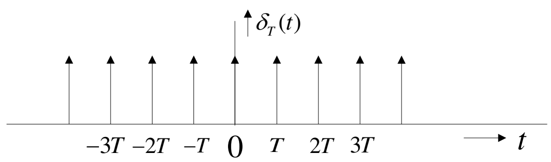

Figure 1: An impulse sequence

   c) If the above impulse sequence is used to sample a bandlimited signal $f(t)$ which has no spectral component above a frequency ω_c rad/sec, show that Fourier transform of the sampled signal $f_s(t)$ consists of replicas of the spectrum $F(\omega)$ of $f(t)$ at regular intervals of the sampling frequency $\omega_s = 2\pi/T$. Explain, with the aid of a diagram, how the minimum value of the sampling frequency is determined if the original signal $f(t)$ is to be recovered without distortion. (10%)

   d) The following signal

$$x(t) = 5\cos(600\pi t)[1 + \cos(3200\pi t)]$$

   is ideally sampled at 4000 samples per second. Plot the spectrum of the sampled signal from 0 Hz to 4000 Hz. Determine the cut-off frequency for the ideal low-pass filter used to reconstruct the signal. (10%)

2. a) An energy signal $x(t)$ is applied to a 1-Ω resistor. If $X(\omega)$ is the Fourier transform of $x(t)$, derive an expression for the normalized energy of the signal in terms of $X(\omega)$. (10%)

   b) A waveform $f(t)$ has a Fourier transform $F(\omega)$ whose magnitude is shown in Fig. 2.

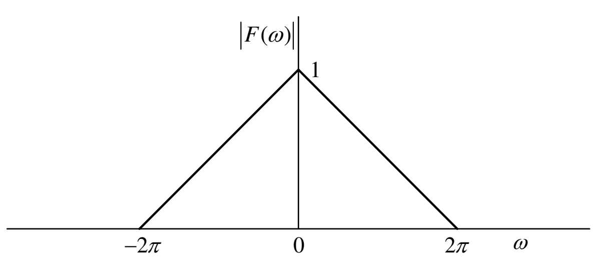

Figure 2:

   (i) Find the normalized energy content of the waveform. (10%)

   (ii) Calculate the frequency ω_1 such that one-half of the normalized energy is in the frequency range $-\omega_1$ to ω_1. (10%)

3. An amplitude modulated wave can be described by the following equation

$$s_{AM}(t) = [A + f_m(t)] \cos \omega_c t$$

where $f_m(t)$ is the baseband signal and $A \cos \omega_c t$ is the carrier.

a) If the baseband signal is given by

$$f_m(t) = mA \cos \omega_s t, \qquad |m| \leq 1$$

calculate the powers in the carrier and the sideband of the AM signal $s_{AM}(t)$. What is the maximum efficiency (in %) of transmission? (10%)

b) The AM signal $s_{AM}(t)$ can be detected by the full-wave rectifier detector as shown in Fig 3.

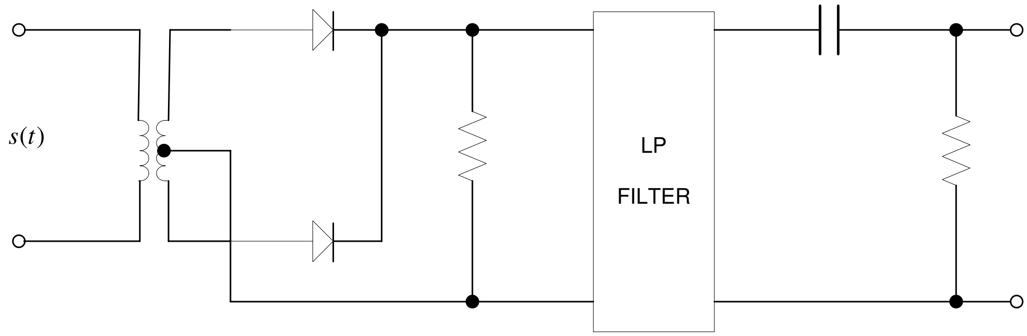

Figure 3:

Sketch the incoming AM signal, the rectified signal and the final output of the low-pass filter in this arrangement. (10%)

c) Describe mathematically the equivalent operation of a full-wave rectifier circuit on the AM signal $s_{AM}(t)$. (5%)

d) From c) above, derive the expressions of the frequency spectra of the above signals. (10%)

e) Sketch the frequency spectra of the above signals. (5%)