EE3TR4 Mid-Term Test 7 March, 2014

1. a) Method 1

$$\mathcal{F}[s_1(t).s_2(t)] = \int_{-\infty}^{\infty}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} S_1(\mu)e^{j\mu t}d\mu\right] s_2(t)e^{-j\omega t}dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_1(\mu)\left[\int_{-\infty}^{\infty} s_2(t)e^{-j(\omega-\mu)t}dt\right]d\mu$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_1(\mu)S_2(\omega-\mu)d\mu = \underline{\frac{1}{2\pi}S_1(\omega)*S_2(\omega)}$$

Method 2

$$\mathcal{F}^{-1}[S_1(\omega)*S_2(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} S_1(\mu)S_2(\omega-\mu)d\mu\right]e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} S_1(\mu)\int_{-\infty}^{\infty} S_2(\omega-\mu)e^{j\omega t}d\omega\, d\mu$$

$$= \int_{-\infty}^{\infty} S_1(\mu)e^{j\mu t}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} S_2(\omega-\mu)e^{j(\omega-\mu)t}d\omega\right]d\mu$$

$$= \int_{-\infty}^{\infty} S_1(\mu)e^{j\mu t}d\mu \, . \, s_2(t) = \underline{2\pi(s_1(t).s_2(t))}$$

$$\therefore \mathcal{F}[s_1(t).s_2(t)] = \frac{1}{2\pi}\left[S_1(\omega)*S_2(\omega)\right]$$

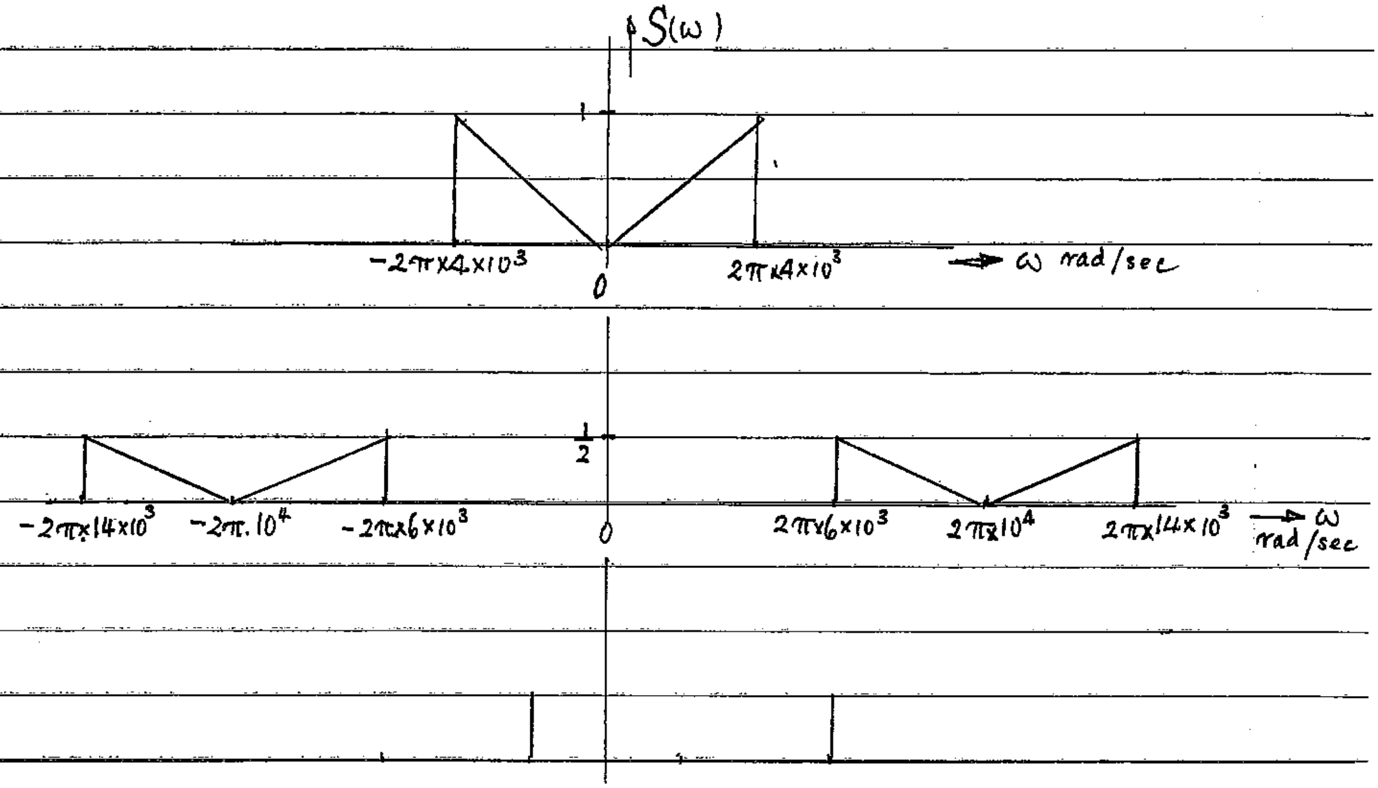
b) For $S_2(\omega) = \delta(\omega-\omega_c)$, then

$$S_1(\omega)*S_2(\omega) = \int_{-\infty}^{\infty} S_1(\mu)S_2(\omega-\mu)d\mu$$

$$= \int_{-\infty}^{\infty} S_1(\mu)\delta(\omega-\omega_c-\mu)d\mu$$

$$= S_1(\omega-\omega_c)\int_{-\infty}^{\infty}\delta(\omega-\omega_c-\mu)d\mu = \underline{S_1(\omega-\omega_c)}$$

since $\delta(\omega-\omega_c-\mu)$ has only value at $\mu=\omega-\omega_c$ and zero elsewhere while $\int_{-\infty}^{\infty}\delta(t)dt = 1$.


S(ω)
1
−2π×4×10³
0
2π×4×10³
ω rad/sec
1/2
−2π×14×10³
−2π.10⁴
−2π×6×10³
0
2π×6×10³
2π×10⁴
2π×14×10³
ω rad/sec

2 a). Nonperiodic signal $x(t)$

Normalized power:

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty}\int_{-T/2}^{T/2} \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{X(\omega)}{T} e^{j\omega t}\,d\omega \,.\, x(t)\,dt$$

$$= \lim_{T\to\infty}\frac{1}{2\pi}\int_{-T/2}^{T/2} \frac{X(\omega)}{T}\left[\int_{-\infty}^{\infty} x(t)\,e^{j\omega t}\,dt\right] d\omega \qquad \text{where } S_x(\omega) \triangleq \lim_{T\to\infty}\frac{|X(\omega)|^2}{T}$$

$$= \lim_{T\to\infty}\frac{1}{2\pi}\int_{-T/2}^{T/2} \frac{X(\omega)\cdot X(-\omega)}{T}\,d\omega = \frac{1}{2\pi}\lim_{T\to\infty}\int_{-T/2}^{T/2}\frac{|X(\omega)|^2}{T}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(\omega)\,d\omega$$

b) $X(\omega) \longrightarrow \boxed{H(\omega)} \longrightarrow Y(\omega)$

$$Y(\omega) = H(\omega)\,.\,X(\omega)$$

$$P_y = \frac{1}{2\pi}\ \cancel{\lim_{T\to\infty}}\int_{-\infty}^{\infty} S_y(\omega)\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\lim_{T\to\infty}\frac{|Y(\omega)|^2}{T}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}|H(\omega)|^2 \lim_{T\to\infty}\frac{|X(\omega)|^2}{T}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}|H(\omega)|^2 S_x(\omega)\,d\omega$$

c) $$R_x(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)\,x(t+\tau)\,dt$$

$$\mathcal{F}[R_x(\tau)] = \int_{-\infty}^{\infty}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)\,x(t+\tau)\,dt\; e^{-j\omega\tau}\,d\tau$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)\,e^{+j\omega t}\int_{-\infty}^{\infty} x(t+\tau)\,e^{-j\omega(t+\tau)}\,d\tau\,dt$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x(t)\,e^{j\omega t}\,dt\left[\int_{-\infty}^{\infty} x(\lambda)\,e^{-j\omega\lambda}\,d\lambda\right]$$

$$= \lim_{T\to\infty} X(-\omega)\,X(\omega)/T = S_x(\omega). \quad \therefore\ \mathcal{F}[R_x(\tau)] \leftrightarrow S_x(\omega)$$

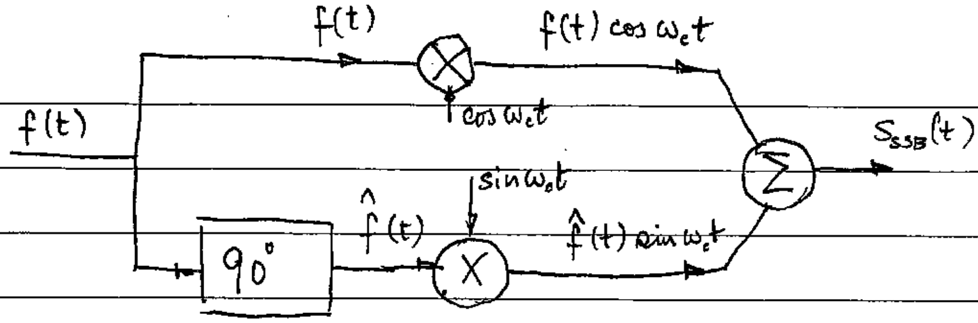
2 d) $$R_v(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} v(t)\, v(t+\tau)\, dt = \begin{cases} K/2 & \tau = 0 \\ 0 & \tau \neq 0 \end{cases}$$

$\therefore$

$$R_v(\tau) = \frac{K}{2}\,\delta(\tau)$$

$$S_v(\omega) = \mathcal{F}\left[\frac{K}{2}\,\delta(\tau)\right] = \frac{K}{2}$$

3 a)



~~b)~~ $s_{SSB}(t) = f(t)\cos\omega_c t \pm \hat{f}(t)\sin\omega_c t$

b) $f(t)\cos\omega_c t \Longleftrightarrow \frac{1}{2}\left[F(\omega+\omega_c) + F(\omega-\omega_c)\right]$

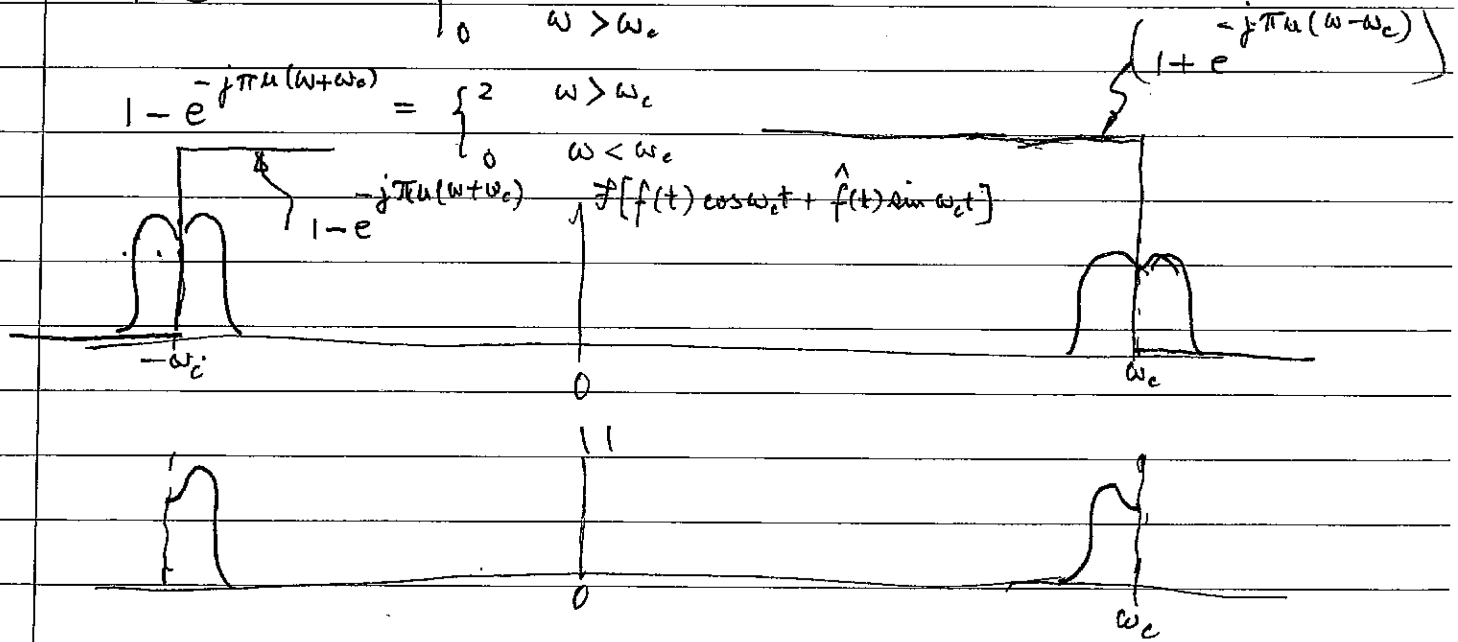
$\hat{f}(t)\sin\omega_c t \Longleftrightarrow -\frac{1}{2}\left[F(\omega+\omega_c)e^{-j\pi u(\omega+\omega_c)} - F(\omega-\omega_c)e^{-j\pi u(\omega-\omega_c)}\right]$

$\therefore \left[f(t)\cos\omega_c t + \hat{f}(t)\sin\omega_c t\right] \Longleftrightarrow \frac{1}{2}F(\omega-\omega_c)\left[1+e^{-j\pi u(\omega-\omega_c)}\right]$

$+\frac{1}{2}F(\omega+\omega_c)\left[1-e^{-j\pi u(\omega+\omega_c)}\right]$

$1+e^{-j\pi u(\omega-\omega_c)} = \begin{cases} 2 & \omega<\omega_c \\ 0 & \omega>\omega_c \end{cases}$

$1-e^{-j\pi u(\omega+\omega_c)} = \begin{cases} 2 & \omega>\omega_c \\ 0 & \omega<\omega_c \end{cases}$



3 c) $s_0(t) = s_{SSB}(t) \cdot \cos[(\omega_c+\Delta\omega)t+\phi]$

$$= \left[f(t)\cos\omega_c t + \hat{f}(t)\sin\omega_c t\right]\cos\left[(\omega_c+\Delta\omega)t+\phi\right]$$
$$= \tfrac{1}{2} f(t)\cos(\Delta\omega t+\phi) - \tfrac{1}{2}\hat{f}(t)\sin(\Delta\omega t+\phi)$$

(i) For $\Delta\omega = 0$

$$s_0(t) = \tfrac{1}{2}\left[f(t)\cos\phi - \hat{f}(t)\sin\phi\right]$$

$$S_0(\omega) = \tfrac{1}{2}\left[F(\omega)\cos\phi - \hat{F}(\omega)\sin\phi\right]$$

But $\hat{F}(\omega) = j\,F(\omega)e^{-j\pi u(\omega)}$

$$S_0(\omega) = \tfrac{1}{2}F(\omega)\left[\cos\phi - je^{-j\pi u(\omega)}\sin\phi\right] = \begin{cases} \tfrac{1}{2}F(\omega)e^{-j\phi} & \omega<0 \ \cancel{\omega>0} \\ \tfrac{1}{2}F(\omega)e^{j\phi} & \omega>0 \end{cases}$$

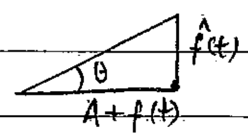
(ii) For $\phi = 0$, $\Delta\omega \neq 0$

$$s_0(t) = \tfrac{1}{2} f\cos\Delta\omega t - \tfrac{1}{2}\hat{f}(t)\sin\Delta\omega t$$

~~Modul~~ Base band modulating a slowly vary sinusoid, plus interference of $\hat{f}(t)\sin\Delta\omega t$

d) $s(t) = \underline{A\cos\omega_c t + s_{SSB}(t)} = [A+f(t)]\cos\omega_c t + \hat{f}(t)\sin\omega_c t$

$$= \sqrt{\{A+f(t)\}^2 + \hat{f}^2(t)}\ \cos(\omega_c t+\theta)$$



The envelope of $s(t)$ is

$$\left\{[A+f(t)]^2 + \hat{f}^2(t)\right\}^{1/2} = A\left[1 + \tfrac{1}{2}\left\{\frac{2f(t)}{A} + \frac{f^2(t)+\hat{f}^2(t)}{A}\right\} - \dots\right]$$

$$\simeq A\left\{1 + \cancel{\tfrac{1}{2}}\frac{f(t)}{A}\right\} \simeq A + f(t)$$

$\therefore$ Envelope detector output $\cong A + f(t)$

After DC block, output $\simeq f(t)$