EE3TR4 **Introduction to Communication Systems** 7 March, 2014

Mid-Term Test *Prof. K.M. Wong*

DURATION OF TEST: 2 hours
No book or notes allowed
Only Cassio FX991 calculators are permitted

1. a) The convolution between two signals $S_1(\omega)$ and $S_2(\omega)$ in the frequency domain is defined as

$$S_1(\omega) * S_2(\omega) = \int_{-\infty}^{\infty} S_1(\mu) S_2(\omega - \mu) d\mu \tag{1}$$

Show that,

$$\mathcal{F}[s_1(t) \cdot s_2(t)] = \frac{1}{2\pi} \{S_1(\omega) * S_2(\omega)\}$$

i.e., the Fourier transform of the product of two time signals $s_1(t)$ and $s_2(t)$ is the convolution of the spectra of the two signals. (10%)

b) In particular, if $S_2(\omega)$ in Eq. (1) is $\delta(\omega - \omega_c)$, show that the result of the convolution is a frequency-shifted version of $S_1(\omega)$. (5%)

c) A signal $s(t)$ the spectrum of which is shown in Fig. 1 is multiplied by a carrier signal $\cos \omega_c t$ yielding a signal $y(t) = s(t) \cos \omega_c t$. Given that $\mathcal{F}[\cos \omega_c t] = \pi[\delta(\omega + \omega_c) + \delta(\omega - \omega_c)]$,

(i) sketch the spectrum $Y(\omega)$ if $\omega_c = 2\pi \times 10^4$ rad/sec. (5%)

(ii) sketch the spectrum $Y(\omega)$ if $\omega_c = 2\pi \times 2 \times 10^3$ rad/sec. (5%)

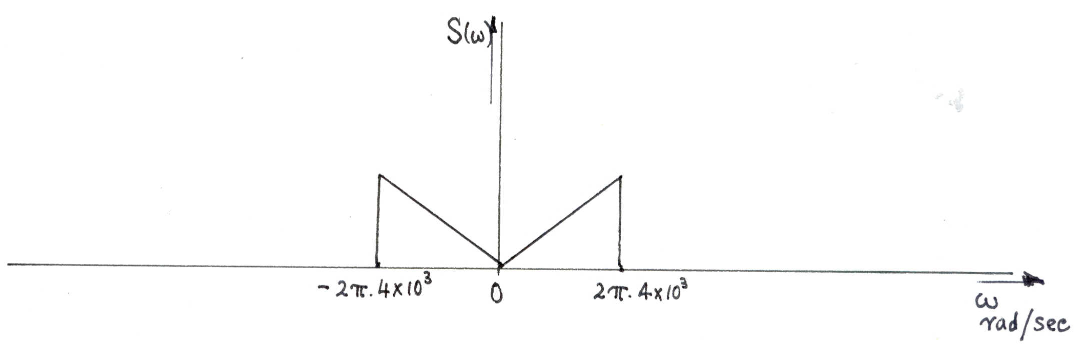

Figure 1: The spectrum of signal $S(\omega)$

2. a) Give an expression for the average power of a non-periodic power signal $x(t)$ over an observation period T_0. If $X(\omega)$ is the Fourier transform of $x(t)$, derive an expression for the normalized power of the signal in terms of $X(\omega)$ and T_0. Hence write down an expression for the *spectral power density* of the signal $\mathcal{S}_x(\omega)$. (10%)

b) The above signal $x(t)$ is passed through a circuit having a transfer function $H(\omega)$ yielding an output $y(t)$. Derive a relationship between the input and output spectral power densities. (5%)

c) The autocorrelation function $R_x(\tau)$ of a *power* signal $x(t)$ is defined as

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)x(t+\tau)dt$$

Show that there is a direct relationship between $R_x(\tau)$ and the spectral power density $\mathcal{S}_x(\omega)$ of $x(t)$. (5%)

d) White Gaussian noise $\nu(t)$ has an auto-correlation function given by

$$R_\nu(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \nu(t)\nu(t+\tau)dt = \begin{cases} K/2 & \tau = 0 \\ 0 & \tau \neq 0 \end{cases}$$

Express this characteristic of $R_\nu(\tau)$ in terms of a mathematical function and hence find the power spectral density $\mathcal{S}_\nu(\omega)$ of white noise. (5%)

3. A 90° phase shift network has a transfer function such that

$$H(\omega) = |H(\omega)|e^{j\theta(\omega)}$$

with the magnitude and phase characteristics given by

$$\begin{aligned} |H(\omega)| &= 1 \\ e^{j\theta(\omega)} &= e^{j\left\{\frac{\pi}{2}-\pi u(\omega)\right\}} = je^{-j\pi u(\omega)} \end{aligned}$$

If the input signal to the 90° phase shift network is $f(t)$, we denote its output by $\widehat{f}(t)$ such that

$$\widehat{f}(t) \leftrightarrow \widehat{F}(\omega) = F(\omega)H(\omega) = jF(\omega)e^{-j\pi u(\omega)}$$

a) If the baseband signal is $f(t)$, design a single sideband modulator having a carrier frequency of ω_c using the above 90° phase shift network. Write down the expression of the single sideband signal $s_{SSB}(t)$ in terms of $f(t)$ and $\widehat{f}(t)$. (15%)

b) Show by taking the Fourier transforms of the signals at the various stages of your modulator, the output signal is indeed a single sideband signal. Comment on how the result is the upper sideband or the lower sideband. (15%)

c) The above SSB signal is to be demodulated at the receiver by multiplying the signal $s_{SSB}(t)$ by a defective carrier $A\cos[(\omega_c + \triangle\omega)t + \phi]$ and low-pass filter. Derive the expression of the signal at the output of the demodulator. Comment on the phase characteristics of the demodulated signal if $\triangle\omega = 0$ and $\phi \neq 0$. (10%)

d) If the above single sideband signal is transmitted together with a *large* carrier such that the resulting signal is $s(t) = A\cos\omega_c t + s_{SSB}(t)$, show that the envelope $s_e(t)$ of the transmitted signal $s(t)$ can be approximated by

$$s_e(t) \approx A\left[1 + \frac{f(t)}{A}\right] = A + f(t)$$

and hence can be demodulated using an envelope detector. (10%)