EE2CI5 Lab 5: Frequency Response

1   Objective

The objective of this laboratory is to investigate the response of a frequency-selective circuit to sinusoidal inputs of different frequencies.

2   Equipment

Function generator
Oscilloscope
Hook-up wire
Resistors: 1kΩ, 4.7kΩ
Capacitors: 1.0µF

3   Pre-Lab Exercises

3.1   Circuit Analysis (6 marks)
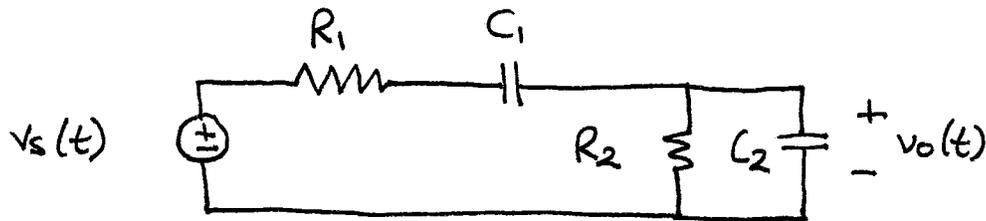
Consider the circuit model in Figure 1.

Figure 1: Circuit model for Lab 5. Here $v_s(t) = A\cos(\omega t + \theta)$.

Let $\mathbf{V}_s$ denote the phasor for the source voltage, $v_s(t)$, and let $\mathbf{V}_o$ denote the phasor for the output voltage, $v_o(t)$. Let $\mathbf{Z}_1$ denote the impedance of the series combination of R_1 and C_1, and let $\mathbf{Z}_2$ denote the impedance of the parallel combination of R_2 and C_2.

3.1.1 **(2 marks)** Show that

$$\mathbf{V}_o = \frac{j\omega R_2 C_1}{1 - \omega^2 R_1 C_1 R_2 C_2 + j\omega(R_1 C_1 + R_2 C_2 + C_1 R_2)} \mathbf{V}_s \tag{1}$$

3.1.2 **(0.5 marks)** Use this expression to find the "frequency response" of the model, $H(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s}$. The magnitude of the frequency response is the ratio of the ratio of the amplitude of the output sinusoid to that of the input sinusoid. The phase of the frequency response is the difference between the phase of the output sinusoid and that of the input sinusoid.

3.1.3 **(1 mark)** What is the magnitude of the frequency response, $|H(\omega)|$?

3.1.4 **(0.5 marks)** Use the expression for $|H(\omega)|$ to determine how the circuit model behaves at low frequencies?

3.1.5 **(0.5 marks)** Use the expression for $|H(\omega)|$ to determine how the circuit model behaves at high frequencies?

3.1.6 **(1 mark)** Use your knowledge of the behaviour of a capacitor at low and high frequencies to provide an intuitive explanation of your answers to 3.1.4 and 3.1.5.

3.1.7 **(0.5 marks)** At what frequency does the magnitude reach its maximum value?

3.2 Numerical Evaluation (1 mark)

Consider the case of the circuit model in Figure 1 in which

- $R_1 = 1\text{k}\Omega$, $R_2 = 4.7\text{k}\Omega$

- $C_1 = C_2 = 1\mu\text{F}$

Verify that the plots in Fig. 2 represent $|H(\omega)|$.

4 Physical Experiment (3 marks)

- For the values of the resistors and capacitors in Section 3.2, construct a physical circuit that matches the model in Figure 1.

- Based on the plots of the magnitude of the frequency response obtained in Section 3.2, select some appropriate frequencies at which you will evaluate the frequency response of the physical circuit.

- For those frequencies, use the oscilloscope to measure the amplitude of the sinusoidal input and the amplitude of the sinusoidal output.

- Use these measurements to calculate the magnitude of the frequency response of the physical circuit at those frequencies.

- Compare these measured values to the values that are obtained by analyzing the model of the circuit in Figure 1. (*Hint:* Make use of the expression in (1).) Postulate plausible reasons for any differences.
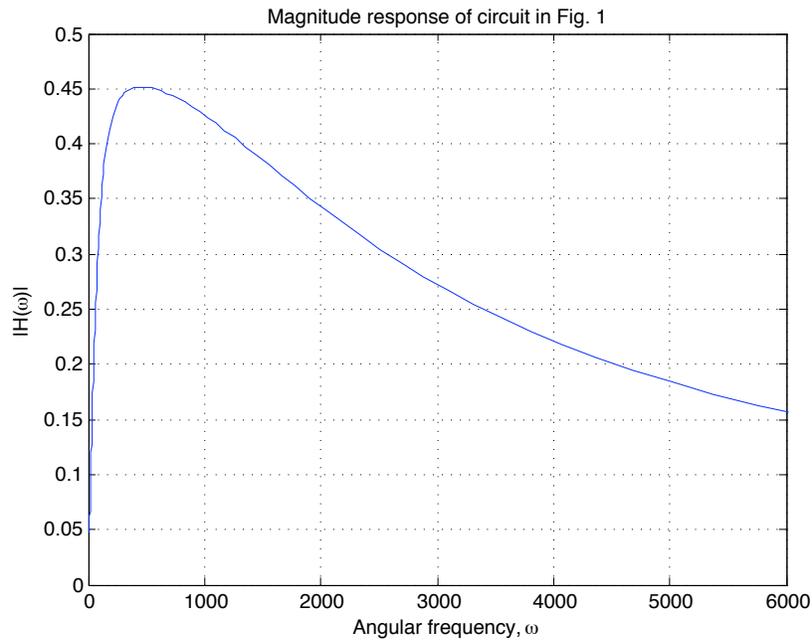
(a) Linear scale
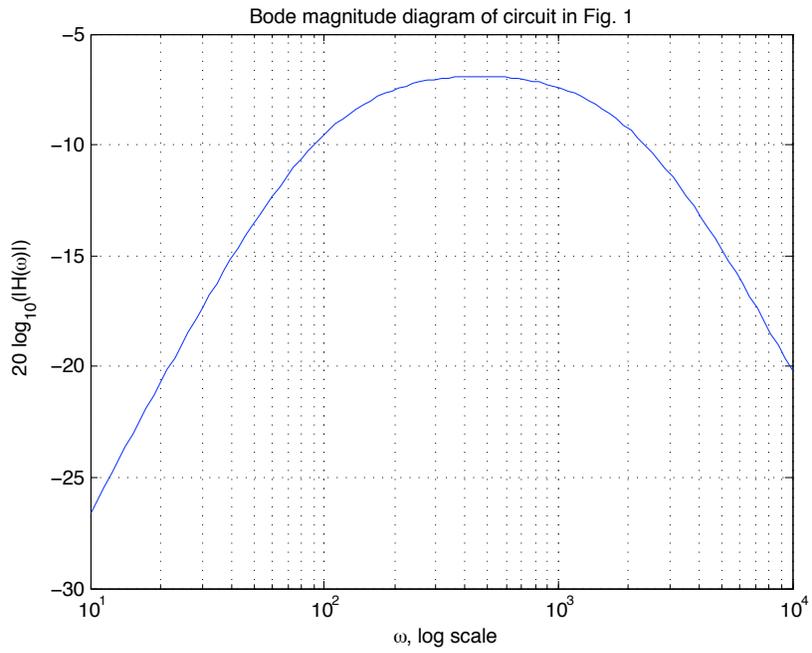
(b) Decibel scale

Figure 2: Magnitude response, $|H(\omega)|$, of the circuit in Fig. 1.