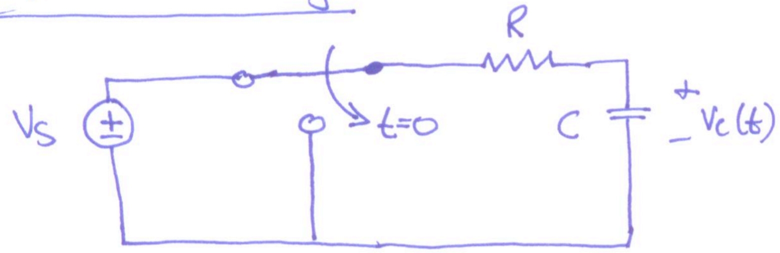
Capacitor Discharge



Assume that the switch has been connected for a long time before $t=0$,

for example, assume that the source was connected before $t = -5RC$

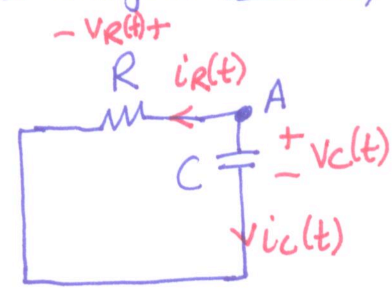
In that case we can assume the circuit is in the steady state.

That means that the capacitor acts as an open circuit to the DC Source

Hence $\quad V_C(t)\big|_{t=0^-} = V_S$

Since the capacitor voltage is continuous, that implies $V_C(t)\big|_{t=0^+} = V_S$

For $t \geq 0$



KVL: $\quad -v_R(t) + V_C(t) = 0$

Capacitor: $\quad i_C(t) = C\,\dfrac{dV_C(t)}{dt}$

Resistor: $\quad i_R(t) = \dfrac{V_R(t)}{R}$

KCL at Node A: $\quad i_R(t) + i_C(t) = 0$

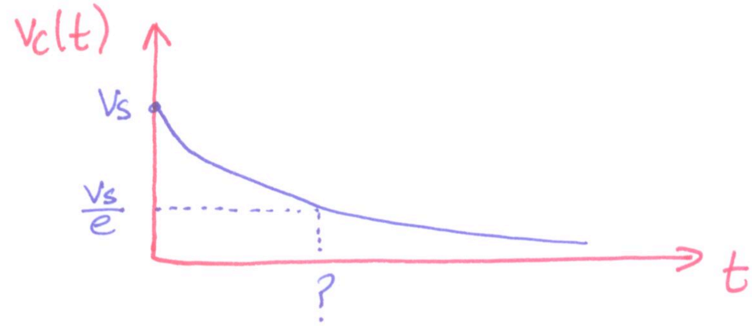
By substitution: $\quad \dfrac{dV_C(t)}{dt} + \dfrac{1}{RC} V_C(t) = 0$

Solving differential equation: $\quad V_C(t) = K e^{-t/RC} \quad , t \geq 0$

Using initial condition $\quad V_C(0^+) = V_S$

$$\Rightarrow V_C(t) = V_S\, e^{-t/RC} \quad , t \geq 0$$

Sketch of $V_c(t)$



How long does it take for the voltage to drop to $\frac{V_s}{e}$?

One time constant, $\tau = RC$

How long does it take for the voltage to drop below 1% of V_s?

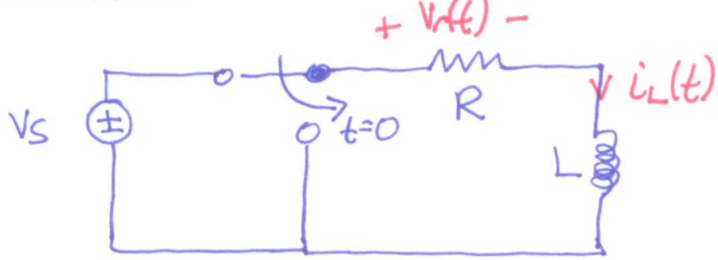
Find t such that $V_s e^{-t/RC} \leq V_s/100$

$$\Rightarrow \quad \frac{t}{RC} \geq \ln(100)$$

$$\Rightarrow \quad t \gtrsim 4.6\,RC = 4.6\,\tau$$

How can this analysis be modified to model a (low-frequency) square wave input?

INDUCTOR



Assume that the source was connected a long time before $t=0$

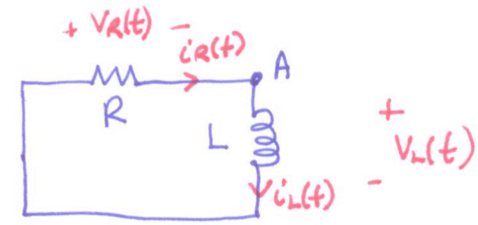
say before $t = -5\,L/R$

Then the circuit can be assumed to be in the steady state
That means the inductor acts as a short circuit for the DC input

Hence $\quad i_L(t)\Big|_{t=0^-} = \dfrac{V_S}{R}$

Since the inductor [redacted] current is continuous, $\quad i_L(t)\Big|_{t=0^+} = \dfrac{V_S}{R}$

For $t \geq 0$



KVL: $\quad V_R(t) + V_L(t) = 0$

Inductor: $\quad V_L(t) = L\,\dfrac{di_L(t)}{dt}$

Resistor: $\quad V_R(t) = R\, i_R(t)$

KCL at Node A: $\quad i_R(t) = i_L(t)$

Substitute: $\quad \dfrac{di_L(t)}{dt} + \dfrac{R}{L}\, i_L(t) = 0$

Generic solution: $\quad i_L(t) = K e^{-t/(L/R)}, \quad t \geq 0.$

Initial Condition: $\quad i_L(0^+) = \dfrac{V_S}{R} \Rightarrow i_L(t) = \dfrac{V_S}{R} e^{-t/(L/R)}, \quad t \geq 0$

$\Rightarrow$ time constant: $\quad \tau = L/R$