Node Voltage Analysis with Voltage Sources.

When the only sources are independent current sources, node voltage analysis boils down to

* picking a reference node
* writing the KCL equation at all other nodes
  If there are N nodes $\Rightarrow$ $N-1$ equations in $N-1$ unknowns
* Solve the linear system

The presence of voltage sources

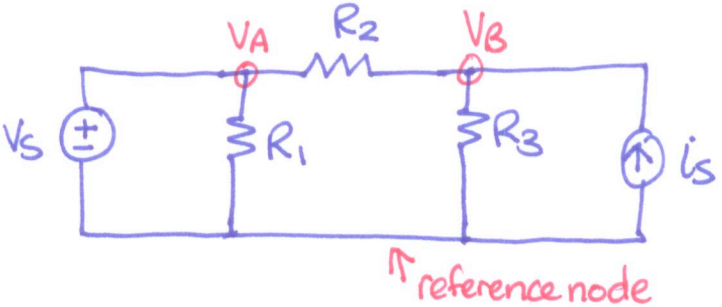
- sometimes makes things easier
- sometimes makes things a little more complicated

This depends on how the sources are connected.

VOLTAGE SOURCES CONNECTED TO THE REFERENCE NODE

- Make things easier
- Immediately know the voltage at the other node to which the source is connected
- We do not need the KCL equation at that node
- Solve the set of KCL equations for the other nodes

EXAMPLE



Using the voltage source, $V_A = V_S$

KCL at node B $\quad \frac{V_A - V_B}{R_2} + i_S = \frac{V_B}{R_3}$

Two equations, two unknowns

Solution:

$$V_B = \frac{R_2 R_3 i_S + R_3 V_S}{R_2 + R_3}$$

VOLTAGE SOURCE CONNECTED BETWEEN NON-REFERENCE NODES

- Make things a little more complicated
- Writing KCLs at non-reference nodes yields an undetermined linear system
- Reason: there is no Ohm's Law for the branch with the voltage source
- Therefore we need to add an equation that equates the voltage difference between the nodes to the value of the source. e.g.,

$V_A - V_B = V_S$

- Can become cumbersome
- Can we streamline the procedure?

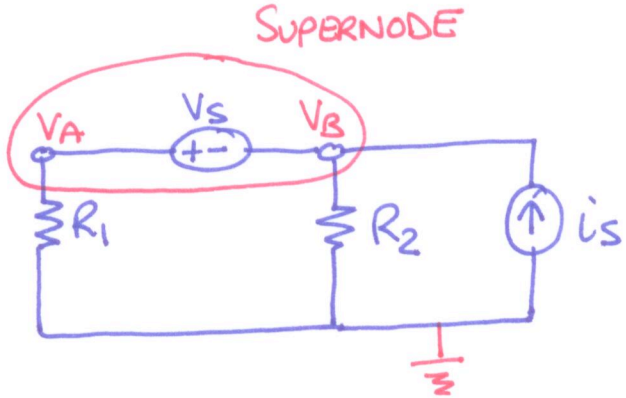
SUPERNODE

Ⓐ Place the voltage source of interest inside a "supernode"

Ⓑ Write the KCL equation for the supernode

Ⓒ Use the source to relate voltages of nodes.

EXAMPLE



KCL @ supernode $\quad i_S = \frac{V_A}{R_1} + \frac{V_B}{R_2}$

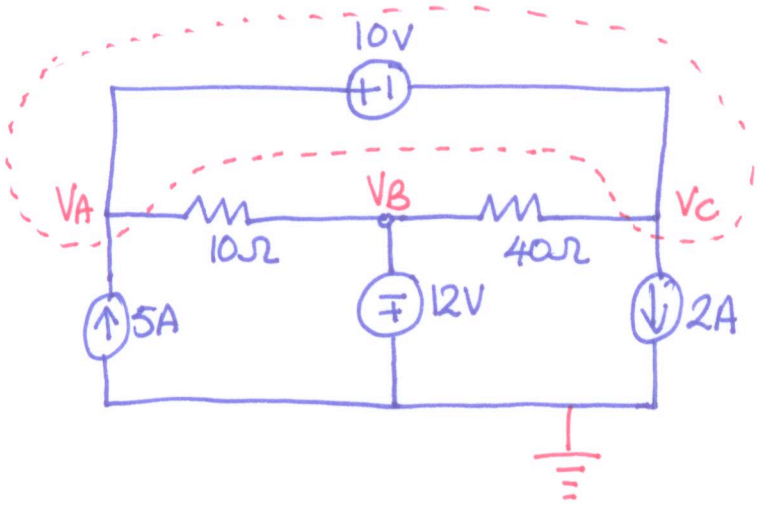
Use source $\quad V_A - V_B = V_S$

Solve :

$$V_B = \frac{R_1 R_2 i_S - R_2 V_S}{R_1 + R_2}$$

What is V_A ?

An example with Two Voltage Sources.



Steps

1. Choose reference node
2. Observe that $V_B = -12V$
3. Place 10V source in a supernode
4. KCL @ supernode :

$$5 = \frac{V_A - V_B}{10} + \frac{V_C - V_B}{40} + 2$$

5. Relate node voltages in supernode.

$$V_A - V_C = 10$$

6. 3 equations, 3 unknowns
7. Solve:

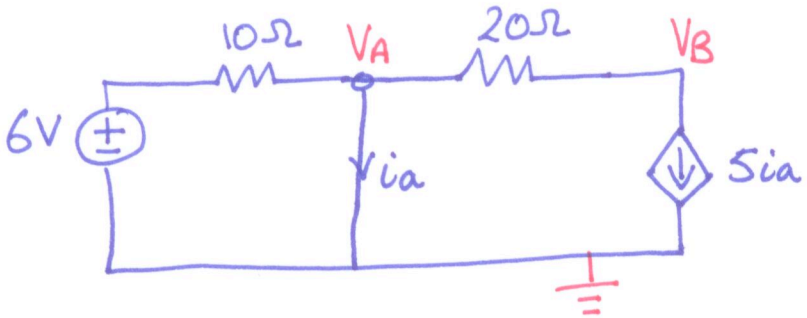
$$V_B = -12V$$
$$V_C = 4V$$
$$V_A = 14V$$

Node Analysis with Dependent Sources.

Step 1 – Express controlling voltage or current in terms of node voltages

Step 2 – Proceed as before

Example



KCL at node A yields : $\frac{6 - V_A}{10} = i_a + \frac{V_A - V_B}{20}$

Short to reference node : $V_A = 0$

KCL at node B : $\frac{V_A - V_B}{20} = 5\, i_a$

3 equations, 3 unknowns

$V_b = -10V$