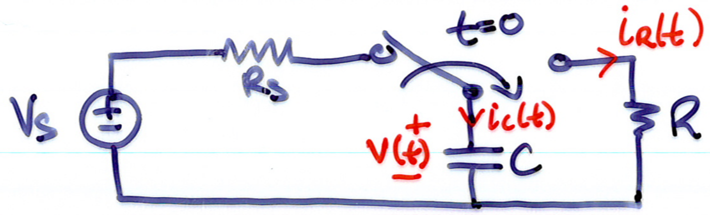
Review of Flash Example



$v(0^-) = V_s$
$\Rightarrow v(0^+) = V_s$

For $t \geq 0$

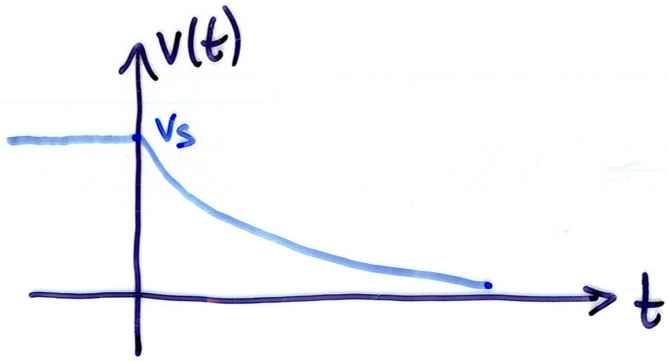
$$\frac{dv(t)}{dt} + \frac{1}{RC} v(t) = 0$$

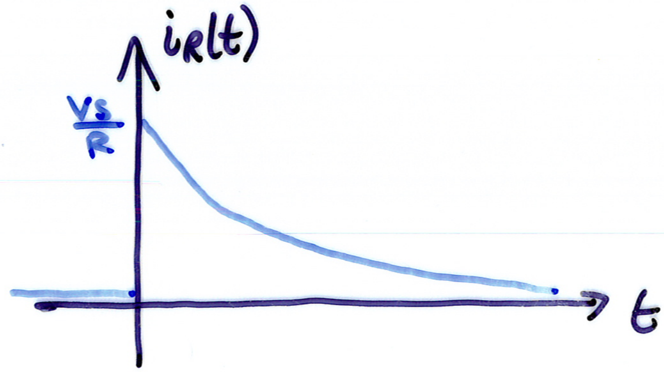
$$\Rightarrow v(t) = V_s \, e^{-t/RC}$$

what about current?

$$i_R(t) = -i_c(t) = -C \frac{dv(t)}{dt}$$

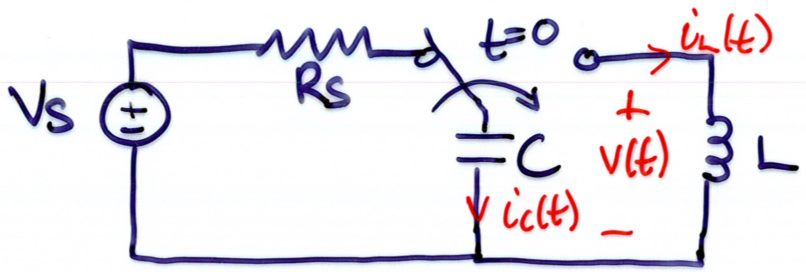
$$= \frac{V_s}{R} e^{-t/RC}$$





WHAT HAPPENS IF WE CHANGE THE CIRCUIT SLIGHTLY?

Now discharge through an inductor



As in previous case,

$$v(0^-) = V_s \Rightarrow v(0^+) = V_s.$$

What about the differences?

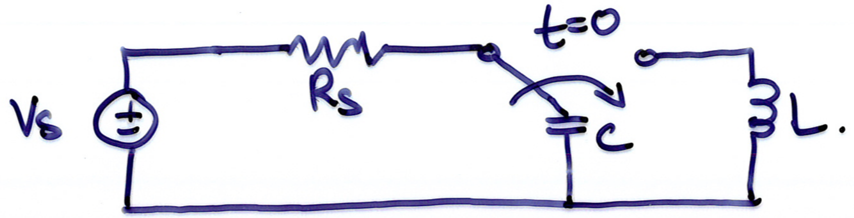
- $i_L(t)$ must be continuous

$$i_L(0^-) = 0 \Rightarrow i_L(0^+) = 0$$

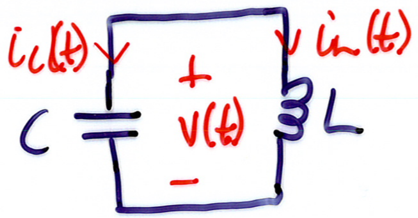
- there a only energy storage elements for $t > 0$

No dissipative elements.

LET'S ANALYZE THE CIRCUIT



~~After~~ For $t \geq 0$ we have.



KCL: $i_C(t) + i_L(t) = 0$

$$\Rightarrow \quad C\frac{dV(t)}{dt} + i_L(0) + \frac{1}{L}\int_0^t v(x)\,dx = 0$$

Differentiate + divide by C

$$\frac{d^2v(t)}{dt^2} + \frac{1}{LC}v(t) = 0$$

Alternatively

$$\frac{d^2v(t)}{dt^2} = -\frac{1}{LC}v(t)$$

Do you know any functions with this property?

How about $f(t) = K\cos(\omega t + \theta)$?

$$\frac{df(t)}{dt} = -K\omega \sin(\omega t + \theta)$$

$$\frac{d^2 f(t)}{dt} = -K\omega^2 \cos(\omega t + \theta)$$

Recall that $\sin(\omega t + \phi) = \cos(\omega t + \phi - \pi/2)$

This suggests that the solution of the equation takes the form.

$$v(t) = K\cos(\omega t + \theta)$$

and indeed we will make this rigorous soon.

First, let's try to determine ω, K and θ.

<u>Aside</u>: Since $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$$K\cos(\omega t + \theta) = [K\cos(\theta)]\cos\omega t - [K\sin(\theta)]\sin\omega t$$

• FINDING THE FREQUENCY

$$\frac{d^2v(t)}{dt^2} = -\frac{1}{LC}\, v(t)$$

Let's put in our postulated solution $v(t) = K\cos(\omega t + \theta)$

$$\Rightarrow \quad -K\omega^2 \cos(\omega t + \theta) = -\frac{1}{LC}\, K\cos(\omega t + \theta)$$

This must hold for all values of t

$$\Rightarrow \quad \omega^2 = \frac{1}{LC}$$

Let's take the positive root: $\omega = \frac{1}{\sqrt{LC}}$

• FINDING K and Θ

- Capacitor voltage is continuous $\Rightarrow v(0^+) = V_s$

$$\Rightarrow \quad K\cos(\theta) = V_s$$

- Inductor current is continuous $\Rightarrow i_L(0^+) = 0$

by KCL $\quad i_L(t) = -C\frac{dV(t)}{dt}$

$$\Rightarrow \quad -CK\omega \sin(\theta) = 0$$

Final Solution

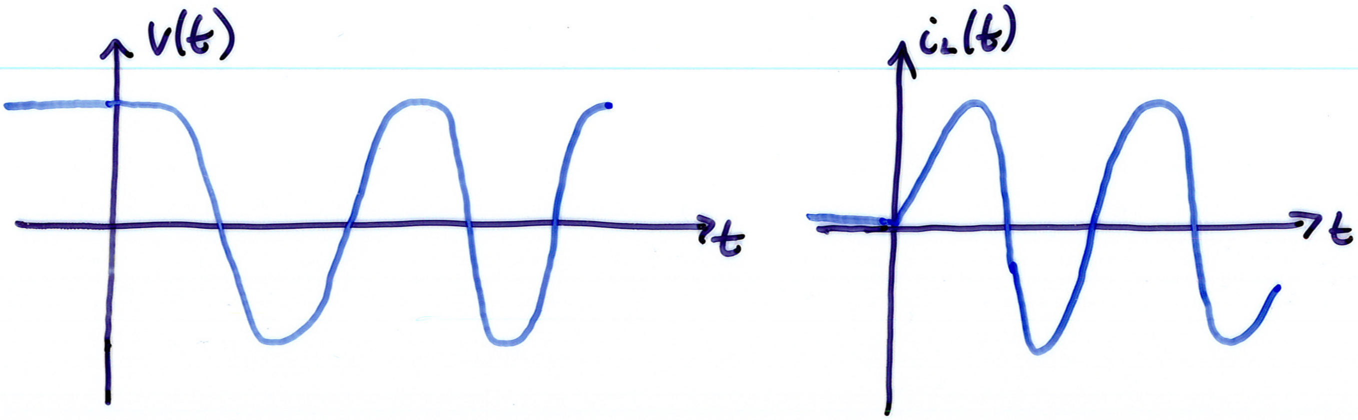
$\theta = 0$

$K = V_s$

$$\Rightarrow \quad v(t) = V_s \cos\left(\frac{1}{\sqrt{LC}}\, t\right)$$

$$\Rightarrow \quad i_L(t) = V_0 \sqrt{\frac{C}{L}}\, \sin\left(\frac{1}{\sqrt{LC}}\, t\right)$$

Sketch. [redacted]



Do these solutions make sense?

What might this circuit be used for?

Notes: frequency only dependent on LC

Amplitude dependent on Vs