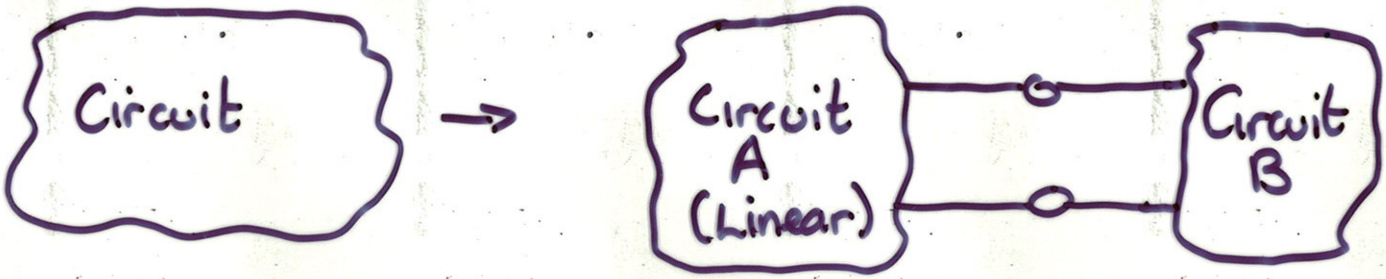
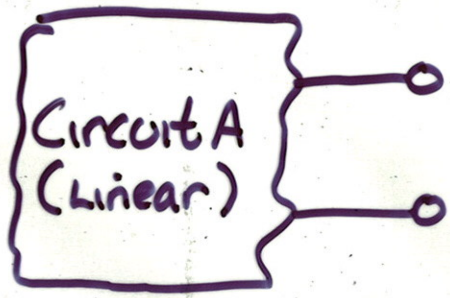
THEVENIN & NORTON EQUIVALENTS

- a structured "divide and conquer" approach
- Partition the circuit



- We would like to build a simplified model for circuit A.
- Very useful if you need to solve several circuits in which A is the same, but B changes
- Example: A models source & distribution network

  B models the load.

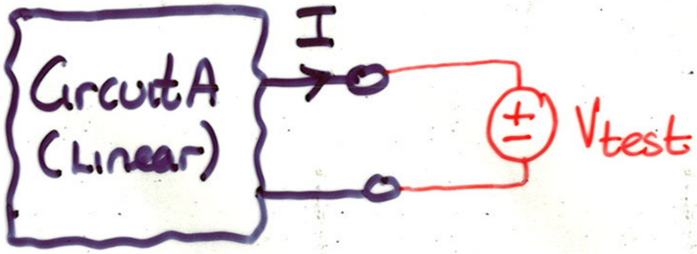


We want a model for Circuit A that is

- simple
- indistinguishable from Circuit A from the perspective of the terminals

How to do this?

- Apply a test voltage
- Apply superposition
- Interpret result in circuit form

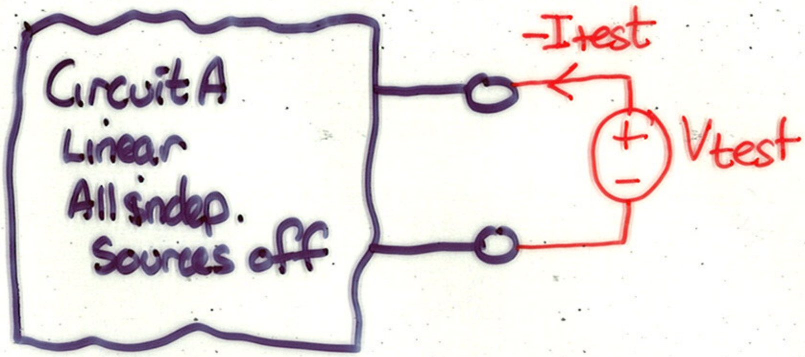


By superposition: $I = I_{test} + I_{sc}$

I_{test} : current due to V_{test} alone; i.e., all independent sources in Circuit A turned off

I_{sc} : current due to sources in Circuit A alone; i.e., V_{test} turned off;
i.e., V_{test} replaced by short circuit

What is $\dfrac{V_{test}}{-I_{test}}$?



This is the equivalent resistance of the circuit Called R_{TH}

$$\Rightarrow I = -\frac{V_{test}}{R_{TH}} + I_{sc}$$

Now consider the case in which $V_{test} = R_{TH} I_{SC}$

$\Rightarrow I = 0$

$\Rightarrow$ terminals act like an open circuit

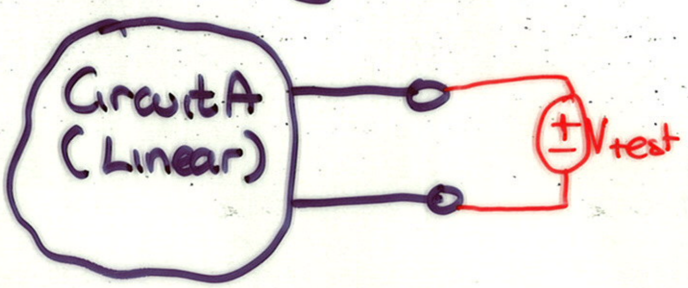
Let's call $R_{TH} I_{SC} = V_{OC}$

$$\Rightarrow \quad I = -\frac{V_{test}}{R_{TH}} + \frac{V_{OC}}{R_{TH}}$$

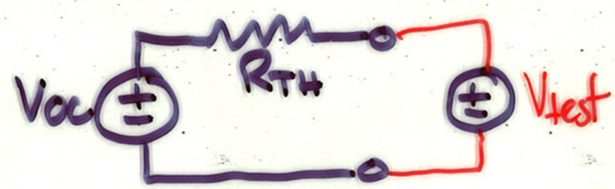
Alternatively, $\quad -V_{OC} + R_{TH} I + V_{test} = 0$

What is the circuit realization of that equation?

Previously



Now.



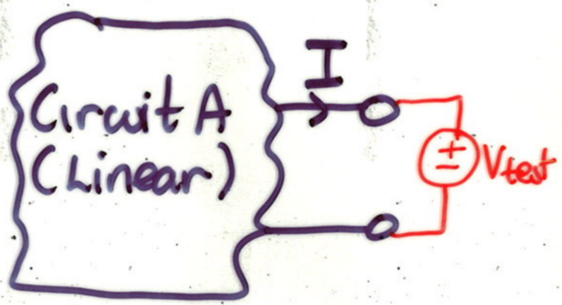
Hence, the Thevenin equivalent of Circuit A is

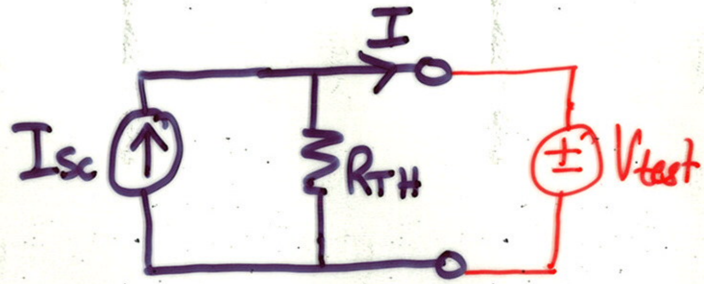
Circuit B cannot tell the difference between Circuit A and its Thevenin equivalent

In the previous analysis, we also had the equation

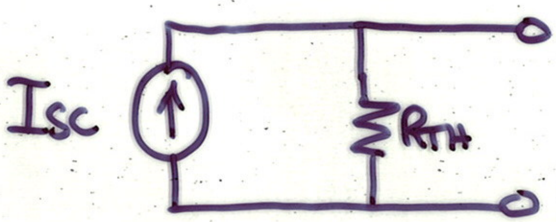
$$I = I_{sc} - V_{test}/R_{TH}$$

What is the circuit realization?





Hence Norton equivalent of Circuit A.



Circuit B cannot tell the difference between Circuit A, the Thevenin equiv and the Norton equiv.

~~From~~ From the perspective of Circuit B, Thevenin & Norton circuits are equivalent

Are they equivalent in a more general setting?

Circuit Analysis with Thev. & Nort. Equivs

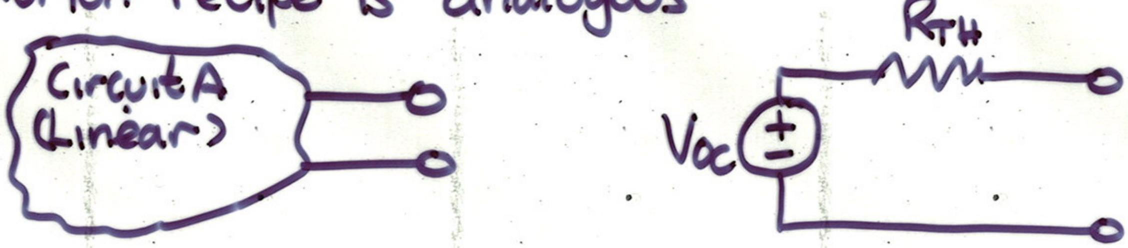
1. Partition the circuit into circuits A and B
   - For all dependent sources, make sure that the controlling current or voltage is in the same partition

2. Construct the Thevenin or Norton equivalent of Circuit A.

3. Analyze Circuit B

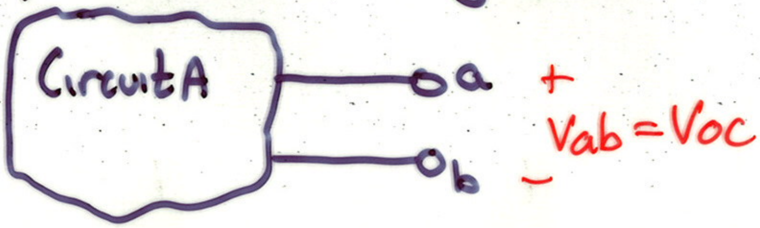
General Recipe for Thevenin

Norton recipe is analogous



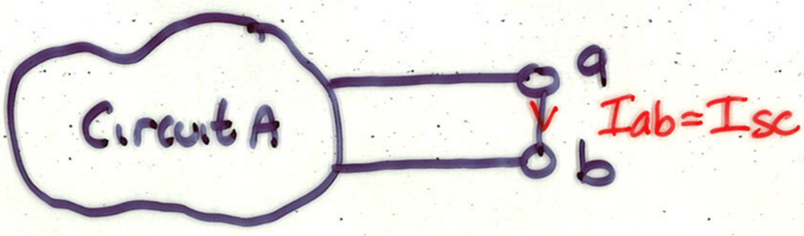
① Find V_{oc}

That is, solve the following circuit for V_{ab}.



② Find I_{sc}

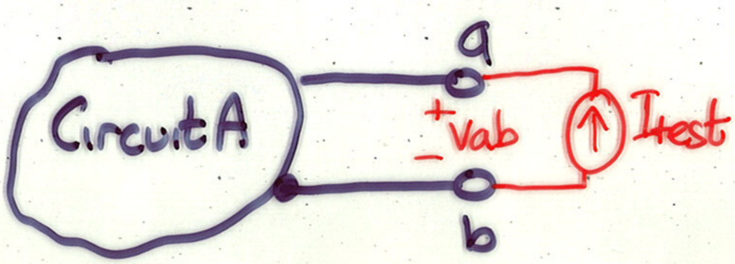
That is, solve the following circuit for I_{ab}



③ Find R_{TH}

If $I_{sc} \neq 0$, $R_{TH} = V_{oc}/I_{sc}$

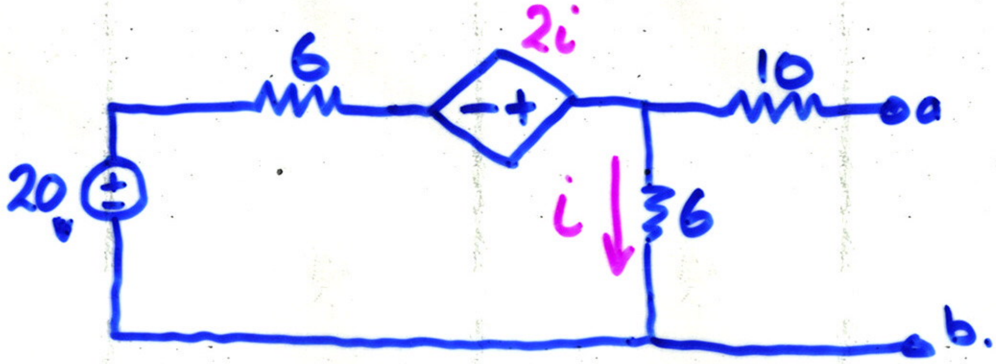
If $I_{sc} = 0$, solve



$R_{TH} = V_{ab}/I_{test}$

Example:

Find the Thevenin equivalent of.



Need to find Voc and isc

For Voc : KVL around left mesh. (no current in right mesh)

$$-20 + 6i - 2i + 6i = 0.$$

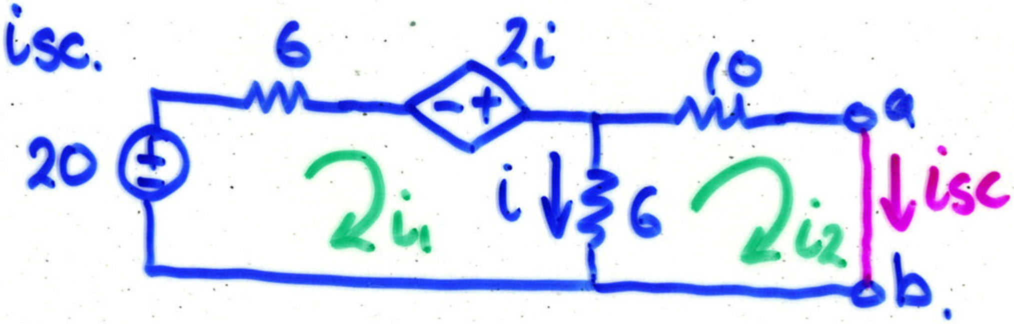
$$\Rightarrow i = 2A$$

KVL around right mesh. (no current in right mesh)

$$-6i + 10 \times 0 + V_{ab} = 0.$$

$$\Rightarrow V_{ab} = V_{OC} = 12V$$

Find isc.



KVL on both meshes

$$-20 + 6i_1 - 2i + 6(i_1 - i_2) = 0$$

$$6(i_2 - i_1) + 10i_2 = 0$$

2 equations, 3 unknowns !!!

$$i = i_1 - i_2$$

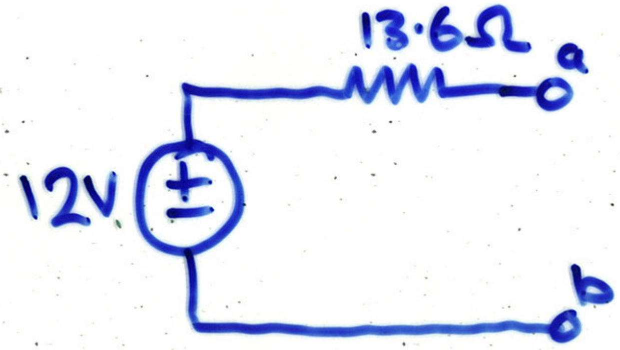
Now can solve

$$10i_1 - 4i_2 = 20$$
$$-6i_1 + 16i_2 = 0.$$

$$\Rightarrow \quad i_2 = i_{sc} = 120/136 \text{ A}.$$

$$\Rightarrow \quad R_t = \frac{V_{oc}}{i_{sc}} = \frac{12}{120/136} = 13.6\,\Omega.$$

⇒ Thevenin equivalent



SPECIAL RECIPE FOR THEVENIN.

- WHEN ONLY INDEPENDENT SOURCES

1. FIND V_{oc}

2. Turn "off" all sources

   R_{TH} is the equivalent resistance looking into the terminals

   (Find this by series and parellel combinations)

- WHEN ONLY DEPENDENT SOURCES

1. $V_{oc} = 0$. why?

2. FIND R_{TH} by the "test-based" method