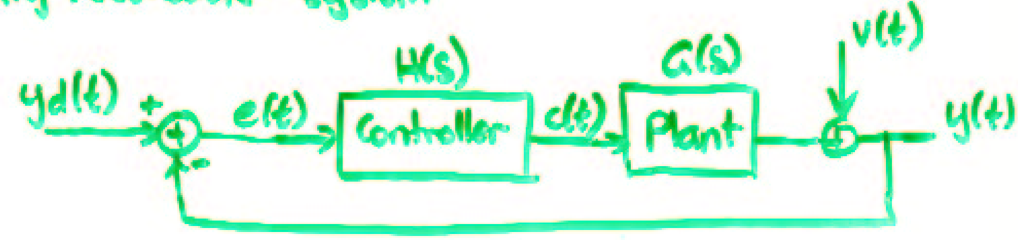
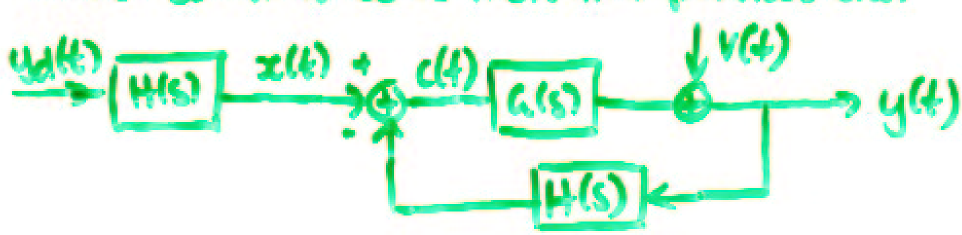
Closed-Loop Control.

A standard model used in control design is the "unity feedback system"



- In this system the idea is to design the controller or compensator H(s) so that y(t) closely follows the command signal (desired signal) $y_d(t)$, even in the presence of disturbances v(t) and uncertainty in the model of the plant.
- The system assumes an ideal sensor, and that there is no significant communication delay between the blocks, but many practical systems can be closely approximated by this model.
- For analysis purposes, we will re-arrange this model so that it looks more like previous one.



Now clear that

$$T(s) = \frac{Y(s)}{Y_d(s)} = \frac{Y(s)}{X(s)} \cdot \frac{X(s)}{Y_d(s)}.$$

Yesterday we found that $\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)H(s)}$

and $\frac{X(s)}{Y_d(s)} = H(s)$

$$\Rightarrow \frac{Y(s)}{Y_d(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}$$

What was our goal?

Make $y(t)$ follow $y_d(t)$

~~If $G(s)H(s)$ is large (compared to 1) for the values of s which are important (these are dictated by yd(t), see below), then:~~

$$\cancel{\frac{Y(s)}{Y_d(s)} \approx 1}$$

$$Y(s) = \frac{G(s)H(s)}{1+G(s)H(s)} Y_d(s) + \frac{1}{1+G(s)H(s)} V(s).$$

If $|G(s)H(s)| \gg 1$ for all values of s for which $|Y_d(s)|$ or $|V(s)|$ is significant, then.

$$\frac{G(s)H(s)}{1+G(s)H(s)} Y_d(s) \approx Y_d(s)$$

and. $$\left| \frac{1}{1+G(s)H(s)} V(s) \right| \ll |V(s)|$$

Hence. $y(t) \approx y_d(t)$

- In practice $v(t)$ is often unknown. It is usually modelled as a stochastic process with known pdf and temporal correlation properties
- Large loop gain also reduces the sensitivity of $T(s)$ to changes in the gain of $G(s)$.
- However, as we will show later, large gain can lead to stability problems.