TIME DOMAIN INPUT-OUTPUT DESCRIPTIONS OF CONTINUOUS-TIME SYSTEMS: THE CONVOLUTION INTEGRA

The intuition and development is the same as in the discrete-time case

Each signal can be written as a weighted sum (in fact an integral) of time-shifted impulses

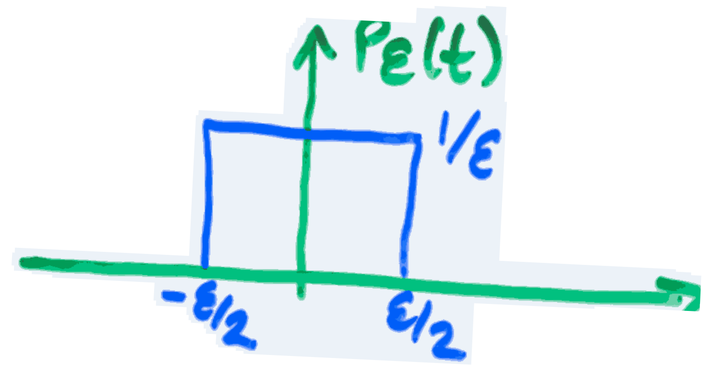
The response to an impulse is called the impulse response

Since the system is linear, the response to a weighted sum of inputs is a weighted sum of the outputs obtained if each input acted alone

4 Since the system is time invariant, the response to a time shifted impulse is a time shifted impulse response

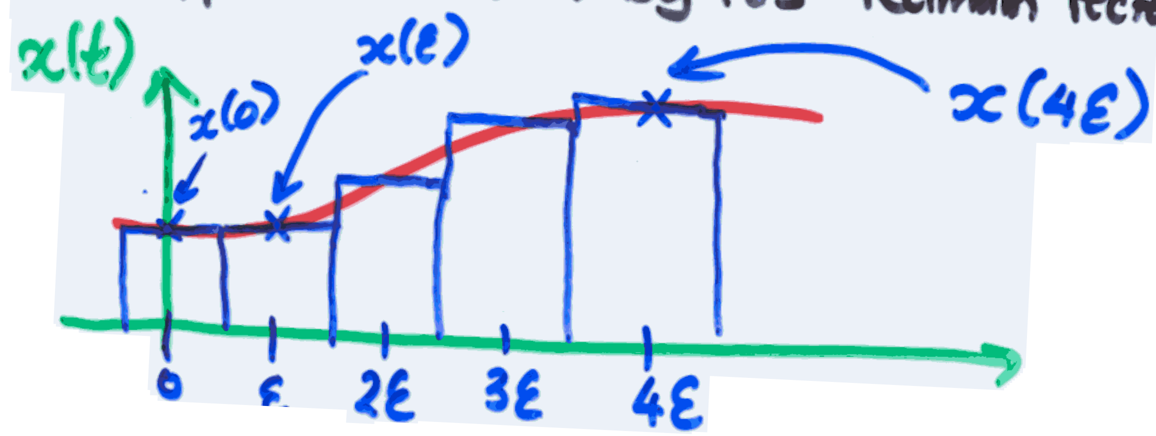
Write $x(t)$ as a weighted sum of impulses

- First recall $p_\varepsilon(t)$



- Note that $p_\varepsilon(t)\cdot\varepsilon$ has a height of 1.

- Now approximate $x(t)$ by its Reiman rectangles.



$x(t)$ $\hat{x}$ $\sum_k$ $k\varepsilon)$ p_ε $t-k$

Now as $\varepsilon \to 0$, $(t) \to$

and $\hat{x}$ $\to x(t)$

$\Rightarrow$ $\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)$ ~

Sanity check

$$x(t_0) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t_0-\tau)\,d\tau.$$

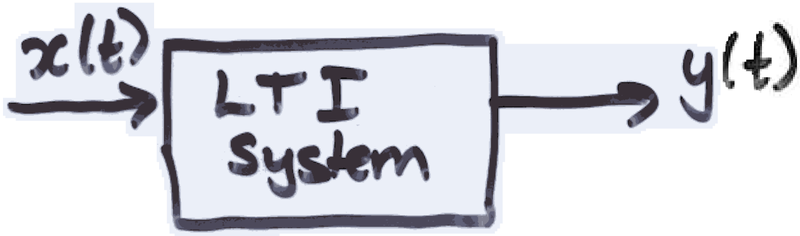
$\delta(t_0-\tau)$ is zero for all τ except $\tau = t_0$.

using the "sifting property" of the impulse

$$\int_{-\infty}^{\infty} x(\tau)\,\delta(t_0-\tau)\,d\tau = x(t_0).$$

Sanity check passed.

$$y[n] = \sum_{k} x[k]\,h[n-k].$$



$$y(t) = \mathcal{H}\{x(t)\}$$

$\mathcal{H}$ is the operator which describes the system.

$$y(t) = \mathcal{H}\left\{\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau\right\}$$

Since $\mathcal{H}$ is linear

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,\mathcal{H}\{\delta(t-\tau)\}\,d\tau$$

Let $h(t)$ denote the impulse response,

i.e., $h(t) = \mathcal{H}\{\delta(t)\}$

Because the system is time invariant,

$$\mathcal{H}\{\delta(t-\tau)\} = h(t-\tau)$$

Hence

$$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau}$$

convolution